SNS COLLEGE OF TECHNOLOGY

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COIMBATORE

DEPARTMENT OF CIVIL ENGINEERING

16CET204 – MECHANICS OF MATERIALS

II YEAR / IV SEMESTER

Unit 1 : ENERGY PRINCIPLES

Topic 3 : Castigliano’s theorem

ENERGY PRINCIPLES

CASTIGLIANO FIRST THEOREM

In the linearly elastic system, the partial derivative of the total strain energy stored in the structure with respect to a load gives displacement at that point in the direction of load.

$$\Delta i = \frac{\partial U}{\partial Pi}$$

Where,

U = Total Strain Energy

$Pi = Mf$ = Loads

$\Delta i = \theta f$ = Deflection displacement

The load may be a force (or) moment

CASTIGLIANO SECOND THEOREM

In the linearly elastic system, the partial derivative of the total strain energy stored in the structure with respect to the displacement at a point is equal to the force at that point.

$$\frac{\partial U}{\partial \delta} = P$$

USES OF CASTIGLIANO THEOREM

1. To determine the displacements of complicated structures.
2. To find the deflection of beams due to shearing or bending if the total strain energy due to shearing forces or bending moments is known.
3. To find the deflections of curved beams, springs etc.

Deflection under loadings

(i) Deflection under axial load :

Strain energy under axial load W,

$$U = \int \frac{1}{2} \frac{W^2\, dx}{AE}$$

$$\therefore \qquad \delta = \frac{\partial U}{\partial W} = \int \frac{W\, dx}{AE}$$

(ii) Deflection under bending:

Strain energy under bending moment M,

$$U = \int \frac{M^2\, dx}{2EI}$$

$$\therefore \qquad \delta = \frac{\partial U}{\partial W} = \frac{\partial U}{\partial M} \times \frac{\partial M}{\partial W} = \int \frac{M}{EI}\, dx \times \frac{\partial M}{\partial W} = \int \frac{M}{EI} \frac{\partial M}{\partial W}\, dx$$

(iii) Deflection under torsion:

Strain energy under torque T,

$$U = \int \frac{T^2\, dx}{2CI_p}$$

$$\therefore \qquad \delta = \frac{\partial U}{\partial W} = \frac{\partial U}{\partial T} \times \frac{\partial T}{\partial W} = \int \frac{T\, dx}{CI_p} \frac{\partial T}{\partial W} = \int \frac{T}{CI_p} \frac{\partial T}{\partial W}\, dx$$

(iv) Deflection under shear :

Strain energy under shear free F,

$$U = \int \frac{F^2\, dx}{2AC}$$

$$\therefore \qquad \delta = \frac{\partial U}{\partial F} = \int \frac{F\, dx}{AC}$$

DEFLECTION UNDER LOADINGS

(v) **Deflection under horizontal shear :**

$$U = \left(\frac{1 + \frac{1}{m}}{E I^2} \right) \int F^2 \, dx \iint \frac{(A \bar{y})^2}{b^2} \, dA$$

where, F is a function of W.

$$\therefore \quad \delta = \frac{\partial U}{\partial W} \quad \text{...(15.39)}$$

(vi) **Rotation under bending :**

$$U = \int \frac{M^2}{2 E I}$$

$$\therefore \quad \phi = \frac{\partial U}{\partial M} = \int \frac{M \, dx}{EI} \quad \text{...(15.40)}$$

(vii) **Rotation under torsion :**

$$U = \int \frac{T^2 \, dx}{2 C I_p}$$

$$\therefore \quad \theta = \frac{\partial U}{\partial T} = \int \frac{T \, dx}{C I_p} \quad \text{...(15.41)}$$

PROBLEMS

Reference

1)R.K.BANSAL STRENGTH OF MATERIALS

2)Er.R.K.RAJPUT STRENGTH OF MATERIALS

THANK YOU