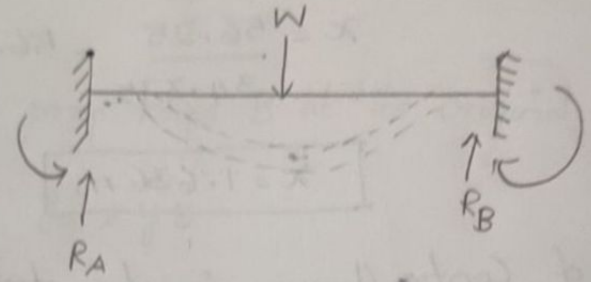
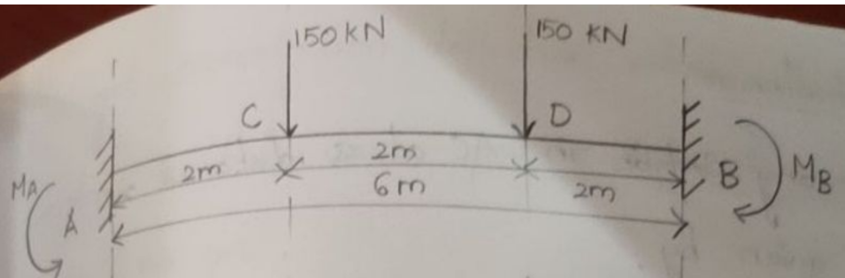
Fixed Beams

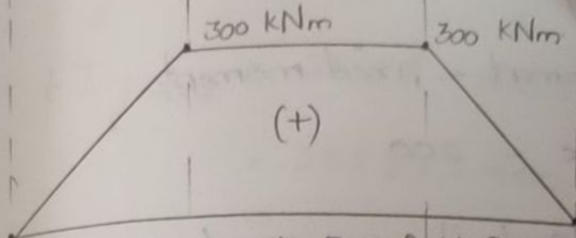
A fixed Beam (also called built-in or encastre beam) is a beam the ends of which are constrained or built-in to remain in horizontal position.



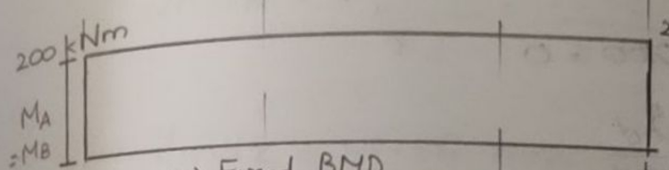
1) A fixed beam of 6m span is loaded with point loads of 150 kN at distance 2m from each support. Draw the B.M & S.F diagrams. Find also the maximum deflection. Take $E = 2\times10^8$ kN/m² & $I = 8\times10^8$ mm⁴

Fig (a) shows the fixed beam AB carrying points loads of 150 kN each. The fixed moments M_A & M_B are equal (due to symmetry). Free & fixed B.M diagrams are also shown in fig (b,c)



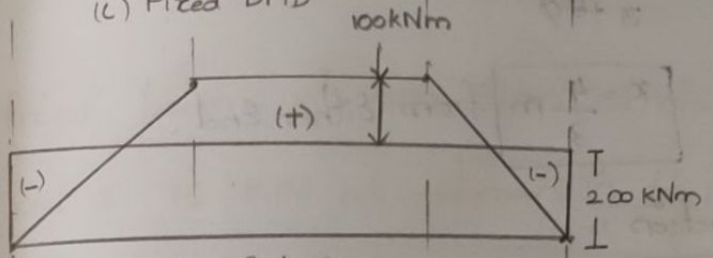


(b) Free B.M.D

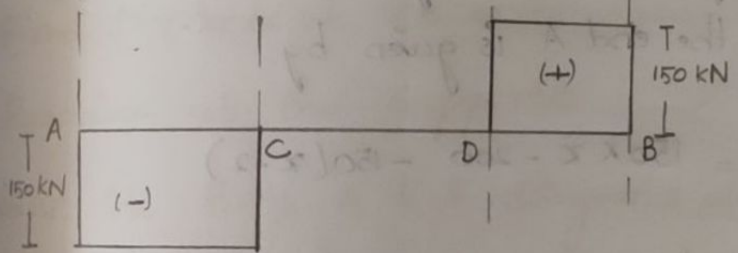


$$\frac{Wab^2}{l^2} + \frac{Wa^2b}{l^2}$$

(c) Fixed BMD



(d) Final BMD



(e) S.F.D

By equating the areas of free & fixed B.M diagrams we have.

$$M_A \times 6 = \frac{1}{2}(6+2) \times 300$$

$$\boxed{M_A = 200 \text{ kNm}}$$

$$\boxed{\therefore M_B = 200 \text{ kNm}}$$

$\therefore$ B.M at Centre $= 300 - 200 = 100$ kNm.

Point of Contraflexure:

B.M at any Section in AC at a distance x from A is given by.

$$M = \text{Free moment} - \text{fixed moment}$$

$$= 150 \times x - 200$$

To get point of contraflexure equate $M=0$

$$150 \times x - 200 = 0$$

$$x = \frac{\cancel{200}^{4}}{_{3}\cancel{150}}$$

$$\boxed{x = \frac{4}{3}\,m}$$ from Either End.

Slope & Deflection :-

The Bending moment at any section between A & D distant x from the end A is given by

$$EI.\frac{d^2y}{dx^2} = 150 \times x - 200 \quad - 150(x-2)$$

Integrating both Sides we get.

$$EI.\frac{dy}{dx} = 75x^2 - 200x + C_1 \quad - 75(x-2)^2$$

when, $x=0, \frac{dy}{dx} = 0 \quad \therefore C_1 = 0$

Integrating again, We get

$$EI.y = 25x^3 - 100x^2 + C_2 \quad - 25(x-2)^3$$

when, $x=0, y=0 \quad \therefore C_2 = 0$

To get maximum deflection which occurs at the centre in this case, put $x = 3$ m in the deflection eqn we get,

$$EI y_{max} = 25 \times 3^3 - 100 \times 3^2 - 25(3-2)^3$$

$$= 25 \times 27 - 900 - 25 = -250$$

$$y_{max} = \frac{-250}{EI} = \frac{-250}{2 \times 10^8 \times 8 \times 10^8 \times 10^{-2}} \text{ m}$$

$$y_{max} = -1.56 \text{ mm}.$$

2) A fixed beam of 6m Span carries point loads of 100 KN & 75 KN as shown in fig. Find the following.

1) Fixed Moments at the ends.

2) Reactions at the Supports.

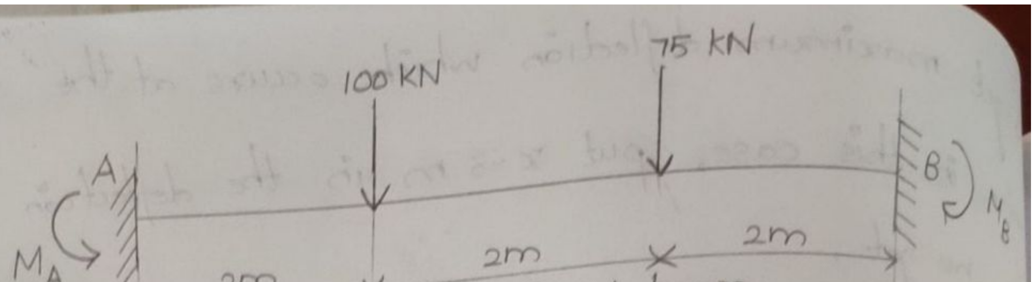
Draw the B.M & S.F Diagrams also.

1) Fixed moment at the left end A.

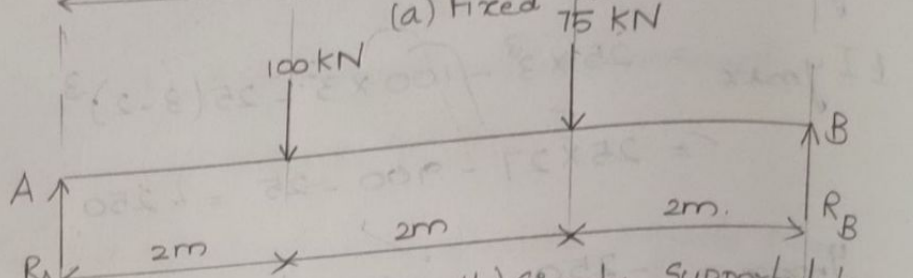
$$M_A = \Sigma \frac{Wab^2}{l^2} = \frac{100 \times 2 \times 4^2}{6^2} + \frac{75 \times 4 \times 2^2}{6^2} = 122.22 \text{ kNm}.$$

Fixed moment at the right end B.

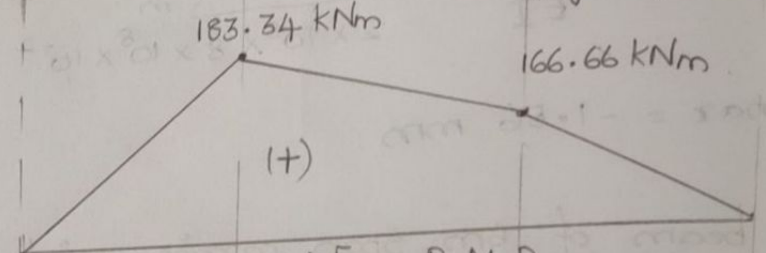
$$M_B = \Sigma \frac{Wa^2b}{l^2} = \frac{100 \times 2^2 \times 4}{6^2} + \frac{75 \times 4^2 \times 2}{6^2} = 111.11 \text{ kNm}.$$



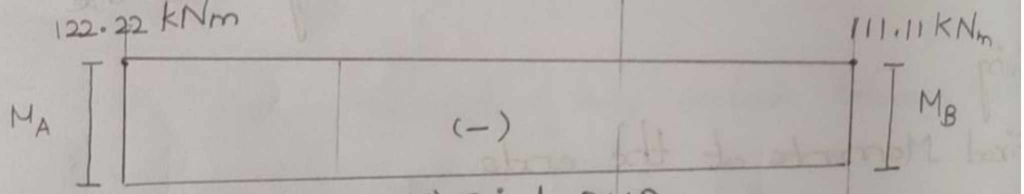
(a) Fixed beam



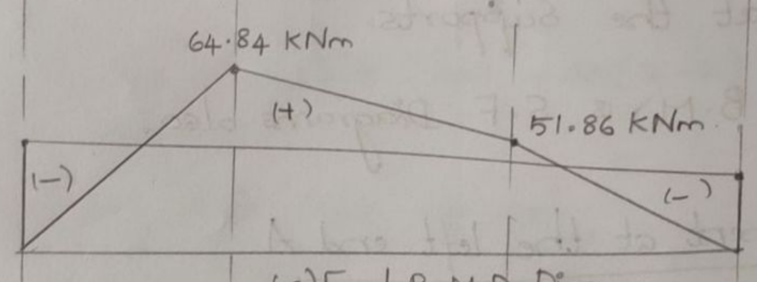
(b) Simply supported beam



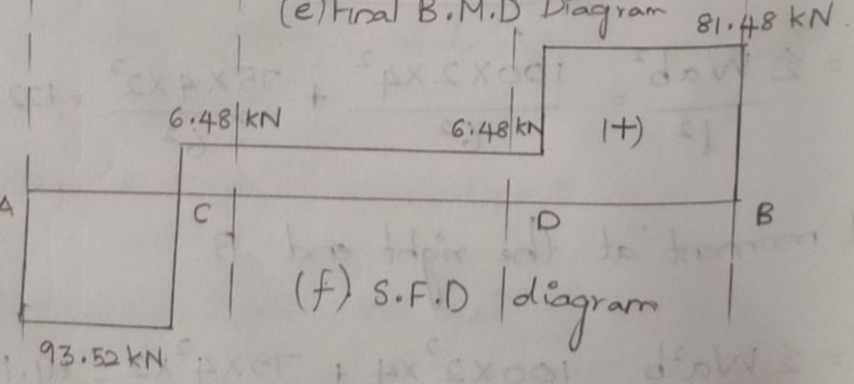
(c) Free B.M.D



(d) Fixed BMD



(e) Final B.M.D Diagram



(f) S.F.D diagram

ii) Reactions at the Supports

Consider the simply supported beam shown in fig (b).

Taking moment about A, We get

$R_B \times 6 = 100 \times 2 + 75 \times 4$

$= 500$

$R_B = \frac{500}{6} = 83.33$ kN.

also, $R_A + R_B = 100 + 75 = 175$ kN

$R_A = 175 - 83.33 = 91.67$ kN

$\boxed{R_A = 91.67 \text{ kN.}}$

B.M at C $= 91.67 \times 2 = 183.34$ kNm.

B.M at D $= 83.33 \times 2 = 166.66$ kNm.

Reactions (R) at each support due to end moments alone,

$$R = \frac{122.22 - 111.11}{6} = 1.85 \text{ kN.}$$

Since, $M_A > M_B$, the reaction R at A is upward & R at B is downward.

∴ Final reaction at A,

$R_{fA} = R_A + R = 91.67 + 1.85 = 93.52$ kN.

Final Reaction at B, $R_{fB} = 83.33 - 1.85 = 81.48$ kN.

(e) is the Final B.M.D & (f) is Final S.F.D.

3) A fixed beam AB of Length 3m carries an Point load of 45 KN at a distance of 2m from A. If the flexural rigidity of the beam is 1×10^4 KN/m². Determine

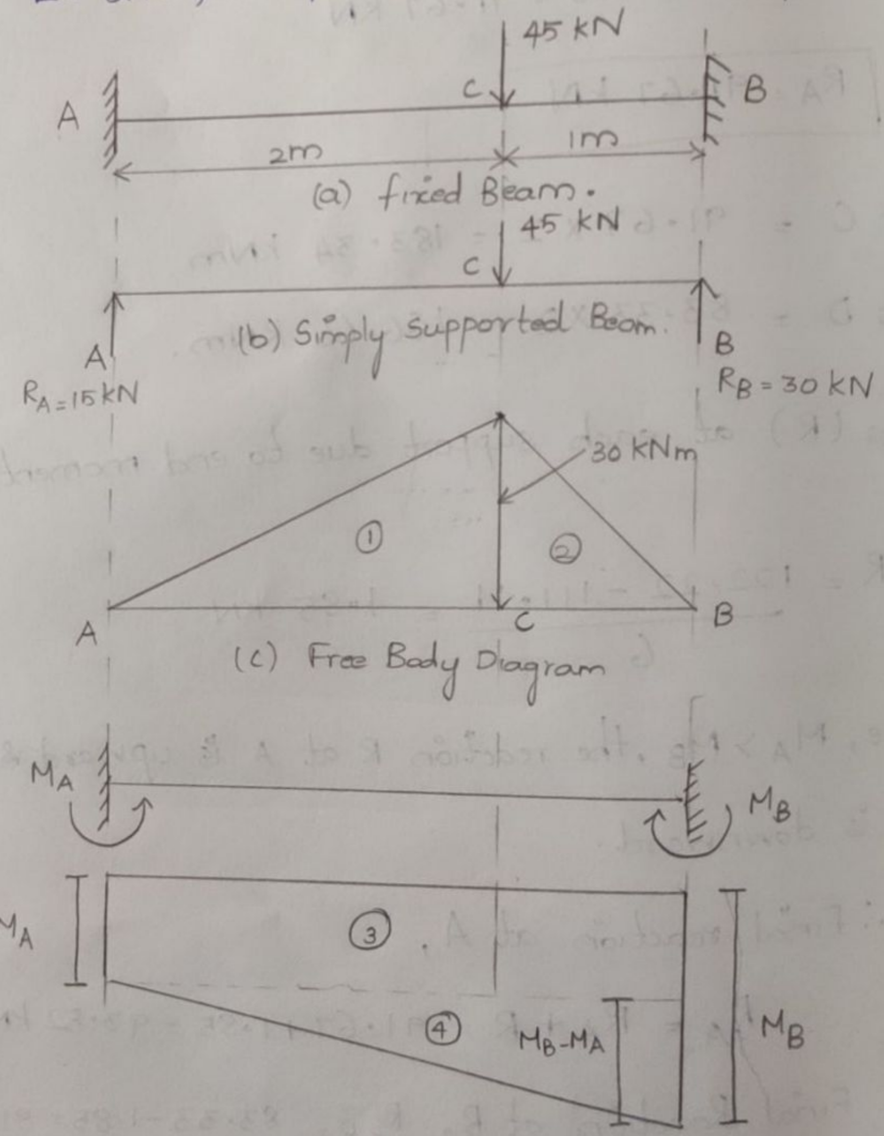
i) Fixed end moments A & B.

ii) Deflection under the Load.

iii) Maximum Deflection.

iv) Position of Maximum Deflection.

$L = 3m$, $P = 45$ KN, $EI = 1\times10^4$ KN/m².



1) Fixed End moments M_A, M_B :-

$$M_A = \frac{Wab^2}{L^2} = \frac{45 \times 2 \times 1^2}{3^2} = 10 \text{ kNm.}$$

$$M_B = \frac{Wa^2b}{L^2} = \frac{45 \times 2^2 \times 1}{3^2} = 20 \text{ kNm.}$$

Refer fig (b)

Taking moment about A.

$$R_B \times 3 - 45 \times 2 = 0$$

$$R_B = 30 \text{ kN}$$

R_A = Total Load − 30 = 45 − 30 = 15 kN.

Final Reaction :-

Here, $M_B > M_A$

$$R = \frac{M_B - M_A}{3} = \frac{20-10}{3} = \frac{10}{3}$$

$$R_{f_A} = 10/3 \text{ kN}$$

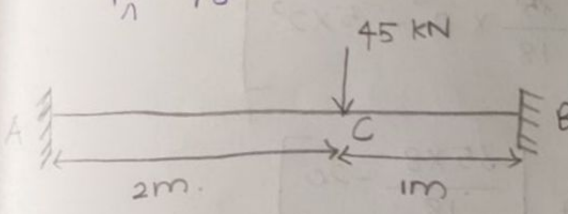


fig (F)

Consider fig (f), fixed Beams are shown in fig

The B.M at any section b/w AC at a distance from A is given by,

$$EI \cdot \frac{d^2y}{dx^2} = R_A \times x - M_A$$

$$EI \frac{d^2y}{dx^2} = \frac{35}{3}x - 10$$

$$EI.\frac{dy}{dx} = \frac{35}{3} \times \frac{x^2}{2} - 10x + C_1$$

When $x=0$ $\frac{dy}{dx} = 0$ $\boxed{\therefore C_1 = 0}$

$$\therefore EI.\frac{dy}{dx} = \frac{35}{6}.\frac{x^2}{2} - 10x \rightarrow (i)$$

$$EI.y = \frac{35}{6} \times \frac{x^3}{3} - \frac{10x^2}{2} + C_2$$

when $x=0$ $y=0$ $\boxed{\therefore C_2 = 0}$

$$EI.y = \frac{35}{18}x^3 - 5x^2 \rightarrow (ii)$$

iii) Deflection Under Load :-

$$y = \frac{1}{EI}\left[\frac{35}{18}x^3 - 5x^2\right]$$

Substitute $x=2$,

$$y = \frac{1}{EI}\left[\frac{35}{18} \times 2^3 - 5 \times 2^2\right]$$

$$= \frac{1}{1 \times 10^4}\left[\frac{35 \times 8}{18} - 20\right]$$

$$\boxed{y = 0.444 \text{ mm.}}$$

iv) Maximum Deflection :

Deflection will be maximum when $\frac{dy}{dx} = 0$

$$\frac{35}{6}x^2 - 10x = 0$$

$$35x^2 - 60x = 0$$

$$x(35x - 60) = 0$$

$$35x = 60$$

$$x = \frac{60}{35}$$

$$\boxed{x = 1.714\ m.}$$

$$EI \cdot y_{max} = \frac{35}{18}(1.714)^3 - 5(1.714)^2$$

$$y_{max} = \frac{1}{EI}\left[\frac{35}{18}(1.714)^3 - 5(1.714)^2\right]$$

$$= \frac{1}{1 \times 10^4}\left[9.79 - 14.69\right]$$

$$\boxed{y_{max} = 0.49\ mm.}$$

iii) Position of Max. Deflection:

Max. Deflection occurs 1.714 m from A

$$\boxed{x = 1.714\ m.}$$