SNS COLLEGE OF TECHNOLOGY

Coimbatore-15
An Autonomous Institution

Accredited by NBA – AICTE and Accredited by NAAC – UGC with ‘A+’ Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECT212 – LINEAR CONTROL SYSTEMS

II YEAR/ IV SEMESTER

UNIT I – CONTROL SYSTEM MODELING

TOPIC 6- TRANSLATIONAL AND ROTATIONAL MECHANICAL SYSTEMS

OUTLINE

- REVIEW ABOUT PREVIOUS CLASS
- DIFFERENTIAL EQUATION MODELLING OF MECHANICAL SYSTEMS
- TRANSLATIONAL MECHANICAL SYSTEMS
    - MASS
    - SPRING
    - DASHPOT OR DAMPER.
- ACTIVITY
- MODELING OF ROTATIONAL MECHANICAL SYSTEMS
    - MOMENT OF INERTIA,
    - TORSIONAL SPRING
    - DASHPOT.
- SUMMARY

DIFFERENTIAL EQUATION MODELLING OF MECHANICAL SYSTEMS

There are two types of mechanical systems based on the type of motion

- Translational mechanical systems
- Rotational mechanical systems

MODELLING OF TRANSLATIONAL MECHANICAL SYSTEMS

Translational mechanical systems move along a **straight line**.
three basic elements.

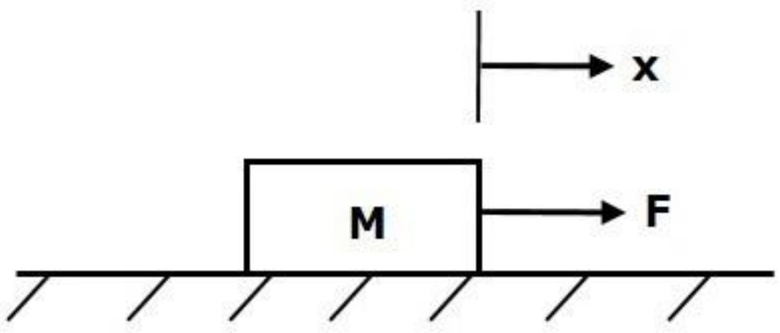
1. mass
2. spring
3. dashpot or damper.

If a force is applied to a translational mechanical system, then it is opposed by opposing forces due to mass, elasticity and friction of the system.

Since the applied force and the opposing forces are in opposite directions, the algebraic sum of the forces acting on the system is zero. Let us now see the force opposed by these three elements individually.

MASS

Mass is the property of a body, which stores **kinetic energy**. If a force is applied on a body having mass **M**, then it is opposed by an opposing force due to mass. This opposing force is proportional to the acceleration of the body. Assume elasticity and friction are negligible.



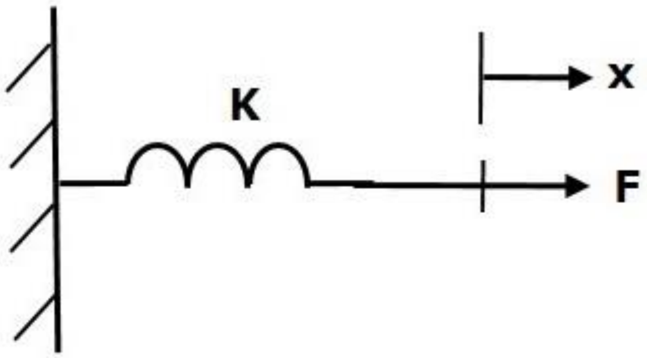
$$F_m \propto a$$

$$F_m = M_a = M\frac{d^2x}{dt^2}$$

$$F = F_m = M\frac{d^2x}{dt^2}$$

Where,
F is the applied force
F_m is the opposing force due to mass
M is mass
a is acceleration
x is displacement

SPRING



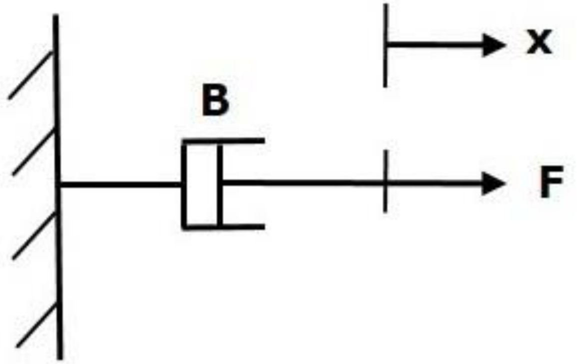
$$F \propto x$$

$$F_k = Kx$$

$$F = F_k = Kx$$

Where,
F is the applied force
F_k is the opposing force due to elasticity of spring
K is spring constant
x is displacement

DASHPOT



$$F_b \propto v$$

$$F_b = Bv = B\frac{dx}{dt}$$

$$F = F_b = B\frac{dx}{dt}$$

Where,
F_b is the opposing force due to friction of dashpot
B is the frictional coefficient
v is velocity
x is displacement

MODELING OF ROTATIONAL MECHANICAL SYSTEMS

Rotational mechanical systems move about a fixed axis.
three basic elements.

1. **moment of inertia,**
2. **torsional spring**
3. **dashpot**.

If a torque is applied to a rotational mechanical system, then it is opposed by opposing torques due to moment of inertia, elasticity and friction of the system.

Since the applied torque and the opposing torques are in opposite directions, the algebraic sum of torques acting on the system is zero. Let us now see the torque opposed by these three elements individually.

ACTIVITY

UPSC IAS Prelims Exam Pattern 1515:

- **The General Studies Paper 1** tests candidates knowledge of History, Geography, Polity, Economy, Environment, General Science and Current Events.
- **The General Studies (CSAT)** tests candidates' mental ability, numerical ability and command over English Language. This paper is qualifying in nature.

Paper	**No. of Questions**	**Total Marks**	**Duration**
Paper I: General Studies	100	150	2 Hours
Paper II: General Studies (CSAT)	80	150	2 Hours

ACTIVITY-IAS QUESTIONS

Consider the following statements:

1. The first case of Coronavirus was reported from China
2. Coronavirus is officially known as 'COVID-19'
3. The Coronavirus is highly contagious disease, but does not spread from human to animal.

Which of the above statement(s) is/are incorrect?

a) 1 only
b) 3 only
c) 2 and 3 only
d) 1 and 3 only

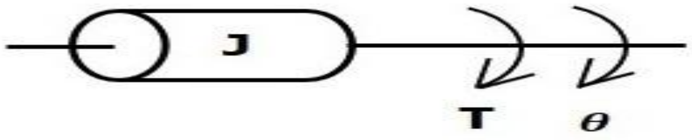
When is the Global Handwashing Day observed every year?

a) August 12
b) September 6
c) October 15
d) November 15

MOMENT OF INERTIA

In translational mechanical system, mass stores kinetic energy. Similarly, in rotational mechanical system, moment of inertia stores **kinetic energy**.

If a torque is applied on a body having moment of inertia **J**, then it is opposed by an opposing torque due to the moment of inertia. This opposing torque is proportional to angular acceleration of the body. Assume elasticity and friction are negligible.



$$T_j \propto \alpha$$

$$\Rightarrow T_j = J\alpha = J\frac{d^2\theta}{dt^2}$$

$$T = T_j = J\frac{d^2\theta}{dt^2}$$

Where,

- **T** is the applied torque
- T_j is the opposing torque due to moment of inertia
- **J** is moment of inertia
- **α** is angular acceleration
- **θ** is angular displacement

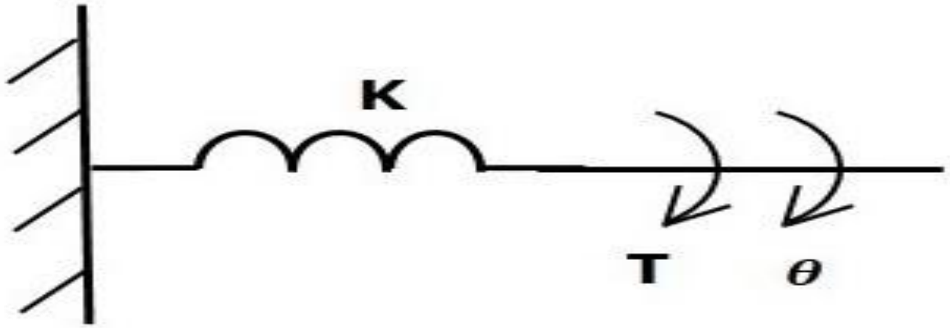
TORSIONAL SPRING

In translational mechanical system, spring stores potential energy. Similarly, in rotational mechanical system, torsional spring stores **potential energy**.

If a torque is applied on torsional spring **K**, then it is opposed by an opposing torque due to the elasticity of torsional spring.

This opposing torque is proportional to the angular displacement of the torsional spring. Assume that the moment of inertia and friction are negligible.

TORSIONAL SPRING



$$T_k \propto \theta$$

$$\Rightarrow T_k = K\theta$$

$$T = T_k = K\theta$$

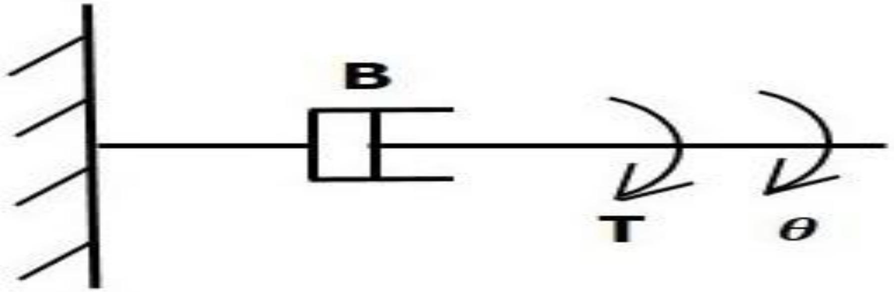
Where,

- **T** is the applied torque
- **T_k** is the opposing torque due to elasticity of torsional spring
- **K** is the torsional spring constant
- **θ** is angular displacement

DASHPOT

If a torque is applied on dashpot **B**, then it is opposed by an opposing torque due to the **rotational friction** of the dashpot.

This opposing torque is proportional to the angular velocity of the body. Assume the moment of inertia and elasticity are negligible.



$$T_b \propto \omega$$

$$\Rightarrow T_b = B\omega = B\frac{d\theta}{dt}$$

$$T = T_b = B\frac{d\theta}{dt}$$

Where,

- T_b is the opposing torque due to the rotational friction of the dashpot
- B is the rotational friction coefficient
- ω is the angular velocity
- θ is the angular displacement

SUMMARY