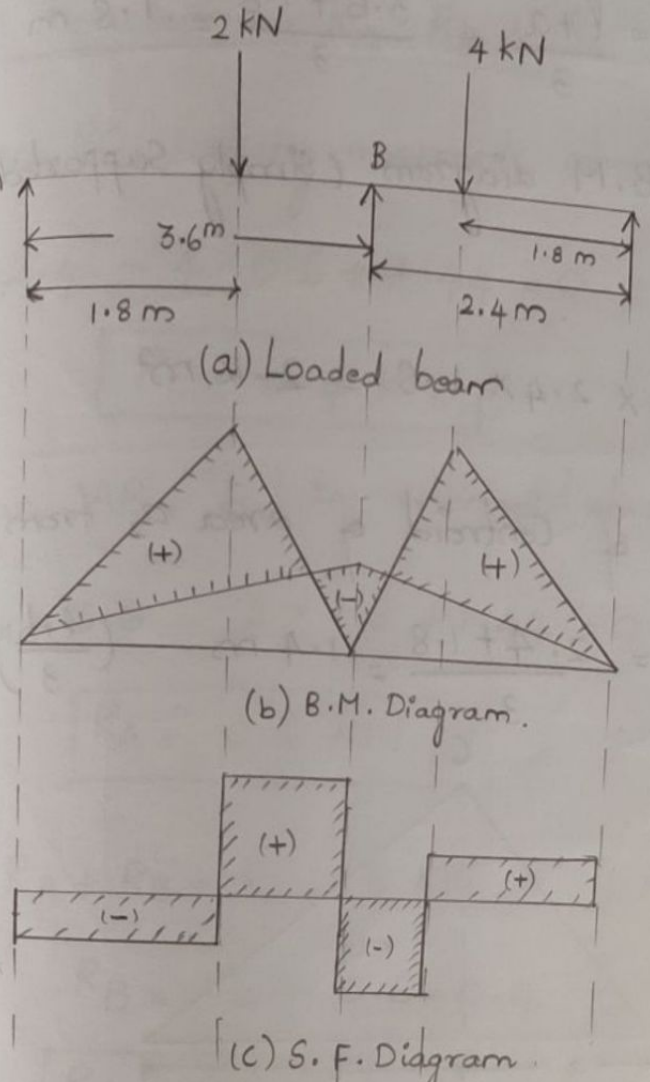
A beam Simply Supported at the Supports A & C and is Continuous over the Support B. Assuming EI is constant. Draw the Bending Moment & shear force Diagrams.



(a) Loaded beam

(b) B.M. Diagram.

(c) S.F. Diagram

M_A → Moment at the Support A.

M_B → Moment at the Support B

M_C → Moment at the Support C

the Bending moments under the Loads 2 kN and 4 kN (treating the Span AB & BC as

Simply Supported) are 1.8 kNm each.

Area of B.M diagram (Simply Supported) for Span AB.

$$a_1 = \frac{1}{2} \times 3.6 \times 1.8 = 3.24\ m^2.$$

Distance of Centroid of area a_1 from A.

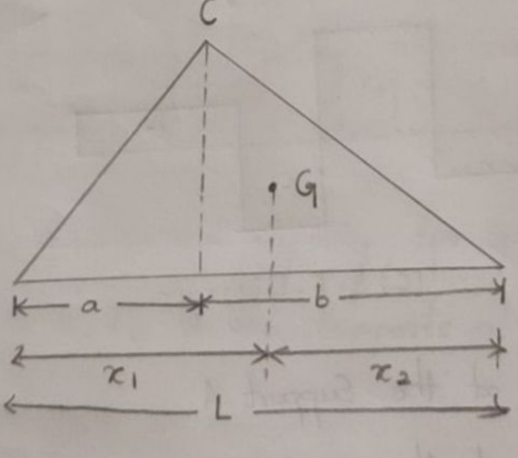
$$\bar{x}_1 = \frac{l+a}{3} = \frac{3.6+1.8}{3} = 1.8\ m.$$

Area of B.M diagram (simply supported) for Span BC.

$$a_2 = \frac{1}{2} \times 2.4 \times 1.8 = 2.16\ m^2$$

Distance of Centroid of area a_2 from C,

$$\bar{x}_2 = \frac{2.4+1.8}{3} = 1.4\ m. \quad \left(\frac{l+b}{3}\right)$$



Since, the beam is freely Supported at A & C the Support moments $M_A = M_C = 0$

Using the relation when there is no Sinking of Supports & EI is Constant. We have.

$$M_a l_1 + 2 M_B (l_1 + l_2) + M_c l_2 + \frac{6a_1 \bar{x}_1}{l_1} + \frac{6a_2 \bar{x}_2}{l_2} = 0$$

$$0 + 2 M_B (3.6 + 2.4) + 0 + \frac{6 \times 3.24 \times 1.8}{3.6} + \frac{6 \times 2.16 \times 1.4}{2.4} = 0$$

$$12 M_B + 9.72 + 7.56 = 0$$

$$\boxed{M_B = -1.44 \text{ kNm}}$$

Support Reactions R_A, R_B & R_C.

For Span BC, taking moments about B, We get

$$R_C \times 2.4 - 4 \times 0.6 + 1.44 = 0$$

$$\boxed{R_C = 0.4 \text{ KN}}$$

For Span AB, taking moment about B, We get

$$R_A \times 3.6 - 2 \times 1.8 + 1.44 = 0$$

$$\boxed{R_A = 0.6 \text{ KN}}$$

$$R_A + R_B + R_C = 6 \text{ KN}$$

$$R_B = 6 - 0.6 - 0.4$$

$$\boxed{R_B = 5 \text{ KN}}$$

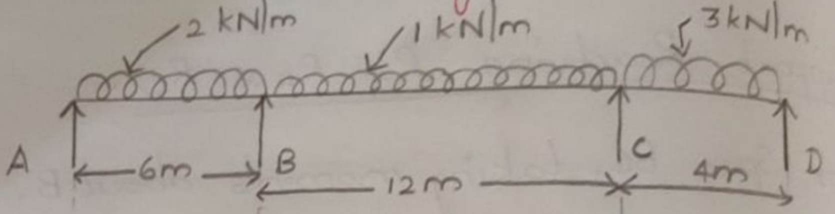
B.M.D & S.F.D are drawn.

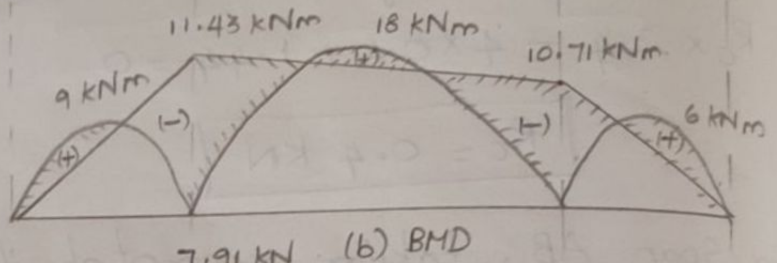
2) A Continuous beam ABCD of Uniform Cross-section is Loaded as shown in fig. Find

i) Bending Moments at Supports B & C

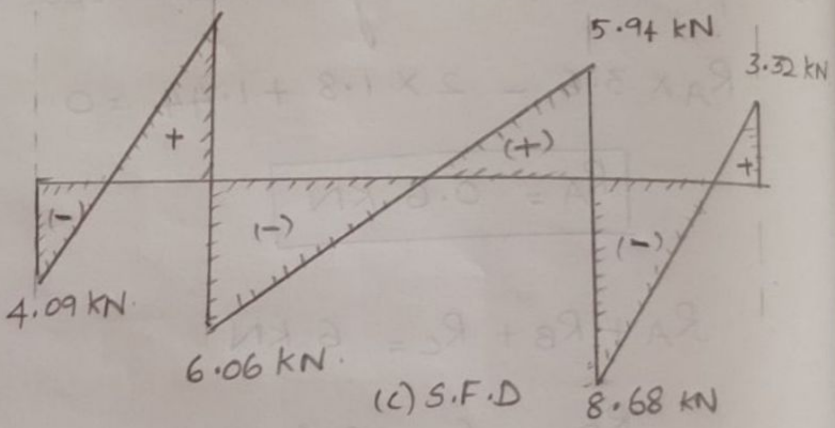
ii) Reactions at the Supports

Draw B.M.D & S.F.D diagrams.





(b) BMD



(c) S.F.D

For Span AB : Max. BM $= \frac{wl^2}{8} = \frac{2 \times 6^2}{8} = 9$ kNm

For Span BC : Max. B.M $= \frac{1 \times 12^2}{8} = 18$ kNm

for Span CD : Max. B.M $= \frac{3 \times 4^2}{8} = 6$ kNm

Applying three moments theorem for Span AB & BC

We get,

$$M_A \times 6 + 2 M_B (6+12) + 12 M_C + \frac{6 a_1 \bar{x}_1}{l_1} + \frac{6 a_2 \bar{x}_2}{l_2} = 0$$

where, $a_1 \bar{x}_1 = \frac{W_1 l_1^3}{12} \times \frac{l_1}{2} = \frac{W_1 l_1^4}{24}$

$$a_2 \bar{x}_2 = \frac{W_2 l_2^3}{12} \times \frac{l_2}{2} = \frac{W_2 l_2^4}{24}$$

$$6 M_A + 36 M_B + 12 M_C + \frac{6 \times 2 \times 6^4}{6 \times 24} + \frac{6 \times 1 \times 12^4}{12 \times 24} = 0$$

$$6 M_A + 36 M_B + 12 M_C + 108 + 432 = 0$$

$$M_A = 0$$

$$36 M_B + 12 M_C = -540$$

$$M_B + 0.333 M_C = -15.$$

Applying three moment theorem to Span BC & CD

We get,

$$M_B \times 12 + 2 M_C (12+4) + M_D \times 4 + \frac{6 \times 1 \times 12^4}{12 \times 24} + \frac{6 \times 3 \times 4^4}{4 \times 24} = 0$$

$$12 M_B + 32 M_C + 4 M_D + 432 + 48 = 0$$

$M_D = 0$ $\quad \therefore M_B + 2.667 M_C = -40.$

From Eqns:

$$M_C = -10.71 \text{ kNm}$$

$$M_B = -11.43 \text{ kNm}.$$

ii) Reactions at Supports.

Taking moments about B, We get

$$R_A + 6 + M_B - 2 \times 6 \times \frac{6}{2} = 0$$

$$R_A \times 6 + 11.43 - 36 = 0$$

$$\boxed{R_A = 4.09 \text{ kN.}}$$

Taking Moments about C, We get.

$$R_A \times 18 + R_B \times 12 - 2 \times 6 \times \left(\frac{6}{3} + 12\right) - 1 \times 12 \times \frac{12}{5}$$

$$+ 10.71 = 0.$$

$$4.09 \times 18 + 12 R_B - 180 - 72 + 10.71 = 0$$

$$\boxed{R_B = 13.97 \text{ kN.}}$$

Taking Moments about C for Span CD,

We get,

$$R_D \times 4 + M_C - 3 \times 4 \times \frac{4}{2} = 0$$

$$4R_D + 10.71 - 24 = 0$$

$$R_D = 3.32 \text{ kN.}$$

$$R_A + R_B + R_C + R_D = 2 \times 6 + 1 \times 12 + 3 \times 4.$$

$$\boxed{R_C = 14.62 \text{ kN.}}$$

The B.M.D & S.F.D are drawn.