4. TENSION MEMBERS

4.1 Introduction

Tension members are linear members in which axial forces act so as to elongate (stretch) the member. A rope, for example, is a tension member. Tension members carry loads most efficiently, since the entire cross section is subjected to uniform stress. Unlike compression members, they do not fail by buckling (see chapter on compression members). Ties of trusses [Fig 4.1(a)], suspenders of cable stayed and suspension bridges [Fig.4.1(b)], suspenders of buildings systems hung from a central core [Fig.4.1(c)] (such buildings are used in earthquake prone zones as a way of minimising inertia forces on the structure), and sag rods of roof purlins [Fig4.1(d)] are other examples of tension members.

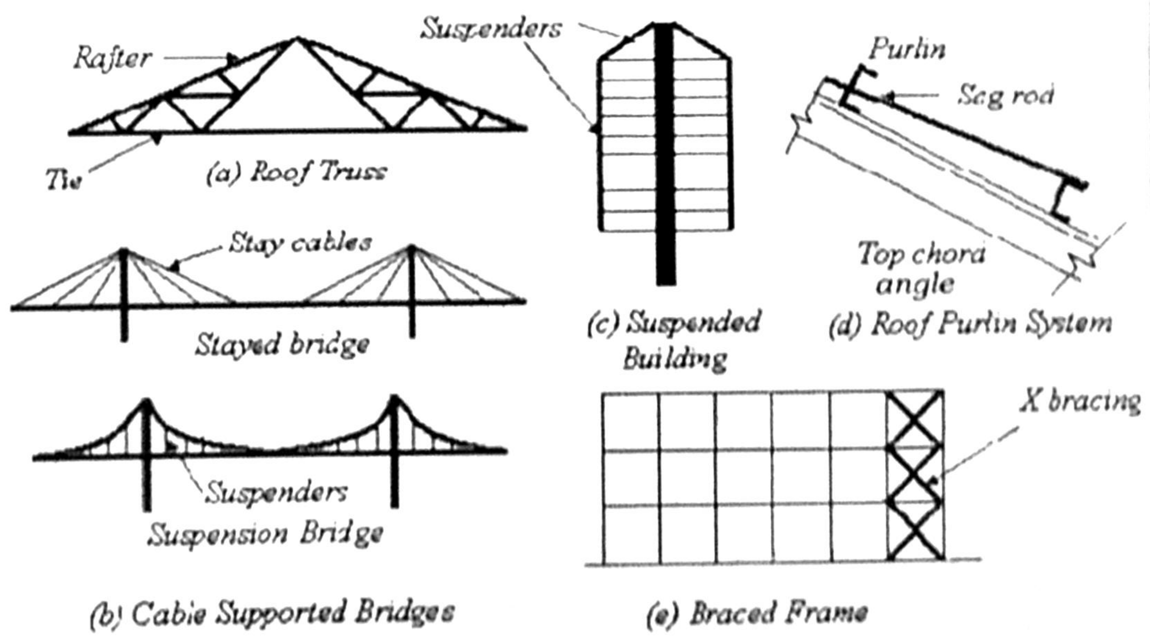


Fig 4.1 Tension members in structures

Tension members are also encountered as bracings used for the lateral load resistance. In X type bracings [Fig 4.1(e)] the member which is under tension, due to lateral load acting in one direction, undergoes compressive force, when the direction of the lateral load is changed and vice versa. Hence, such members may have to be designed to resist tensile and compressive forces.

The tension members can have a variety of cross sections. The single angle and double angle sections [Fig4.2 (a)] are used in light roof trusses as in industrial buildings. The tension members in bridge trusses are made of channels or I sections, acting individually or built-up [Figs.4.2(c) and 2(d)]. The circular rods [Fig.4.2 (d)] are used in bracings designed to resist loads in tension only. They buckle at very low compression and are not considered effective. Steel wire ropes [Fig.4.2 (e)] are used as suspenders in the cable suspended bridges and as main stays in the cable-stayed bridges.

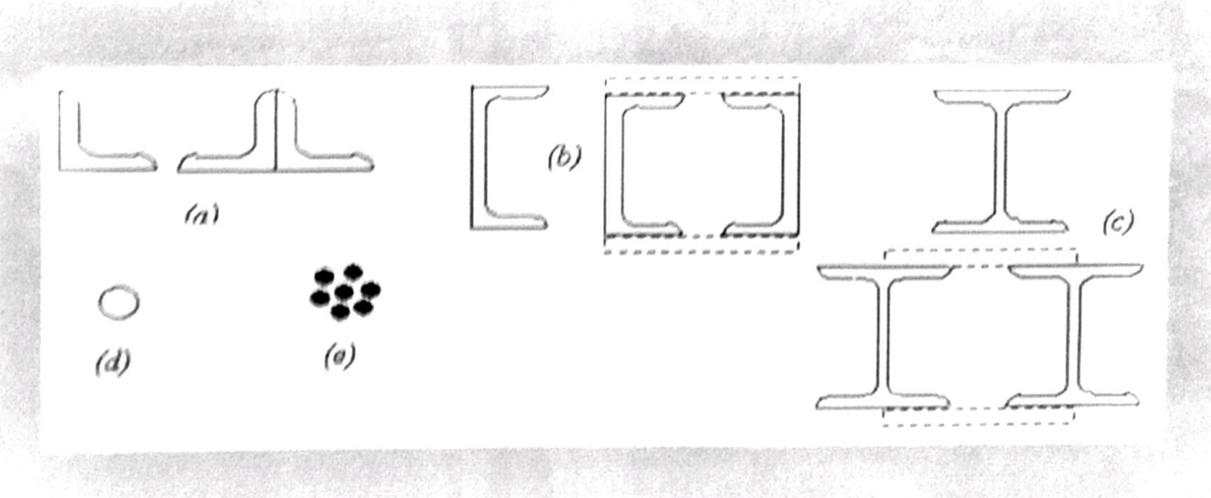


Fig 4.2 Cross sections of tension members

4.2 Behaviour of tension members

Since axially loaded tension members are subjected to uniform tensile stress, their load deformation behaviour (Fig.4.3) is similar to the corresponding basic material stress strain behaviour. Mild steel members (IS: 2062 & IS: 226) exhibit an elastic range (a-b) ending at yielding (b). This is followed by yield plateau (b-c). In the Yield Plateau the load remains constant as the elongation increases to nearly ten times the yield strain. Under further stretching the material shows a smaller increase in tension with elongation (c-d), compared to the elastic range. This range is referred to as the strain hardening range. After reaching the ultimate load (d), the loading decreases as the elongation increases (d-e) until rupture (e). High strength steel tension members do not exhibit a well-defined yield point and a yield plateau (Fig.4.3). The 0.2% offset load, T, as shown in Fig.4.3 is usually taken as the yield point in such cases.

Load-elongation of tension member to view click here

Fig. 4.3 Load – elongation of tension members

4.2.1 Design strength due to yielding of gross section

Although steel tension members can sustain loads up to the ultimate load without failure, the elongation of the members at this load would be nearly 10-15% of the original length and the structure supported by the member would become unserviceable. Hence, in the design of tension members, the yield load is usually taken as the limiting load. The corresponding design strength in member under axial tension is given by (C1.62)

$$T_d = f_y A / \gamma_{m0} \qquad \textit{(4.1)}$$

Where, f_y is the yield strength of the material (in MPa), A is the gross area of cross section in mm^2 and γ_{m0} is the partial safety factor for failure in tension by yielding. The value of γ_{m0} according to IS: 800 is 1.10.

4.2.2 Design strength due to rupture of critical section

Frequently plates under tension have bolt holes. The tensile stress in a plate at the cross section of a hole is not uniformly distributed in the Tension Member: Behaviour of Tension Members elastic range, but exhibits stress concentration adjacent to the hole [Fig 4.4(a)]. The ratio of the maximum elastic stress adjacent to the hole to the average stress on the net cross section is referred to as the Stress Concentration Factor. This factor is in the range of 2 to 3, depending upon the ratio of the diameter of the hole to the width of the plate normal to the direction of stress.

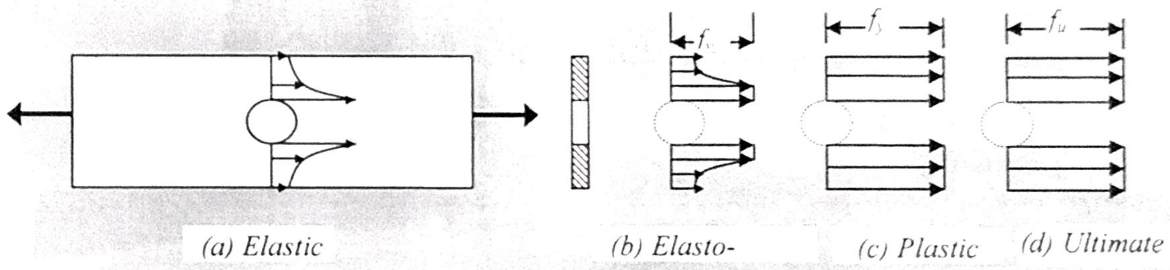


Fig. 4.4 Stress distribution at a hole in a plate under tension

In statically loaded tension members with a hole, the point adjacent to the hole reaches yield stress, f_y, first. On further loading, the stress at that point remains constant at the yield stress and the section plastifies progressively away from the hole [Fig.4.4(b)], until the entire net section at the hole reaches the yield stress, f_y, [Fig.4.4(c)]. Finally, the rupture (tension failure) of the member occurs when the entire net cross section reaches the ultimate stress, f_u, [Fig.4.4 (d)]. Since only a small length of the member adjacent to the smallest cross section at the holes would stretch a lot at the ultimate stress, and the overall member elongation need not be large, as long as the stresses in the gross section is below the yield stress. Hence, the design strength as governed by net cross-section at the hole, T_{dn}, is given by (C1.6.3)

$$P_{tn} = 0.9 f_u A_n / \gamma_{m1} \qquad (4.2)$$

Where, f_u is the ultimate stress of the material, A_n is the net area of the cross section after deductions for the hole [Fig.4.4 (b)] and γ_{m1} is the partial safety factor against ultimate tension failure by rupture ($\gamma_{m1} = 1.25$). Similarly threaded rods subjected to tension could fail by rupture at the root of the threaded region and hence net area, A_n, is the root area of the threaded section (Fig.4.5).

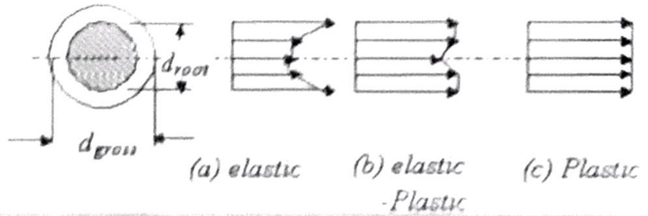


Fig 4.5 Stress in a threaded rod

The lower value of the design tension capacities, as given by Eqn.4.1 and 4.2, governs the design strength of a plate with holes.

Frequently, plates have more than one hole for the purpose of making connections. These holes are usually made in a staggered arrangement [Fig.4.6 (a)]. Let us consider the two extreme arrangements of two bolt holes in a plate, as shown in Fig.4.6 (b) & 4.6(c). In the case of the arrangement shown in Fig.4.6 (b), the gross area is reduced by two bolt holes to obtain the net area. Whereas, in arrangement shown in Fig.4.6c, deduction of only one hole is necessary, while evaluating the net area of the cross section. Obviously the change in the net area from the case shown in Fig.4.6(c) to Fig.4.6 (b) has to be gradual. As the pitch length (the centre to centre distance between holes along the direction of the stress) p, is decreased, the critical cross section at some stage changes from straight section [Fig.4.6(c)] to the staggered section 1-2-3-4 [Fig.4.6(d)]. At this stage, the net area is decreased by two bolt holes along the staggered section, but is increased due to the inclined leg (2-3) of the staggered section. The net effective area of the staggered section 1-2-3-4 is given by

$$A_n = \left(b - 2d + p^2/4g\right)t \qquad \textit{(4.3)}$$

Where, the variables are as defined in Fig.4.6 (a). In Eqn.4.3 the increase of net effective area due to inclined section is empirical and is based on test results. It can be seen from Eqn.4.3 that as the pitch distance, p, increases and the gauge distance, g, decreases, the net effective area corresponding to the staggered section increases and becomes greater than the net area corresponding to single bolt hole. This occurs when

$$p^2/4g > d \qquad \textit{(4.4)}$$

When multiple holes are arranged in a staggered fashion in a plate as shown in Fig.4.6 (a), the net area corresponding to the staggered section in general is given by

$$A_{net} = \left(b - nd + \sum \frac{p^2}{4g}\right)t \qquad \textit{(4.5)}$$

Where, n is the number of bolt holes in the staggered section [n = 7 for the zigzag section in Fig.4.6 (a)] and the summation over $p^2/4g$ is carried over all inclined legs of the section [equal to n-1 = 6 in Fig.4.6 (a)].

Normally, net areas of different staggered and straight sections have to be evaluated to obtain the minimum net area to be used in calculating the design strength in tension.

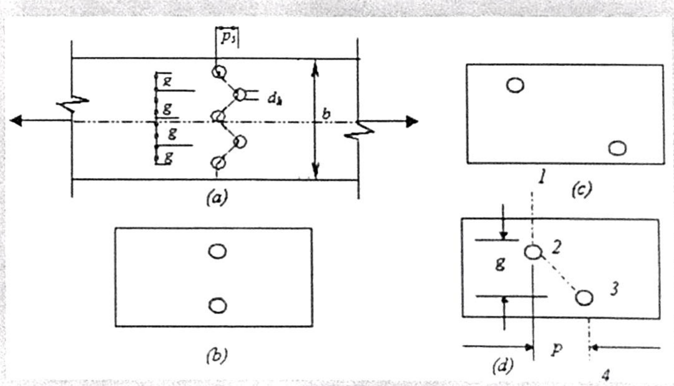


Fig 4.6 Plates with bolt hole under tension

4.2.3 Design strength due to block shear

A tension member may fail along end connection due to block shear as shown in Fig 4.7. The corresponding design strength can be evaluated using the following equations. The block shear strength T_{db}, at an end connection is taken as the smaller of (C1 6.4)

$$T_{db} = \left(A_{vg} f_y / \left(\sqrt{3} \gamma_{m0} \right) + f_u A_{tn} / \gamma_{m1} \right) \qquad \textbf{(4.6)}$$

or

$$T_{db} = \left(f_u A_{vn} / \left(\sqrt{3} \gamma_{m1} \right) + f_y A_{tg} / \gamma_{m0} \right) \qquad \textbf{(4.7)}$$

Where, A_{vg}, A_{vn} = minimum gross and net area in shear along a line of transmitted force, respectively (1-2 and 4 –3 as shown in Fig 4.6 and 1-2 as shown in Fig 4.7), A_{tg}, A_{tn} = minimum gross and net area in tension from the hole to the toe of the angle or next last row of bolt in plates, perpendicular to the line of force, respectively (2-3) as shown in Fig 4.7 and f_u, f_y = ultimate and yield stress of the material respectively

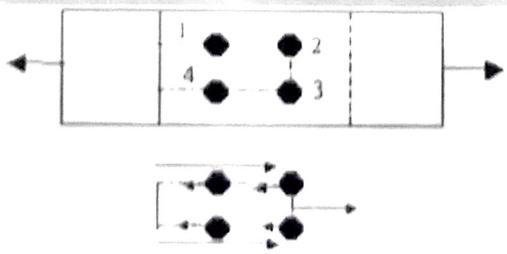


Fig 4.7 Block shearing failure plates

4.2.4 Angles under tension

Angles are extensively used as tension members in trusses and bracings. Angles, if axially loaded through centroid, could be designed as in the case of plates.

However, usually angles are connected to gusset plates by bolting or welding only one of the two legs (Fig. 4.8). This leads to eccentric tension in the member, causing non-uniform distribution of stress over the cross section. Further, since the load is applied by connecting only one leg of the member there is a shear lag locally at the end connections.

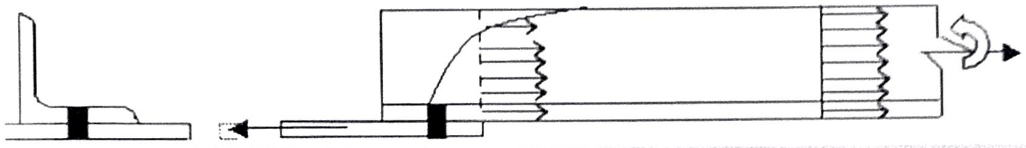


Fig 4.8 Angles eccentrically loaded through gussets

Kulak and Wu (1997) have reported, based on an experimental study, the results on the tensile strength of single and double angle members. Summary of their findings is:

- The effect of the gusset thickness, and hence the out of plane stiffness of the end connection, on the ultimate tensile strength is not significant.
- The thickness of the angle has no significant influence on the member strength.
- The effects of shear lag, and hence the strength reduction, is higher when the ratio of the area of the outstanding leg to the total area of cross-section increases.
- When the length of the connection (the number of bolts in end connections) increases, the tensile strength increases up to 4 bolts and the effect of further increase in the number of bolts, on the tensile strength of the member is not significant. This is due to the connection restraint to member bending caused by the end eccentric connection.

- Even double angles connected on opposite sides of a gusset plate experience the effect of shear lag.

Based on the test results, Kulak and Wu (1997) found that the shear lag due to connection through one leg only causes at the ultimate stage the stress in the outstanding leg to be closer only to yield stress even though the stress at the net section of the connected leg may have reached ultimate stress. They have suggested an equation for evaluating the tensile strength of angles connected by one leg, which accounts for various factors that significantly influence the strength. In order to simplify calculations, this formula has suggested that the stress in the outstanding leg be limited to f_y (the yield stress) and the connected sections having holes to be limited to f_u (the ultimate stress).

The strength of an angle connected by one leg as governed by tearing at the net section is given by (C1.6.3.3)

$$T_{tn} = \left(A_{nc} f_u / \gamma_{m1} + \beta A_0 f_y / \gamma_{m0} \right) \qquad \textbf{(4.8)}$$

Where, f_y and f_u are the yield and ultimate stress of the material, respectively. An_{nc} and A_o, are the net area of the connected leg and the gross area of the outstanding leg, respectively. The partial safety factors $\gamma_{m0} = 1.10$ and $\gamma_{m1} = 1.25$ β accounts for the end fastener restraint effect and is given by

$$\beta = 1.4 - 0.035(w/t)(f_u/f_y)(b_s/L) \qquad \textbf{(4.9)}$$

Where w and b_s are as shown in Fig 4.9. L = Length of the end connection, i.e., distance between the outermost bolts in the joint along the length direction or length of the weld along the length direction

Alternatively, the tearing strength of net section may be taken as

$$T_{dn} = \alpha A_n f_u / \gamma_{m1} \qquad \textbf{(4.10)}$$

Where, $\alpha = 0.6$ for one or two bolts, 0.7 for three bolts and 0.8 for four or more bolts in the end connection or equivalent weld length. A_n = net area of the total cross section, A_{nc}= net area of connected leg, A_{go}= gross area of outstanding leg, t = thickness of the leg

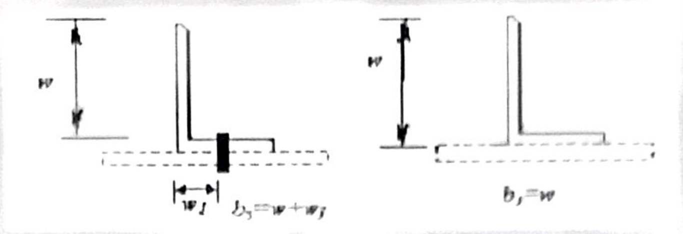


Fig 4.9 Angles with ended connections

$\beta = 1.0$, if the number of fasteners is ≤ 4,

$\beta = 0.75$ if the number of fasteners = 3 and

"Above is not recommended in code anywhere" $\beta = 0.5$, if number of fasteners = 1 or 2

In case of welded connection, $\beta = 1.0$

The strength η as governed by yielding of gross section and block shear may be calculated as explained for the plate. The minimum of the above strengths will govern the design.

The efficiency, of an angle tension member is calculated as given below

$$\eta = F_d / (A_g f_y / \gamma_{m0}) \qquad \textbf{(4.11)}$$

Depending upon the type of end connection and the configuration of the built-up member, the efficiency may vary between 0.85 and 1.0. The higher value of efficiency is obtained in the case of double angles on the opposite sides of the gusset connected at the ends by welding and the lower value is usual in the bolted single angle tension members. In the case of threaded members the efficiency is around 0.85

In order to increase the efficiency of the outstanding leg in single angles and to decrease the length of the end connections, some times a short length angle at the ends are connected to the gusset and the outstanding leg of the main angle directly, as shown in Fig 4.10 Such angles are referred to as lug angles The design of such connections should confirm to the codal provisions given in C1.10.12

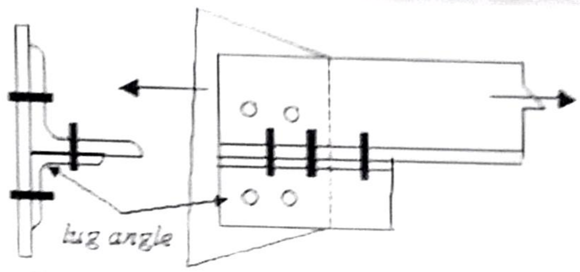


Fig 4.10 Tension member with lug

4.3 Design of tension members

In the design of a tension member, the design tensile force is given and the type of member and the size of the member have to be arrived at. The type of member is usually dictated by the location where the member is used. In the case of roof trusses, for example, angles or pipes are commonly used. Depending upon the span of the truss, the location of the member in the truss and the force in the member either single angle or double angles may be used in roof trusses. Single angle is common in the web members and the double angles are common in rafter and tie members of a roof truss.

Plate tension members are used to suspend pipes and building floors. Rods are also used as suspenders and as sag rods of roof purlins. Steel wires are used as suspender cables in bridges and buildings. Pipes are used in roof trusses on aesthetic considerations, in spite of fabrication difficulty and the higher cost of such tubular trusses. Built-up members made of angles, channels and plates are used as heavy tension members, encountered in bridge trusses.

4.3.1 Trial and error design process

The design process is iterative, involving choice of a trial section and analysis of its capacity. This process is discussed in this section. Initially, the net effective area required is calculated from the design tension and the ultimate strength of the material as given below.

$$A_n = F_t / (f_u / \gamma_{m1}) \qquad \textit{(4.12)}$$

Using the net area required, the gross area required is calculated, allowing for some assumed number and size of bolt holes in plates, or assumed efficiency index in the case of angles and threader rods. The gross area required is also checked against that required from the yield strength of the gross sections as given below.

$$A_g = T_d / (f_y / \gamma_{m0}) \quad (4.13)$$

A suitable trial section is chosen from the steel section handbook to meet the gross area required. The bolt holes are laid out appropriately in the member and the member is analyses to obtain the actual design strength of the trial section. The design strength of the trial section is evaluated using Eqs.4.1 to 4.5 in the case of plates and threaded bars and using Eqs.4.6 in the case of angle ties. If the actual design strength is smaller than or too large compared to the design force, a new trial section is chosen and the analysis is repeated until a satisfactory design is obtained.

4.3.2 Stiffness requirement

The tension members, in addition to meeting the design strength requirement, frequently have to be checked for adequate stiffness. This is done to ensure that the member does not sag too much during service due to self-weight or the eccentricity of end plate connections. The IS: 800 imposes the following limitations on the slenderness ratio of members subjected to tension:

(a) In the case of members that are normally under tension but may experience compression due to stress reversal caused by wind / earthquake loading $l/r \leq 250$.

(b) In the case of members that are designed for tension but may experience stress reversal for which it is not designed (as in X bracings). $l/r \leq 350$

(c) In the case of members subjected to tension only. $l/r \leq 400$

In the case of rods used as a tension member in X bracings, the slenderness ratio limitation need not be checked for if they are pre-tensioned by using a turnbuckle or other such arrangement.

4.4 Summary

The important factors to be considered while evaluating the tensile strength are the reduction in strength due to bolt holes and due to eccentric application of loads through gusset plates attached to one of the elements. The yield strength of the gross area or the ultimate strength of the net area may govern the tensile strength. The effect of connecting the end gusset plate to only one of the elements of the cross section can be empirically accounted for by the reduction in the effectiveness of the out standing leg, while calculating the net effective area. The iterative method has to be used in the design of tension members.

4.5 References

1. IS 800-2005 (Draft) 'Code of practice for general construction in steel', Bureau of Indian Standards, New Delhi.

2. Teaching resources for structural steel design (Volume 1 to 3), INSDAG publication, Calcutta2000.

3. Dowling P.J., Knowles P and Owens GW., ' Structural Steel Design' , The Steel construction Institute, 1988.

4. Owens GW and Knowles PR, 'Steel Designers Manual', fifth edition, Blackwell science 1992

Structural steel design project

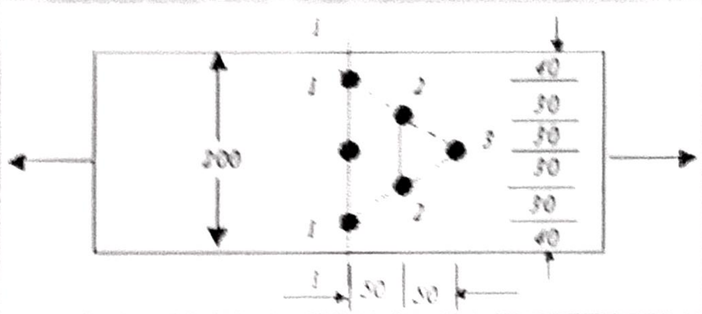
Worked example 1

Problem 1:

Determine the desig tensile strength of the plate (200 X 10 mm) with the holes as shown below, if the yield strength and the ultimate strength of the steel used are 250 MPa and 420 MPa and 20 mm diameter bolts are used.

f_y = 250 MPa

f_u = 420 MPa



Calculation of net area, A_{net}:

A_n Results you need, click here

P_t is lesser of

(i) $A_g f_y / \gamma M_0 = \dfrac{200*10*250/1.15}{1000} = 434.8 kN$

(ii) $0.9 A_n f_u / \gamma M_1 = \dfrac{0.9*1342*420/1.25}{1000} = 405.8 kN$

$P_t = 405.8 kN$

Efficiency of the plate with holes = $\dfrac{P_t}{A_g f_y / \gamma M_0} = \dfrac{409.8}{434.8} = 0.93$

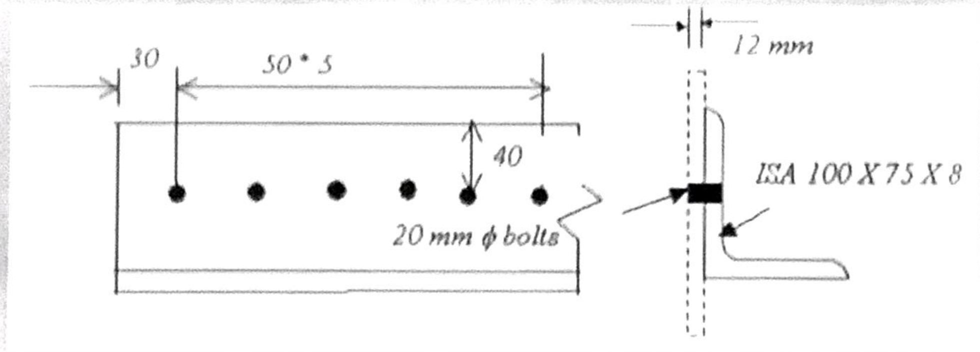
Structural steel design project

Worked example 2

Problem 2:

Analysis of single angle tension members

A single unequal angle 100 X 75 X 8 mm is connected to a 12 mm thick gusset plate at the ends with 6 nos. 20 mm diameter bolts to transfer tension. Determine the design tensile strength of the angle. (a) if the gusset is connected to the 100 mm leg, (b) if the gusset is connected to the 75 mm leg, (c) if two such angles are connected to the same side of the gusset through the 100 mm leg. (d) if two such angles are connected to the opposite sides of the gusset through 100 mm leg.



a) The 100mm leg bolted to the gusset :

A_{nc} = (100 - 8/2 - 21.5) *8 = 596 mm^2

A_o = (75 - 8/2) * 8 = 568 mm^2

A_g = ((100-8/2) + (75 – 8/2)) * 8 =1336 mm^2

Strength as governed by tearing of net section:

Since the number of bolts = 4; $\beta = 1.0$

$$P_t = A_{nc} f_u / \gamma_{m1} + \beta A_o f_y / \gamma_{m0}$$

= 596 * 420/1.25 + 1.0 * 568 * 250 / 1.15

= 323734 N (or) 323.7 kN

Strength as governed by yielding of gross section:

$P_t = A_g f_y / \gamma_{m0}$

$= 1336 * 250 / 1.15 = 290435$ N (or) 290.4 kN

Block shear strength

V_g – Grass "shearing"

t_n – Tearing net

$P_v = \left(0.62\, A_g\, f_y / \gamma_{m0} + A_n\, f_u / \gamma_{m1}\right)$ (Shear yield + tensile fracture)

= 0.62 * (5 *50 +30)* 8 * 250/1.15 + (40-21.5/2) * 8 * 420/1.25

= 380537 N = 380.5 kN

or

$P_v = \left(0.62\, A_n\, f_u / \gamma_{m1} + A_g\, f_y / \gamma_{m0}\right)$ (Shear fracture + tensile yield)

= (0.62 (5 * 50 + 30 –5.5 * 21.5) * 8 * 420 / 1.25 + 40 * 8 * 250/ 1.15)

= 339131 N = 339.1

The design tensile strength of the member = 290.4 kN

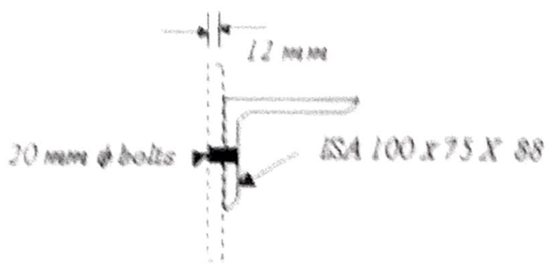
The efficiency of the tension member, is given by

$$\eta = \frac{P_t}{A_g f_y} = \frac{290.4 * 1000}{(100 + 75 - 8) * 8 * 250/1.15} = 1.0$$

b) The 75 mm leg is bolted to the gusset:

$A_{nc} = (75 - 8/2 - 21.5) * 8 = 396\ mm^2$

$A_o = (100 - 8/2) * 8 = 768\ mm^2$



Strength as governed by tearing of net section:

Since the number of bolts = 6, $\beta = 1.0$

$P_t = A_{nc} f_u / \gamma_{m1} + \beta A_0 f_y / \gamma_{m0}$

= 396 * 420/1.25 + 1.0 * 768 *250 / 1.15

= **300123 N (or) 300.1 kN**

Strength as governed by yielding of gross section:

$P_t = A_g f_y / \gamma_{m0}$

= 1336 * 250 / 1.15 = **290435 N (or) 290.4 kN**

Block shear strength:

$P_v \leq (0.62 A_{vg} f_y / \gamma_{m0} + A_{tn} f_u / \gamma_{m1})$

= 0.62 * (5 *50 +30)* 8 * 250/1.15 + (35-21.5/2) * 8 * 420/1.25

= 367097 N = **367.1 kN**

$P_v \leq (0.62 A_{vn} f_u / \gamma_{m1} + A_{tg} f_y / \gamma_{m0})$

= (0.62 (5 * 50 + 30 –5.5 *x 21.5) * 8 * 420 / 1.25 + 35 *8 * 250/ 1.15

= 330435 N = **330.4 kN**

The design tensile strength of the member = **290.4 kN**

Even though the tearing strength of the net section is reduced, the yielding of the gross section still governs the design strength.

The efficiency of the tension member is as before 1.0

Note: The design tension strength is more some times if the longer leg of an unequal angle is connected to the gusset (when the tearing strength of the net section governs the design strength).

An understanding about the range of values for the section efficiency, η, is useful to arrive at the trial size of angle members in design problems.

(c & d)The double angle strength would be twice single angle strength as obtained above in case (a)

P_t = 2 * 290.4 = 580.8 kN

5. COMPRESSION MEMBERS

5.1 Introduction

Column, top chords of trusses, diagonals and bracing members are all examples of compression members. Columns are usually thought of as straight compression members whose lengths are considerably greater than their cross-sectional dimensions.

An initially straight strut or column, compressed by gradually increasing equal and opposite axial forces at the ends is considered first. Columns and struts are termed **"long"** or **"*short*"** depending on their proneness to buckling. If the strut is **"short"**, the applied forces will cause a compressive strain, which results in the shortening of the strut in the direction of the applied forces. Under incremental loading, this shortening continues until the column yields or **"squashes"**. However, if the strut is **"long"**, similar axial shortening is observed only at the initial stages of incremental loading. Thereafter, as the applied forces are increased in magnitude, the strut becomes **"unstable"** and develops a deformation in a direction normal to the loading axis and its axis is no longer straight. (See Fig.5.1). The strut is said to have "buckled".

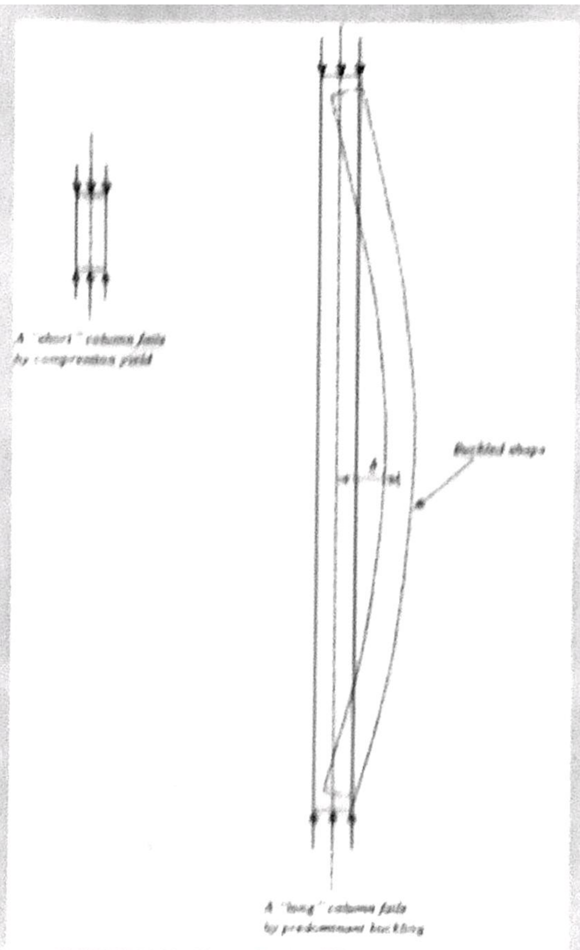


Fig 5.1 'short' vs 'long' columns

Short Columns **Long Columns**

Buckling behaviour is thus characterized by large deformations developed in a direction (or plane) normal to that of the loading that produces it. When the applied loading is increased, the buckling deformation also increases. Buckling occurs mainly in members subjected to compressive forces. If the member has high bending stiffness, its buckling resistance is high. Also, when the member length is increased, the buckling resistance is decreased. Thus the buckling resistance is high when the member is short or "**stocky**" (i.e. the member has a high bending stiffness and is short) conversely, the buckling resistance is low when the member is long or "***slender***"

Structural steel has high yield strength and ultimate strength compared with other construction materials. Hence compression members made of steel tend to be slender compared with reinforced concrete or prestressed concrete compression members. Buckling is of particular interest while employing slender steel members. Members fabricated from steel plating or sheeting and subjected to compressive stresses also experience local buckling of the plate elements. This chapter introduces buckling in the context of axially compressed struts and identifies the factors governing the buckling behaviour. Both global and local buckling is instability phenomena and should be avoided by an adequate margin of safety.

Traditionally, the design of compression members was based on Euler analysis of ideal columns which gives an upper band to the buckling load. However, practical columns are far from ideal and buckle at much lower loads. The first significant step in the design procedures for such columns was the use of Perry Robertsons curves. Modern codes advocate the use of multiple-column curves for design. Although these design procedures are more accurate in predicting the buckling load of practical columns, Euler's theory helps in the understanding of the behaviour of slender columns and is reviewed in the following sections.

5.4 Strength of compression members in practice

The highly idealized straight form assumed for the struts considered so far cannot be achieved in practice. Members are never perfectly straight and they can never be loaded exactly at the centroid of the cross section. Deviations from the ideal elastic plastic behaviour defined by Fig. 5 are encountered due to strain hardening at high strains and the absence of clearly defined yield point in some steel. Moreover, residual stresses locked-in during the process of rolling also provide an added complexity.

Thus the three components, which contribute to a reduction in the actual strength of columns (compared with the predictions from the "ideal" column curve) are

(i) Initial imperfection or initial bow.

(ii) Eccentricity of application of loads.

(iii) Residual stresses locked into the cross section.

5.4.1 The effect of initial out-of-straightness

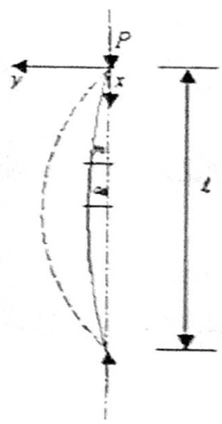


Fig 5.7 Pin -ended strut with initial imperfection

A pin-ended strut having an initial imperfection and acted upon by a gradually increasing axial load is shown in Fig 5.7. As soon as the load is applied, the member experiences a bending moment at every cross section, which in turn causes a bending deformation. For simplicity of calculations, it is usual to assume the initial shape of the column defined by

$$y_0 = a_0 \sin \frac{\pi x}{l} \qquad (5.8)$$

where a_o is the maximum imperfection at the centre, where $x = l / 2$. Other initial shapes are, of course, possible, but the half sine-wave assumed above corresponding to the lowest mode shape, represents the greatest influence on the actual behaviour, and hence is adequate.

Provided the material remains elastic, it is possible to show that the applied force, P, enhances the initial deflection at every point along the length of the column by a multiplier factor, given by

$$MF = \frac{1}{1 - \left(\frac{P}{P_{cr}}\right)} \qquad (5.9)$$

The deflection will tend to infinity, as P is tends to P_{cr} as shown by curve-A, in Fig. 5.8. However the column will fail at a lower load P_f when the deflection becomes large enough. The corresponding stress is denoted as f_f

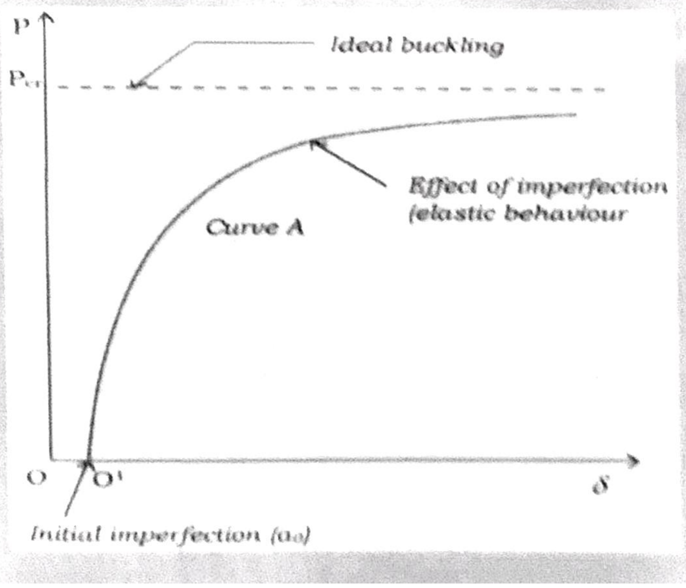


Fig 5.8 Theoretical and actual load deflection response of a strut with initial imperfection

If a large number of imperfect columns are tested to failure, and the data points representing the values of the mean stress at failure plotted against the slenderness (λ) values, the resulting lower bound curve would be similar to the curve shown in Fig. 5.9.

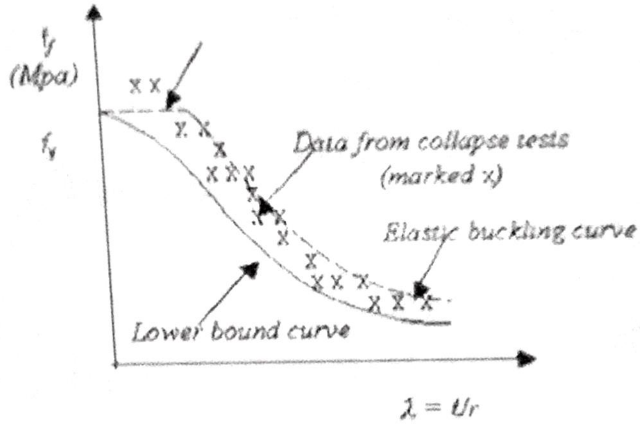


Fig 5.9 Strength curves for strut with initial imperfection

For very stocky members, the initial out of straightness – which is more of a function of length than of cross sectional dimensions – has a very negligible effect and the failure is at plastic squash load. For a very slender member, the lower bound curve is close to the elastic critical stress (f_{cr}) curve. At intermediate values of slenderness the effect of initial out of straightness is very marked and the lower bound curve is significantly below the f_y line and f_{cr} line.

5.4.2 The effect of eccentricity of applied loading

As has already been pointed out, it is impossible to ensure that the load is applied at the exact centroid of the column. Fig. 5.10 shows a straight column with a small eccentricity (e) in the applied loading. The applied load (P) induces a bending moment (P.e) at every cross section. This would cause the column to deflect laterally, in a manner similar to the initially deformed member discussed previously. Once again the greatest compressive stress will occur at the concave face of the column at a section midway along its length. The load-deflection response for purely elastic and elastic-plastic behaviour is similar to those described in Fig. 5.8 except that the deflection is zero at zero load.

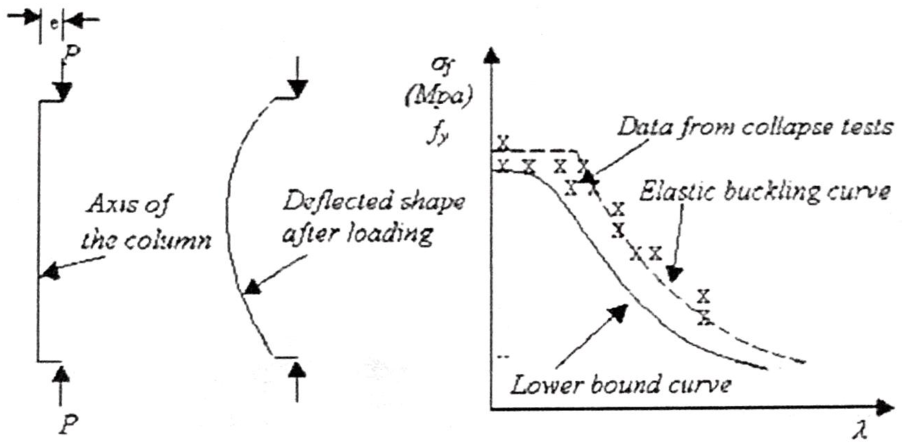


Fig 5.10 Strength curves for *eccentrically loaded columns*

The form of the lower bound strength curve obtained by allowing for eccentricity is shown in Fig. 5.10. The only difference between this curve and that given in Fig. 5.9 is that the load carrying capacity is reduced (for stocky members) even for low values of λ.

5.4.3 The effect of residual stress

As a consequence of the differential heating and cooling in the rolling and forming processes, there will always be inherent residual stresses. A simple explanation for this phenomenon follows. Consider a billet during the rolling process when it is shaped into an I section. As the hot billet shown in Fig. 5.11(a) is passed successively through a series of rollers, the shapes shown in 5.11(b), (c) and (d) are gradually obtained. The outstands (b-b) cool off earlier, before the thicker inner elements (a-a) cool down.

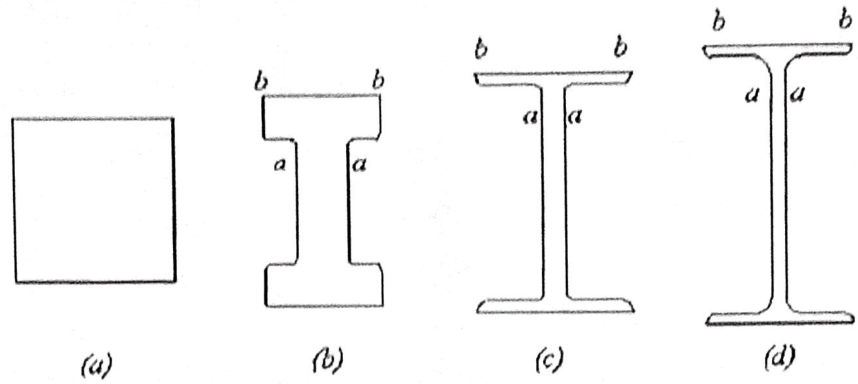


Fig 5.11 Various stages of rolling a steel grider

As one part of the cross section (b-b) cools off, it tends to shrink first but continues to remain an integral part of the rest of the cross section. Eventually the thicker element (a) also cools off and shrinks. As these elements remain composite with the edge elements, the differential shrinkage induces compression at the outer edges (b). But as the cross section is in equilibrium – these stresses have to be balanced by tensile stresses at inner location (a). These stress called residual stresses, can sometimes be very high and reach upto yield stress.

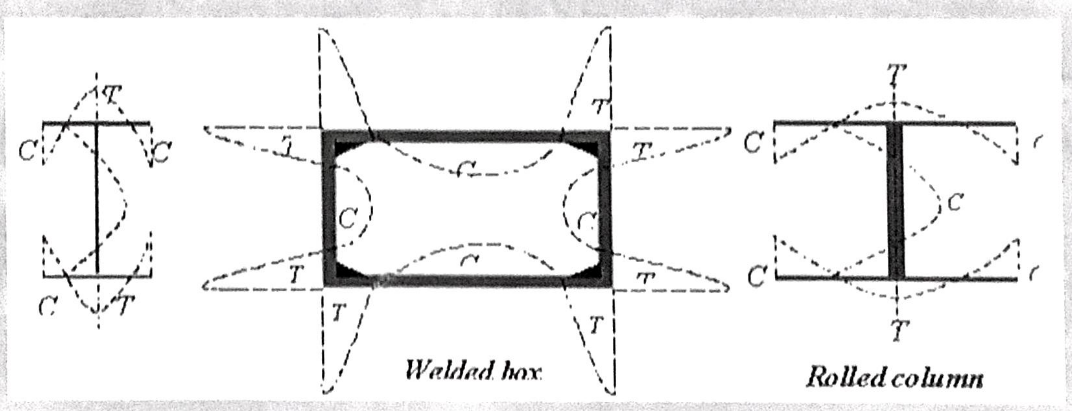


Fig. 5.12 Distribution of residual stresses

Consider a short compression member (called a "stub column", having a residual stress distribution as shown in Fig. 5.12. When this cross section is subjected to an applied uniform compressive stress (f_a) the stress distribution across the cross section can be obtained by superposing the applied stress over the residual stress f_r, provided the total stress nowhere reaches yield, the section continues to deform elastically. Under incremental loading, the flange tips will yield first when $[(f_a + f_r) = f_y]$. Under further loading, yielding will spread inwards and eventually the web will also yield. When $f_a = f_y$, the entire section will have yielded and the column will get squashed.

Only in a very stocky column (i.e. one with a very low slenderness) the residual stress causes premature yielding in the manner just described. The mean stress at failure will be f_y, i.e. failure load is not affected by the residual stress. A very slender strut will fail by buckling, i.e. $f_{cr} << f_y$. For struts having intermediate slenderness, the premature yielding at the tips reduces the effective bending stiffness of the column; in this case, the column will buckle elastically at a load below the elastic critical load and the plastic squash load. The column strength curve will thus be as shown in Fig. 5.13.

Notice the difference between the buckling strength and the plastic squash load is most pronounced when

$\lambda = l / r = \pi (E / f_y)^{1/2}$

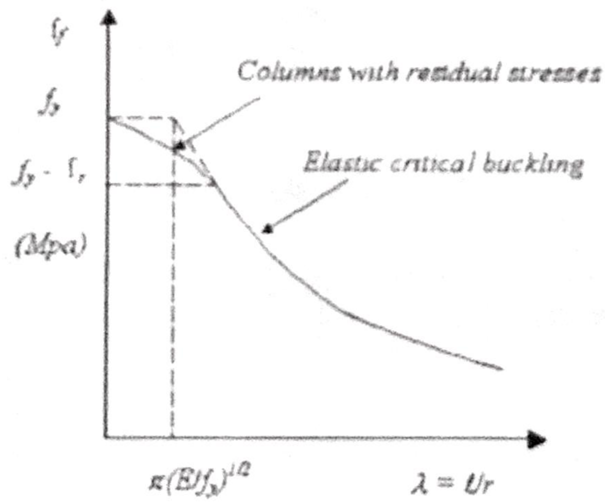


Fig 5.13 Strength curve for columns having residual stress

5.4.4 The effect of strain-hardening and the absence of clearly defined yield point

If the material of the column shows strain harderning after an yield platean, the onset of first yield will not be affected, but the collapse load may be increased. Designers tend to ignore the effect of strain hardening which in fact provides an additional margin of safety.

High strength steels generally have stress-strain curves without a clear yield point. At stresses above the limit of proportionality (f_p), the material behaviour is non linear and on unloading and reloading the material is linear-elastic. Most high strength structural steels have an ultimate stress beyond which the curve becomes more or less horizontal. Some steels do not have a plastic plateau and exhibit strain-hardening throughout the inelastic range. In such cases, the yield stress is generally taken as the 0.2% proof stress, for purposes of computation.

5.4.5 The effect of all features taken together

In practice, a loaded column may experience most, if not all, of the effects listed above i.e. out of straightness, eccentricity of loading, residual stresses and lack of clearly defined yield point and strain hardening occurring simultaneously. Only strain hardening tends to raise the column strengths, particularly at low slenderness values. All other effects lower the column strength values for all or part of the slenderness ratio range.

When all the effects are put together, the resulting column strength curve is generally of the form shown in Fig. 5.14. The beneficial effect of strain hardening at low slenderness values is generally more than adequate to provide compensation for any loss of strength due to small, accidental eccentricities in loading. Although the column strength can exceed the value obtained from the yield strength (f_y), for purposes of structural design, the column strength curve is generally considered as having a cut off at f_y, to avoid large plastic compressive deformation. Since it is impossible to quantify the variations in geometric imperfections, accidental eccentricity, residual stresses and material properties, it is impossible to calculate with certainty, the greatest reduction in strength they might produce in practice. Thus for design purposes, it may be impossible to draw a true lower bound column strength curve. A commonly employed method is to construct a curve on the basis of specified survival probability. (For example, over 98% of the columns to which the column curve relates, can be expected - on a statistical basis – to survive at applied loads equal to those given by the curve). All design codes provide column curves based on this philosophy. Thus a lower band curve (Fig 5.14) or a family of such curves is used in design.

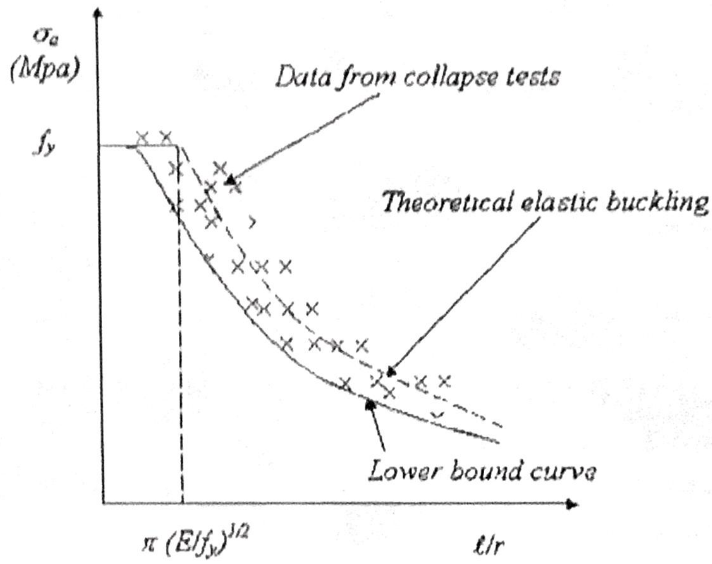


Fig. 5.14 Column strength curves for struts used in practice

5.9 Steps in the design of axially loaded columns

The procedure for the design of an axially compressed column is as follows:

(i) Assume a suitable trial section and classify the section in accordance with the classification in chapter.

(ii) Arrive at the effective length of the column by suitably considering the end conditions.

(iii) Calculate the slenderness ratios (λ values) in both minor and major axes direction and also calculate λ_o using the formula given below:

$$\lambda_0 = 0.2\pi\sqrt{E/f_y}$$

(iv) Calculate f_{cd} values along both major and minor axes from equation 12

(v) Compute the load that the compression member can resist (p_d=$A_e f_{cd}$)

(vi) Calculate the factored applied load and check whether the column is safe against the given loading. The most economical but safe section can be arrived at by trial and error, i.e. repeating the above process.

The following values are suggested for initial choice of members:

(i) Single angle size: 1/30 of the length of the strut (L / r ~ 150)

(ii) Double angle size: 1/35 of the length of strut (L / r ~ 100-120)

(iii) Circular hollow sections diameter = 1/40 length (L / r ~ 100)

5.11 Concluding remarks

The elastic buckling of an ideally straight column pin ended at both ends and subjected to axial compression was considered. The elastic buckling load was shown to be dependent on the slenderness ratio (/r) of the column. Factors affecting the column strengths (viz. initial imperfection, eccentricity of loading, residual stresses and lack of well-defined elastic limit) were all individually considered. Finally a generalized column strength curve (taking account of all these factors) has been suggested, as the basis of column design curves employed in Design Practices. The concept of **"effective length"** of the column has been described, which could be used as the basis of design of columns with differing boundary conditions.

The phenomenon of Elastic Torsional and Torsional-flexural buckling of a perfect column were discussed conceptually. The instability effects due to torsional buckling of slender sections are explained and discussed.

Design of columns using multiple column curves as given in the code; was discussed. Built-up fabricated members frequently employed (when rolled sections are found inadequate) were discussed in detail. Design guidance is provided for laced/battened columns. Steps in the design of axially loaded column were listed.