→ one of matrix method to determine the unknown forces.

Static indeterminacy :- without movement

Kinematic inditerminacy :- with movement.

STATIC EQUILIBRIUM EQUATIONS :- Based on Sir Isaac Newton's law of motion,

(i) Sum of all forces in any axis is zero.

$$\Sigma F_x = \Sigma F_y = \Sigma F_z = 0.$$

(ii) Sum of all moments above any axis is equal to zero.

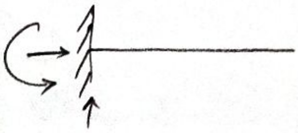
$$\Sigma M_x = \Sigma M_y = \Sigma M_z = 0.$$

DETERMINATE STRUCTURE :- Equations of static equilibrium ($\Sigma F_x = \Sigma F_y = \Sigma M = 0$) are sufficient to determine unknown forces and ~~members~~ moments in a member.

INDETERMINATE STRUCTURE :- Equations of static equilibrium ($\Sigma F_x = \Sigma F_y = \Sigma M = 0$) are not sufficient to determine unknown forces and moments in a member.

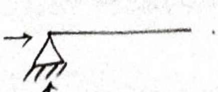
TYPES OF SUPPORT :-

(i) Fixed support



No. of reactions = 3.

(2) Hinged Support.



No. of reactions = 2.

(3) Roller support.

No. of reactions = 1.

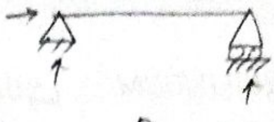
(4) Free end

No. of reactions = 0.

DEGREE OF STATIC INDETERMINACY (D_s) :-

D_s = Total unknowns − Equilibrium equations.

(i) Determinate and Stable.



$D_s = 0$

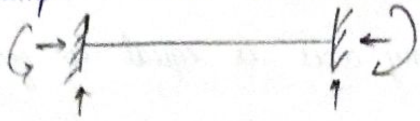
[(ii)] Example :- Simply supported beam.

$D_s = 3-3$
$= 0.$

R = unknown reactions.
r = equations

(ii) $D_s > 0$, Indeterminate and Stable.

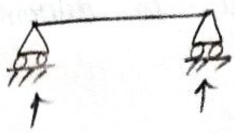
Example :- Fixed beam.



$D_s = 6-3$
$= 3.$

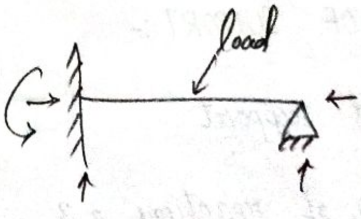
(iii) $D_s < 0$, Determinate and Stable.

Example :- Both ends are roller.



$D_s = 2-3$
$= -1.$

Q. Find out the indeterminacy of a given beam of both vertical and general loading conditions :
(upward & downward direction loads)



General loading :-

$R = 5,\ r = 3\ (\Sigma V = 0, \Sigma H = 0, \Sigma M = 0)$

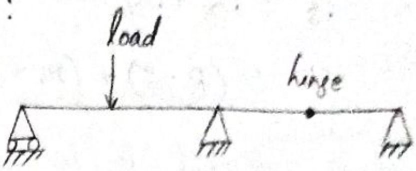
$D_s = 5-3$
$= 2.$

Vertical loading :-

$R = 3,\ \underline{r = 2}\ (\Sigma V = 0, \Sigma M = 0)$

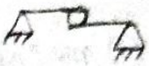
$D_s = 3-2$
$= 1.$

Q2. Determine the degree of indeterminacy of a given beam for general & vertical loading condition :



$D_s = R - r - h.$

General loading :- $R = 5, r = 3.$



$D_s = 5 - 3 - 1$

$= 1.$

$h = m - 1.$
No. of members joint at hinge joint.

~~Vertical loading :- $R = \cancel{5}\,3,\ r = 2.$~~

~~$D_s = \cancel{5}\,3 - 2 - 1.$~~

~~$= \cancel{2}\,0.$~~

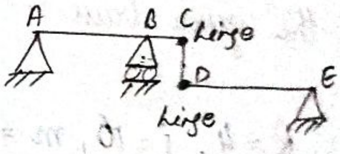
Vertical loading :- $R = 3, r = 2, h = 1.$

$D_s = 3 - 2 - 1$

$= 0.$

Date:- 24/03/2022

Q3. Determine the static indeterminacy of a given beam :-



General loading :- $= (m-1) + (m-1) = 2$

$R = 5,\ r = 3,\ h = (2-1) + (2-1) = 2.$

$D_s = R - r - h.$

$= 5 - 3 - 2$

$= 0.$ (Determinate & Stable).

Vertical loading :- $R = 3,\ r = 2,\ h = 2.$

$D_s = 3 - 2 - 2$

$= -1.$

DETERMINACY OF TRUSSES :- $m = 2j - 3$.

(Pin jointed frames)

$$D_s = \underset{\text{external}}{D_{se}} + \underset{\text{internal}}{D_{si}} \qquad j: \text{joints}.$$

$$= (R - 3) + (m - 2j + 3).$$

If $m = 0$, sufficient to be able to carry loads.

(i) $m = 2j - 3 \rightarrow$ Perfect truss.

(ii) $m < 2j - 3 \rightarrow$ Deficient truss.

(iii) $m > 2j - 3 \rightarrow$ Redundant truss.

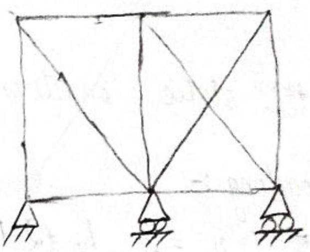
There are two types of indeterminancy

(i) Internal Indeterminancy.

(ii) External Indeterminancy.

$$\boxed{D_s = R + m - 2j}$$

Q. Find the total static indeterminancy of the given truss :-



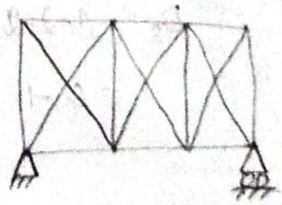
$R = 4$, $j = 6$, $m = 10$

$D_s = 4 + 10 - 12$.

$= 2$.

Q. Find the static indeterminacy of the given truss :-

$R = 3$, $j = 8$, $m = 16$.



$D_s = R - 2j + m$.

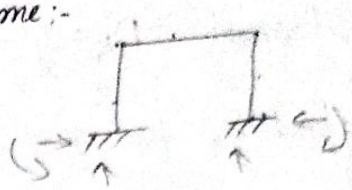
$= 3 - (2 \times 8) + 16$.

$= 3$.

PLANE FRAMES :- (~~Truss~~) Rigid jointed frames.

$D_s = 3m + R - 3j$

Q. Find the static indeterminacy of a given frame :-



$m = 3,\ R = 6,\ j = 4.$

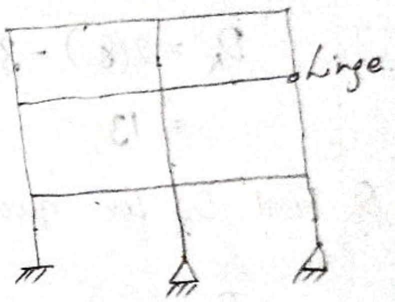
$D_s = (3\times3) + 6 - (3\times4).$

$= 9 + 6 - 12.$

$= 3.$

Date :- 25/03/2022
FRIDAY.

Q. Find the static indeterminacy of the given frame :-



$D_s = \cancel{m}\ R - 3j + 3m.$

$R = 7,\ \cancel{P}\ m = 15,\ j = 12,\ h = 3 - 1 = 2.$

$D_s = 7 - (3\times12) + (15)\cdot3 - 2$

$= \cancel{22}\ 52 - 38$

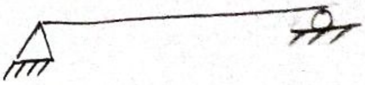
$$\begin{array}{r} 52 \\ 38 \\ \hline 14 \end{array}$$

$= 14$

KINEMATIC INDETERMINANCY :-

For beams, $D_K = 3j - R.$

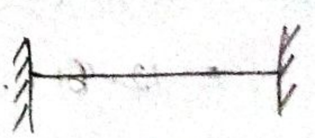
Q. Find D_K for the given beam :-



$D_K = 3(2) - 3.$

$= 3.$

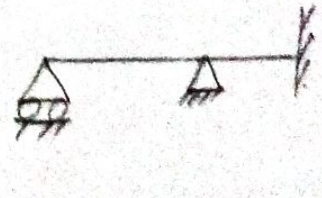
Q. Find D_K for fixed beam :-



$D_K = 3(2) - 6.$

$= 0.$

Q. Find D_K for fixed beam :-



$D_K = 3(3) - 6.$

$= 3.$

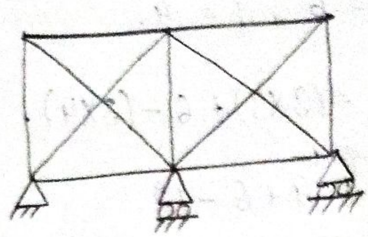
Beam $\quad D_K = 3j - R \qquad D_S = R - r.$

Truss. $\quad D_K = 2j - R + h \qquad \cancel{D_S = m\,2j + R}$

$D_S = m - 2j + R.$

Frames. $\quad D_K = 3j - (m+R) + h \qquad D_S = 3j - (m+R) - h.$

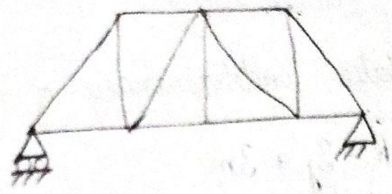
Q. Find D_K for given truss :-



$D_K = 2j - R.$

$= 2(6) - 4.$

$= 8.$

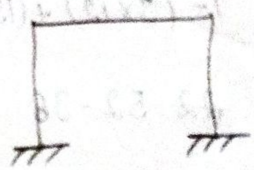


Q. Find D_K for given truss :-

$D_K = 2(8) - 3.$

$= 13.$

Q. Find D_K for given frame :-

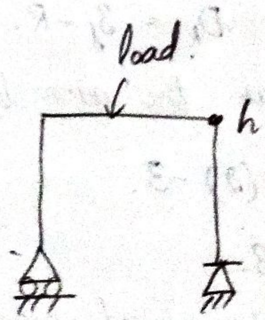


$D_K = 3j - (m+R) + h$

$= 3(4) - (3+6)$

$= 12 - 9$

$= 3.$



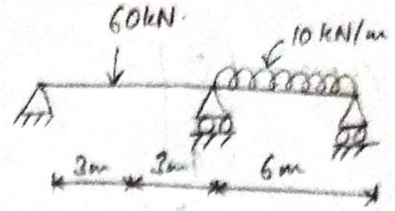
Q. Find D_K for given frame :-

$D_K = 3(4) - (3+3) + 1.$

$= 12 - 6 + 1.$

$= 7.$

Q. Analyse the continuous beam by flexibility matrix method :-



Ans:- Step 1:- Static Indeterminacy

$R = 3,\ r = 2.$

$D_s = R - r$

$= 3-2$

$= 1.$

Statically Indeterminate to 1°.

M_B is considered as redundant.

Step 2:- Fixed End Moment.

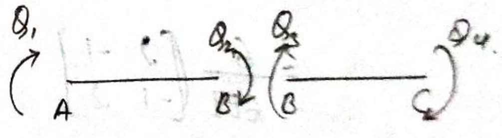
$MF_{AB} = \frac{-wl}{8} = -\frac{60 \times 6}{8} = -45\ kNm.$

$MF_{BA} = \frac{wl}{8} = \frac{60 \times 6}{8} = 45\ kNm.$

$MF_{BC} = -\frac{wl^2}{12} = -\frac{10 \times 6 \times 6}{12} = -30\ kNm.$

$MF_{CB} = \frac{wl^2}{12} = \frac{10 \times 6 \times 6}{12} = 30\ kNm.$

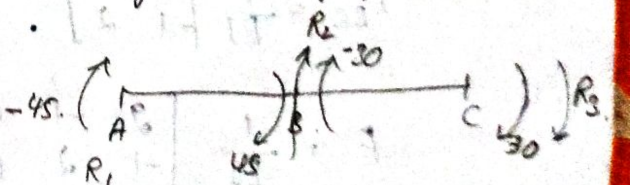
Step 3 :- Internal Member Force Matrix.



$$Q = \begin{bmatrix} -45 \\ 45 \\ -30 \\ 30 \end{bmatrix}$$

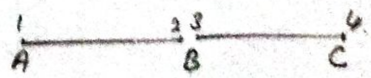
Step 4:- Equal and Joint Moment (moment acting at every joint).

$$R = \begin{bmatrix} 45 \\ -15 \\ -30 \end{bmatrix}.$$



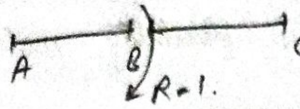
Step 5:- Force transformation matrix.

Apply unit force at every joints.



$$\{bR_1\} = \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$\{bR_2\} = \begin{Bmatrix}0\\1\\0\\0\end{Bmatrix}$.



$(bR_3) = \begin{Bmatrix}0\\0\\0\\1\end{Bmatrix}$

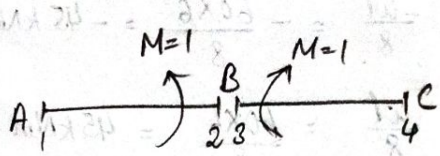
$\cancel{\{bR_4\} = \begin{Bmatrix}0\\0\\0\\1\end{Bmatrix}}$.

$\{bR\} = \begin{Bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 0\\0 & 0 & 1\end{Bmatrix}$.

Intermediate → some moments.
Simply supported ends having no moments

6. Moment Transformation Matrix (bx)

$bx = \begin{Bmatrix}0\\-1\\1\\0\end{Bmatrix}$ -1 → anti-clockwise. 1 → clockwise.



7. Force matrix :- $F = \frac{l}{6EI}\begin{bmatrix}2 & -1\\-1 & 2\end{bmatrix}$.

$F_{AB} = \frac{l}{6EI}\begin{bmatrix}2 & -1\\-1 & 2\end{bmatrix}$.

$= \frac{6}{6EI}\begin{bmatrix}2 & -1\\-1 & 2\end{bmatrix}$

$= \frac{1}{EI}\begin{bmatrix}2 & -1\\-1 & 2\end{bmatrix}$.

$F_{BC} = \frac{1}{EI}\begin{bmatrix}2 & -1\\-1 & 2\end{bmatrix}$.

$$F = \frac{1}{EI}\begin{array}{c c}
 & \begin{matrix}1 & 2 & 3 & 4\end{matrix}\\
\begin{matrix}1\\2\\3\\4\end{matrix} & \begin{bmatrix}2 & -1 & 0 & 0\\-1 & 2 & 0 & 0\\0 & 0 & 2 & -1\\0 & 0 & -1 & 2\end{bmatrix}
\end{array}$$

8. Flexibility Matrix :-

$$F_{xx} = [b_x]^T [F][b_x].$$

$$= \{0 \ -1 \ 1 \ 0\} \frac{1}{EI} \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}_{(4\times4)} \begin{Bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{Bmatrix}_{(4\times1)}.$$

$$= \{0 \ -1 \ 1 \ 0\} \frac{1}{EI} \begin{bmatrix} 0+1+0+0 \\ 0-2+0+0 \\ 0+0+2+0 \\ 0+0-1+0 \end{bmatrix}_{4\times4}.$$

$$= \{0 \ -1 \ 1 \ 0\}_{(1\times4)} \frac{1}{EI} \begin{bmatrix} 1 \\ -2 \\ 2 \\ -1 \end{bmatrix}_{(4\times1)}.$$

$$= \frac{1}{EI} \times 4.$$

$$= \frac{4}{EI}.$$

9. Redundant Force Matrix :-

$$X = -[F_{xx}]^T [F_{xR}][R].$$

$$[F_{xR}] = [b_x]^T [F][b_R].$$

$$= [0 \ -1 \ 1 \ 0] \times \frac{1}{EI} \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}_{(4\times4)} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{(4\times3)}.$$

$$= [0 \ -1 \ 1 \ 0] \begin{bmatrix} 2+0+0+0 & 0-1+0+0 & 0+0+0+0 \\ -1+0+0+0 & 0+2+0+0 & 0+0+0+0 \\ 0 & 0 & -1 \\ 0 & 0 & 2 \end{bmatrix} \frac{1}{EI}.$$

$$= [0 \ -1 \ 1 \ 0] \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 2 \end{bmatrix}_{(4\times3)} \frac{1}{EI}$$

$$= \frac{1}{EI} [1 \ -2 \ -1].$$

$$= \cancel{\frac{1}{EI} \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}}.$$

$$X = -[F_{xx}]^T [F_{xR}][R]. \qquad [1\ -2\ -1]\begin{bmatrix}45\\-15\\30\end{bmatrix}$$

$$= -\underset{(3\times1)}{\begin{bmatrix}45\\-15\\-30\end{bmatrix}} \times \frac{1}{EI}\underset{(1\times3)}{[1\ \ -2\ \ -1]} \times \frac{EI}{4}.$$

$$= [45+30+30] \times \frac{1}{4}.$$

$$= -\frac{105}{4}$$

$$= -26.25.$$

Step 10 :- Internal member force.

$$Q = [b_R][R] + [b_x][x] + FEM.$$

$$= \underset{(4\times3)}{\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\\0&0&1\end{bmatrix}}\underset{(3\times1)}{\begin{bmatrix}45\\-15\\-30\end{bmatrix}} + \underset{(4\times1)}{\begin{bmatrix}0\\-1\\1\\0\end{bmatrix}}\underset{(1\times1)}{[(-26.25)]} + \begin{bmatrix}-45\\45\\-30\\30\end{bmatrix}.$$

$$= \begin{bmatrix}45\\-15\\0\\-30\end{bmatrix} + \begin{bmatrix}0\\26.25\\-26.25\\0\end{bmatrix} + \begin{bmatrix}-45\\45\\-30\\30\end{bmatrix}.$$

$$= \begin{bmatrix}0\\56.25\\-56.25\\0\end{bmatrix} (56.25)$$

Step 11:- Final Moments.

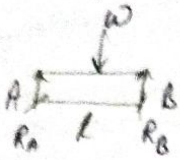
$$\begin{bmatrix}M_{AB}\\M_{BA}\\M_{BC}\\M_{CB}\end{bmatrix} = \begin{bmatrix}0\\56.25\\-56.25\\0\end{bmatrix}$$

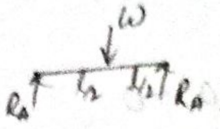
Date:- 29-08-2022.

Step 12 :- Maximum Bending Moment (creates a moment).

Parabolic shape for udl load

For point load, maximum B.M. = $\frac{wl}{4}$





$\Sigma V = 0$

$R_A + R_B = W$

$\Sigma M_B = 0.$

$R_A \times l - W \times l/2 = 0$

$R_A = W/2.$

$R_B + W/2 = W$

$R_B = W/2.$

$\Sigma M_B = 0.$

$\Sigma M_B = \frac{W}{2} \times l/2$

$= \frac{Wl}{4}$

$M_{AB} = \frac{wl}{4} = \frac{60 \times 6}{4} = 90\,kNm.$

$M_{BC} = \frac{wl^2}{8} = \frac{10 \times 6^2}{4} = 45\,kNm.$

Step 13 :- Shear Force.

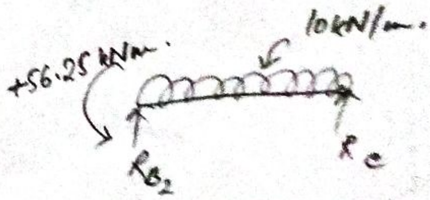
For udl,
load x distance = total intensity of load

$\Sigma V = 0.$

$R_A + R_B + R_C - 60 = 0.$

$R_A + R_B + R_C = 60 + (6 \times 10).$

$R_A + R_B + R_C = 120\,kN.$



$\Sigma V = 0$

$R_{B2} + R_C = 10 \times 6.$

$R_{B2} + R_C = 60$

$\Sigma M_B = 0.$

$R_{BC} \times 6 + 56.25 - (10 \times 6 \times 6/2) = 0.$

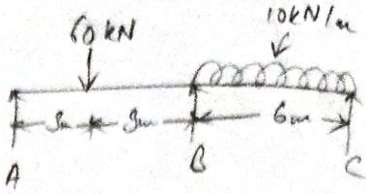
$6R_{BC} + 56.25 - 180 = 0.$

$R_{BC} = 20.625\,kN.$

$R_{B2} = 60 - 20.63.$

$= 39.37\,kN.$

$\Sigma M_A = 0.$



$R_C \times 12 + (R_B \times 6) - (10 \times 6 \times (6/2 + 6)) - (60 \times 3) = 0.$

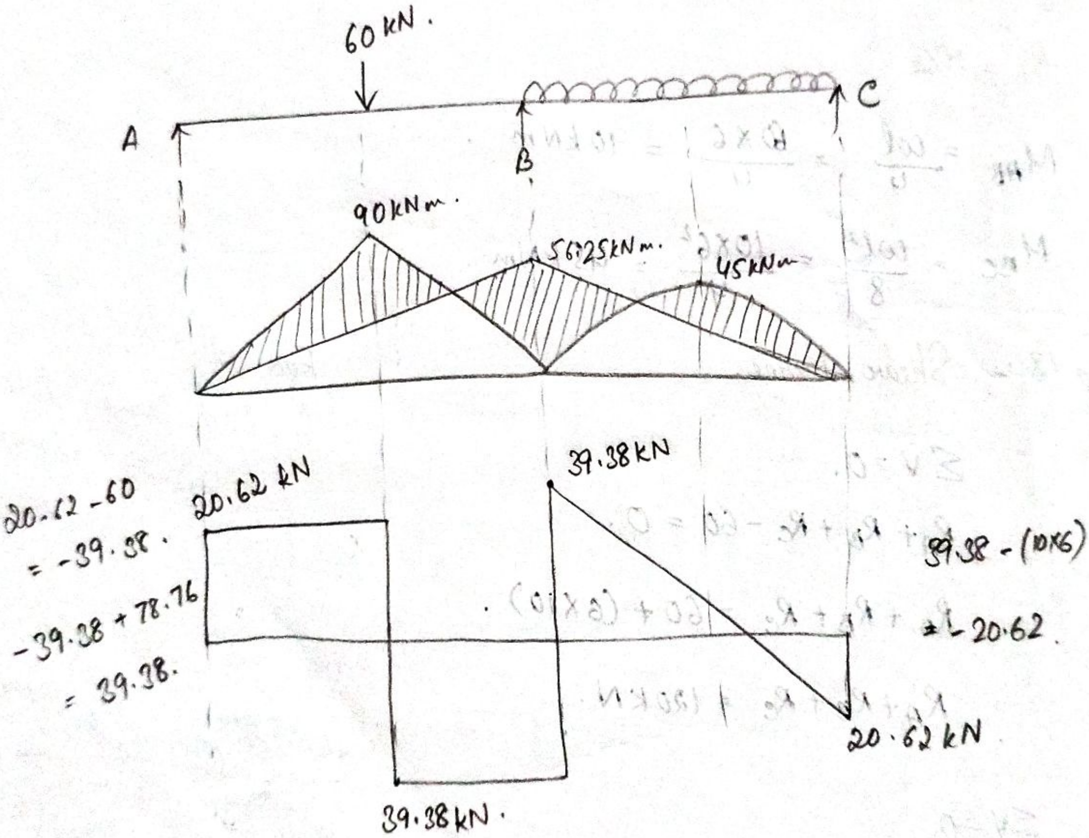
$20.62 \times 12 + 6R_B - 540 - 180 = 0$

$247.44 + 6R_B = 720$

$R_B = 78.76 \text{ kN}.$

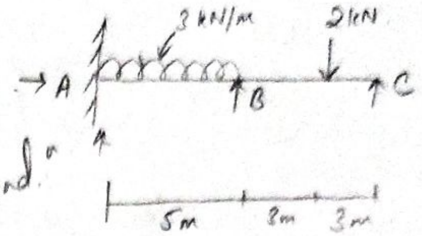
$R_A = 120 - 78.76 - 20.62.$

$= 20.62 \text{ kN}.$



Date:- 4/04/2022.

Q2 Analyse the continuous beam as shown the figure by flexibility matrix method :-



Vertical loading condⁿ

Step 1 :- $D_S = R - r$

$= 4 - 2$

$= 2.$

$R = 4, \; r = 2$

M_A and M_B are identified as redundance

Step 2 :- Fixed End Moment

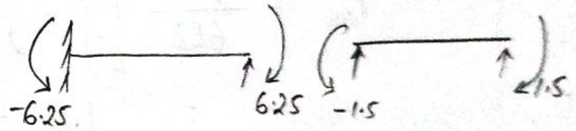
$$MF_{AB} = -\frac{wl^2}{12} = -\frac{3\times5^2}{12} = -6.25 \text{ kNm}.$$

$$MF_{BA} = \frac{wl^2}{12} = \frac{3\times5^2}{12} = 6.25 \text{ kNm}.$$

$$MF_{BC} = -\frac{wl}{8} = -\frac{2\times6}{8} = -1.5 \text{ kNm}.$$

$$MF_{CB} = \frac{wl}{8} = \frac{2\times6}{8} = 1.5 \text{ kNm}.$$

Step 3 :- Internal member force action :-

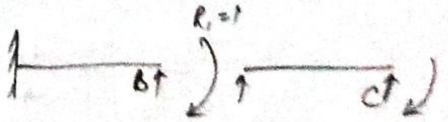


$$Q = \begin{Bmatrix} -6.25 \\ 6.25 \\ -1.5 \\ 1.5 \end{Bmatrix}$$

Step 4 :- Equivalent joint moment :-

$$R = \begin{Bmatrix} R_1 \\ R_2 \end{Bmatrix} = \begin{bmatrix} -4.75 \\ -1.5 \end{bmatrix}.$$

Step 5 :- Force transformation matrix :-

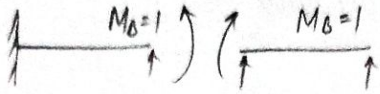


$$bR_1 = \begin{Bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{Bmatrix} \qquad bR_2 = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{Bmatrix}.$$

$$bR = \begin{Bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{Bmatrix}.$$

Step 6 :- Moment transformation matrix :

$$bR_1 = \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{Bmatrix}.$$



$$bR_2 = \begin{Bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{Bmatrix}.$$

$$b_X = \begin{Bmatrix} 1 & 0 \\ 0 & -1 \\ 0 & 0 \\ 0 & 0 \end{Bmatrix}.$$

Step 7 :- Force Matrix.

$$F_{AB} = \frac{l}{6EI} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}.$$

$$= \frac{5}{6EI} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}.$$

$$= \frac{1}{EI} \begin{bmatrix} 1.66 & -0.83 \\ -0.83 & 1.66 \end{bmatrix}.$$

$$F_{BC} = \frac{l}{6EI} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}.$$

$$= \frac{6}{6EI} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}.$$

$$F = \frac{1}{EI} \begin{bmatrix} 1.66 & -0.83 & 0 & 0 \\ -0.83 & 1.66 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

Step 8 :- Flexibility Matrix

$$[F_{xx}] = [b_x]^T [F] [b_x]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix} \frac{1}{EI} \begin{bmatrix} 1.66 & -0.83 & 0 & 0 \\ -0.83 & 1.66 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$[F_{xx}] = \frac{1}{EI} \begin{bmatrix} 1.66 & 0.83 \\ 0.83 & 3.66 \end{bmatrix}$$

$$[F_{xx}]^{-1} = |A| = 6.07 - 0.68$$

$$= 5.39$$

$$[F_{xx}]^{-1} = \frac{EI}{5.39} \begin{bmatrix} 3.66 & -0.83 \\ -0.83 & 1.66 \end{bmatrix}$$

Step 9 :- Redundant force Matrix

$$[X] = -[F_{xx}]^{-1} [F_{xR}] [R]$$

$$[F_{xR}] = [b_x]^T [F] [b_F]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix} \frac{1}{EI} \begin{bmatrix} 1.66 & -0.83 & 0 & 0 \\ -0.83 & 1.66 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \frac{1}{EI} \begin{bmatrix} -0.83 & 0 \\ -1.66 & -1 \end{bmatrix}$$

$$[X] = \begin{bmatrix} -1.22 \\ -2.26 \end{bmatrix}$$

Step 10 :- Internal member force :-

$$[Q] = [b_R][R] + [b_x][X] + FEM$$

$$= \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -4.75 \\ -1.5 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -1.22 \\ -2.26 \end{bmatrix} + \begin{bmatrix} -6.25 \\ 6.25 \\ -1.5 \\ 1.5 \end{bmatrix} = \begin{bmatrix} -7.47 \\ 3.7 \\ -3.7 \\ 0 \end{bmatrix}$$

Step 11 :- Final Moment.

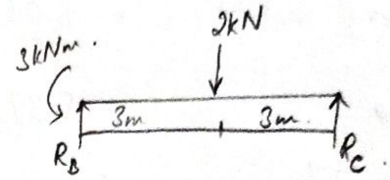
$$\begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \end{bmatrix} = \begin{bmatrix} -7.47 \\ 3.7 \\ -3.7 \\ 0 \end{bmatrix}$$

Step 12 :- Maximum Bending Moment.

$$M_{AB} = \frac{wl^2}{8} = \frac{3 \times 5^2}{8} = 9.37 \text{ kNm}.$$

$$M_{BC} = \frac{wl}{4} = \frac{2 \times 6}{4} = 3 \text{ kNm}.$$

Step 13 :- Shear Force.



$\Sigma V = 0$

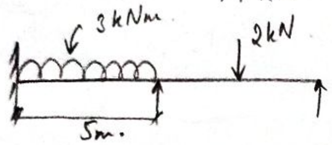
$R_A + R_B + R_C = (3 \times 2) + 5$

$= 17 \text{ kN}.$

Span BC :-

$R_C \times 6 = (2 \times 3) - 3.7.$

$R_C = 0.38 \text{ kN}.$



Span ABC :-

$R_C \times 11 + (R_B \times 5) = (2 \times 8) + (3 \times 5 \times 5/2).$

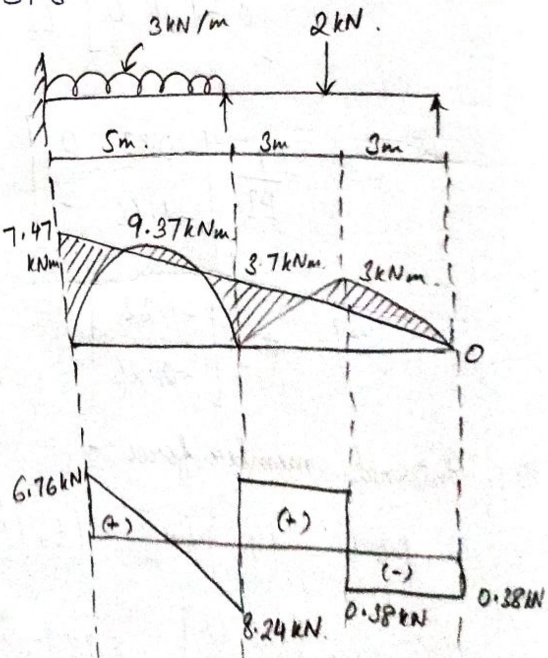
$(11 \times 0.38) + (R_B \times 5) = 16 + 37.5$

$R_B = 9.86 \text{ kN}.$

$R_A + R_B + R_C = 17.$

$R_A + 9.86 + 0.38 = 17.$

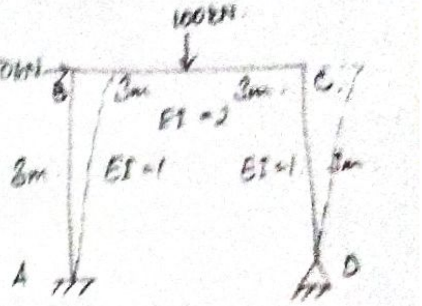
$R_A = 6.76 \text{ kN}.$



Date:- 5/04/2022

Q3. Analyze the frame as shown in fig. by flexibility matrix method :-

Step 1:- Static Indeterminacy.



(General loading condition)

$$D_s = R - r$$
$$= 5 - 3$$
$$= 2$$

D as redundant
as it has two forces.

Support D is selected as an redundant.

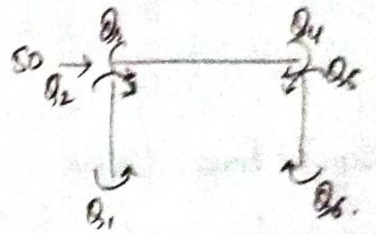
Step 2:- Fixed End Moments.

$$MF_{AB} = MF_{BA} = MF_{CD} = MF_{CD} = 0$$

$$MF_{BC} = -\frac{wl}{8} = -\frac{100\times6}{8} = -75\,kNm$$

$$MF_{CB} = \frac{wl}{8} = \frac{100\times6}{8} = 75\,kNm$$

Step 3:- Internal member force action :-



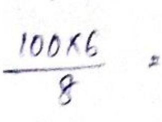
$$Q = \cancel{\begin{Bmatrix}50\\75\\-75\end{Bmatrix}} \quad \begin{Bmatrix}0\\0\\-75\\75\\0\\0\end{Bmatrix}$$

Step 4:- ~~Equivalent~~ Equal & joint moment.

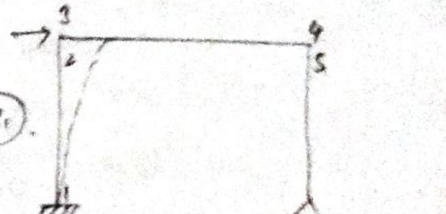
$$R = \begin{Bmatrix}50\\75\\-75\end{Bmatrix}$$

Step 5:- Force transformation matrix.

$$\{bR_1\} = \begin{Bmatrix}3\\0\\0\\0\\0\\0\end{Bmatrix}$$



$$\{bR_2\} = \begin{Bmatrix}0\\0\\1\\0\\0\\0\end{Bmatrix}$$



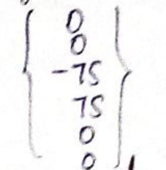
(bR_1)

moment will be created at fix end moment.

pin jointed not created.

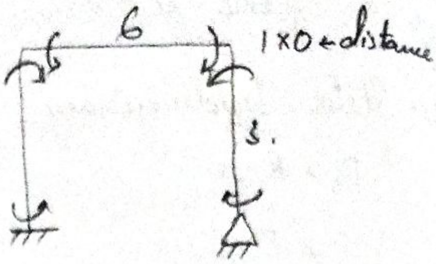
Moment - F x distance

$$\{bR_3\} = \begin{Bmatrix}0\\0\\0\\1\\0\\0\end{Bmatrix}$$

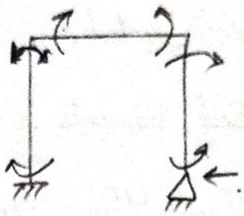


$$\{bR\} = \begin{bmatrix}3 & 0 & 0\\0 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$$

Step 6 :- Moment transformation matrix :-



$$bx_1 = \begin{Bmatrix} -6 \\ 6 \\ 6 \\ 0 \\ 0 \\ 0 \end{Bmatrix}.$$



$$bx_2 = \begin{Bmatrix} 0 \\ -3 \\ 3 \\ -3 \\ 3 \\ 0 \end{Bmatrix}$$

$$bx = \begin{Bmatrix} -6 & 0 \\ 6 & -3 \\ -6 & 3 \\ 0 & -3 \\ 0 & 3 \\ 0 & 0 \end{Bmatrix}.$$

Step 7 :- Force Matrix .

$$F_{AB} = \frac{l}{6EI}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}.$$

$$= \frac{3}{6EI}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}.$$

$$= \frac{3}{6\times 1}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix}.$$

$$F_{BC} = \frac{l}{6EI}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}.$$

$$= \frac{6}{6\times 2}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix}.$$

$$F_{CD} = \frac{3}{6\times 1}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix}.$$

$$F = \begin{bmatrix} 1 & -0.5 & 0 & 0 & 0 & 0 \\ -0.5 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -0.5 & 0 & 0 \\ 0 & 0 & -0.5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -0.5 \\ 0 & 0 & 0 & 0 & -0.5 & 1 \end{bmatrix}.$$

F_x Step 8 :- Flexibility Matrix.

$$F_{xx} = [b_x]^T [F] [b_x].$$

$$= \begin{bmatrix} -6 & 6 & -6 & 0 & 0 & 0 \\ 0 & -3 & 3 & -3 & 3 & 0 \end{bmatrix} \underset{(6\times6)}{\begin{bmatrix} 1 & -0.5 & 0 & 0 & 0 & 0 \\ -0.5 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -0.5 & 0 & 0 \\ 0 & 0 & -0.5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -0.5 \\ 0 & 0 & 0 & 0 & -0.5 & 1 \end{bmatrix}} \underset{(6\times2)}{\begin{bmatrix} -6 & 0 \\ 6 & -3 \\ -6 & 3 \\ 0 & -3 \\ 0 & 3 \\ 0 & 0 \end{bmatrix}}$$

$$= \underset{(2\times6)}{\begin{bmatrix} -6 & 6 & -6 & 0 & 0 & 0 \\ 0 & -3 & 3 & -3 & 3 & 0 \end{bmatrix}} \begin{bmatrix} -6-3. & 4.5. \\ 3+6 & -3. \\ -6. & 3+1.5. \\ \cancel{0}3. & -3-1.5 \\ 0. & \cancel{1.5}3. \\ 0 & -1.5 \end{bmatrix}.$$

$$= \begin{bmatrix} -6 & 6 & -6 & 0 & 0 & 0 \\ 0 & -3 & 3 & -3 & 3 & 0 \end{bmatrix} \cancel{\begin{bmatrix} -9 & 1.5 \\ 9. & -3. \\ -6. & 4.5 \\ 0 & -4.5 \\ 0 & -1.5 \\ 0 & \end{bmatrix}} \begin{bmatrix} -9 & 1.5. \\ 9 & -3. \\ -6 & 4.5 \\ 3 & -4.5 \\ 0 & 3. \\ 0 & -1.5 \end{bmatrix}$$

$$= \cancel{\begin{bmatrix} +54+54+\overset{36}{84} & -9-18-27. \\ -27+18 & 9+13.5+93.5+9 \end{bmatrix}}.$$

$$\longrightarrow \quad = \begin{bmatrix} 54+54+36. & -9-18-27. \\ -27-18-9 & 9+13.5+9+13.5 \end{bmatrix}$$

$$= \begin{bmatrix} 144 & -54 \\ -54 & 45 \end{bmatrix}.$$

Step 4 :-

$$[F_{xx}] = \begin{bmatrix} 144 & -54 \\ -54 & 45 \end{bmatrix}.$$

$$[A^{-1}] = \frac{1}{|A|}(\text{adj } A).$$

$$[F_{xx}]^{-1} = \underbrace{\frac{1}{3564}}_{=0.00028} \begin{bmatrix} 45 & 54 \\ 54 & 144 \end{bmatrix} = \begin{bmatrix} 0.012 & 0.015 \\ 0.015 & 0.040 \end{bmatrix}.$$

Step 9 :- Flexibility matrix).

$$[F_{xR}] = (b_x)^T (F)(b_R).$$

$$= \begin{bmatrix} -6 & 6 & -6 & 0 & 0 & 0 \\ 0 & -3 & 3 & -3 & 3 & 0 \end{bmatrix} \begin{bmatrix} 1 & -0.5 & 0 & 0 & 0 & 0 \\ -0.5 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -0.5 & 0 & 0 \\ 0 & 0 & -0.5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -0.5 \\ 0 & 0 & 0 & 0 & -0.5 & 1 \end{bmatrix}_{(6\times6)} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{(6\times3)}$$

$$= \begin{bmatrix} -6 & 6 & -6 & 0 & 0 & 0 \\ 0 & -3 & 3 & -3 & 3 & 0 \end{bmatrix}_{(2\times6)} \begin{bmatrix} 3 & 0 & 0 \\ -1.5 & 0 & 0 \\ 0 & 1 & -0.5 \\ 0 & -0.5 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{(6\times3)}$$

$$= \begin{bmatrix} -18 & -18 & 0 \\ 0 & 9+1.5 & -3 \end{bmatrix}.$$

$$= \begin{bmatrix} -18 & -18 & 0 \\ 0 & 10.5 & -3 \end{bmatrix}.$$

$$= \begin{bmatrix} -18-9 & -6 & 3 \\ 13.5 & 3+1.5 & -1.5-3 \end{bmatrix}$$

$$= \begin{bmatrix} -27 & -6 & 3 \\ 4.5 & 45 & -4.5 \end{bmatrix}.$$

Step 10:- Internal member force.

$$[Q_i] = [b_R][R] + [b_x][x] + FEM.$$

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 50 \\ 75 \\ -75 \end{bmatrix} + \begin{bmatrix} -6 & 0 \\ 6 & -3 \\ -6 & 3 \\ 0 & -3 \\ 0 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 10.8 \\ -5.63 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -75 \\ 75 \\ 0 \\ 0 \end{bmatrix}$$

$$[X] = -[F_{XX}]^{-1}[F_{XR}][R]$$

$$= -\begin{bmatrix} 0.012 & 0.015 \\ 0.015 & 0.040 \end{bmatrix} \underset{(2\times3)}{\begin{bmatrix} -27 & -6 & 3 \\ 45 & 4.5 & -4.5 \end{bmatrix}} \underset{(3\times1)}{\begin{bmatrix} 50 \\ 75 \\ -75 \end{bmatrix}}.$$

$$= -\begin{bmatrix} 0.012 & 0.015 \\ 0.015 & 0.040 \end{bmatrix} \begin{bmatrix} -1350-450-225. \\ 225+337.5+337.5 \end{bmatrix}.$$

$$= -\underset{(2\times2)}{\begin{bmatrix} 0.012 & 0.015 \\ 0.015 & 0.040 \end{bmatrix}} \underset{(2\times1)}{\begin{bmatrix} -2025. \\ 900 \end{bmatrix}}$$

$$= -\begin{bmatrix} -24.3+13.5 \\ -30.37+36. \end{bmatrix}.$$

$$(X) = \begin{bmatrix} 10.8 \\ -5.63 \end{bmatrix}.$$

10. Internal member force :-

$$[Q] = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 50 \\ 75 \\ -75 \end{bmatrix} + \begin{bmatrix} -6 & 0 \\ 6 & -3 \\ -6 & 3 \\ 0 & -3 \\ 0 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 10.8 \\ -5.63 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -75 \\ 75 \\ 0 \\ 0 \end{bmatrix}.$$

$$= \begin{bmatrix} 150 \\ 0 \\ 75 \\ -75 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -64.8 \\ +64.8+16.89 \\ -64.8-16.89 \\ 16.89 \\ -16.89 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -75 \\ 75 \\ 0 \\ 0 \end{bmatrix}$$

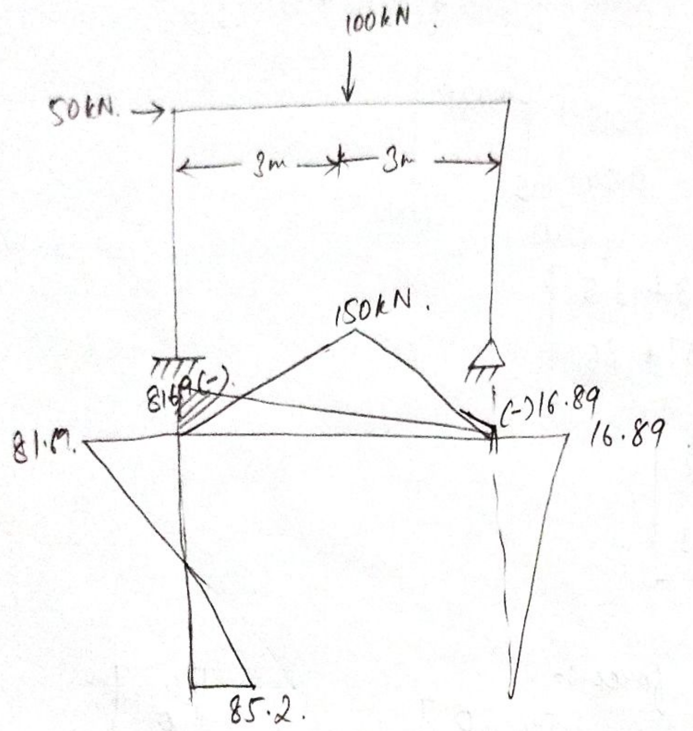
$$= \begin{bmatrix} 150 \\ 0 \\ 75 \\ -75 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -64.8 \\ 81.69 \\ -81.69 \\ +16.89 \\ -6.89 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -75 \\ 75 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 85.2. \\ 81.69 \\ -81.69. \\ 16.89 \\ -16.89 \\ 0 \end{bmatrix}$$

Step 11:- Final Moment.

$$\begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \\ M_{CD} \end{bmatrix} = \begin{bmatrix} 85.2 \\ 81.69 \\ -81.69 \\ -16.89 \\ 16.89 \\ 0 \end{bmatrix}$$

M_{DC}

Step 12:- Maximum Bending Moment.

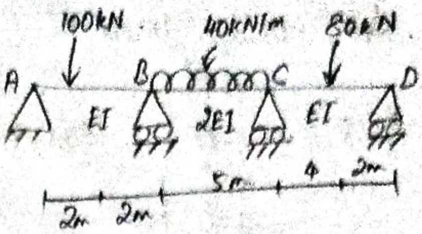


$$\text{Max. B.M.} = \frac{wl}{4} = \frac{100 \times 6}{4}$$

$$= 150 \text{ kNm}$$

Date:- 11/04/2021

Q. Analyse the continuous beam as shown in fig. by flexibility matrix method :



(Vertical Loading Condition).

Ans:- Step 1: Static Indeterminacy.

$D_s = R - r$

$= (5-3)\ 4-2$

$= 2$

M_B and M_C are considered to be redundant.

$\frac{wab^2}{l^2} \Big/ \frac{wa^2b}{l^2}$

Step 2:- Fixed End Moments.

max B.M. $= \frac{wab}{l}$

$\left[\cancel{AF}\ M_{FAB} = \frac{wab^2}{l^2} = 100\times \right]$

$M_{FAB} = -\frac{wl}{8} = -\frac{100\times4}{8} = -50$ kNm

$M_{FBA} = \frac{wl}{8} = 50$ kNm.

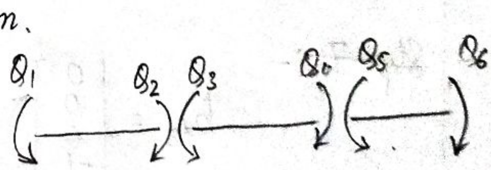
$M_{FBC} = -\frac{wl^2}{12} = -\frac{40\times5^2}{12} = -83.3$ kNm.

$M_{FCB} = \frac{wl^2}{12} = 83.3$ kNm.

$M_{FCD} = -\frac{wab^2}{l^2} = \frac{80\times4\times2^2}{6^2} = -35.56$ kNm.

$M_{FDC} = \frac{wa^2b}{l^2} = \frac{80\times4^2\times2}{6^2} = 71.1$ kNm.

Step 3:- Internal ~~memf~~ member force action.



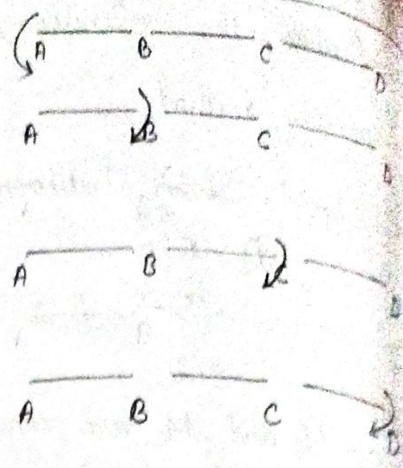
$$Q = \begin{bmatrix} -50 \\ 50 \\ -83.3 \\ 83.3 \\ -35.56 \\ 71.1 \end{bmatrix}$$

Step 4:- Equivalent joint moment.

$$R = \begin{bmatrix} 50 \\ 33.3 \\ -47.8 \\ -71.1 \end{bmatrix}$$

Step 5 :- Force transformation matrix.

Apply unit force at every points.

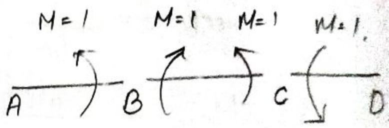


$$\{bR_1\} = \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \qquad \{bR_2\} = \begin{Bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}.$$

$$\{bR_3\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{Bmatrix} \qquad \{BR_4\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{Bmatrix}.$$

$$\{bR\} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Step 6 :- Moment transformation matrix (bx).



$$b_{x_1} = \begin{bmatrix} 0 \\ -1 \\ 1 \\ -0 \\ 0 \\ 0 \end{bmatrix}.$$

~~Step 7~~ :-

$$b_{x_2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} \qquad bx = \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

Step 7 :- Force Matrix.

$$f_{AB} = \frac{l}{6EI}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \frac{4}{6EI}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \frac{1}{EI}.\begin{bmatrix} 1.34 & -0.67 \\ -0.67 & 1.34 \end{bmatrix}$$

$$f_{BC} = \frac{5}{6EI\times 2}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1.66 & -0.83 \\ -0.83 & 1.66 \end{bmatrix}\frac{1}{2EI} \quad \begin{bmatrix} 0.83 & -0.42 \\ -0.42 & 0.83 \end{bmatrix}$$

$$= (2.76 - \cancel{0.69})EI = 2.07EI.$$

$$f_{CD} = \frac{6}{6EI}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} \cancel{1.66}\ 2 & \cancel{-0.83}\ -1 \\ -1 & 2 \end{bmatrix}$$

$$F = \frac{1}{EI}\begin{bmatrix} 1.34 & -0.67 & 0 & 0 & 0 & 0 \\ -0.67 & 1.34 & 0 & 0 & 0 & 0 \\ 0 & 0 & \overset{0.83}{\cancel{1.66}} & \overset{-0.42}{\cancel{-0.83}} & 0 & 0 \\ 0 & 0 & \overset{-0.42}{\cancel{-0.83}} & \overset{0.83}{\cancel{1.66}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

Step 8 :- Flexibility Matrix .

$$F_{xx} = [b_x]^T [F] [b_x] .$$

$$= \frac{1}{EI}\begin{bmatrix} 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix} \underset{(6\times 6)}{\begin{bmatrix} 1.34 & -0.67 & 0 & 0 & 0 & 0 \\ -0.67 & 1.34 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.83 & -0.42 & 0 & 0 \\ 0 & 0 & -0.42 & 0.83 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}} \underset{(6\times 2)}{\begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}}$$

$$= \frac{1}{EI}\underset{(2\times 6)}{\begin{bmatrix} 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix}} \underset{(6\times 2)}{\begin{bmatrix} 0.67 & 0 \\ -1.34 & 0 \\ 0.83 & 0.42 \\ -0.42 & -0.83 \\ 0 & 2 \\ 0 & -1 \end{bmatrix}}$$

$$= \frac{1}{EI}\begin{bmatrix} 1.34 + 0.83 & 0.42 \\ 0.42 & 0.83 + 2 \end{bmatrix}$$

$$[F_{xx}] = \frac{1}{EI}\begin{bmatrix} 2.17 & 0.42 \\ 0.42 & 2.83 \end{bmatrix} \qquad (6.14 - 0.18)$$

~~Step 9.~~

$$[A] = \frac{1}{|A|} (\text{adj } A) .$$

$$= \frac{EI}{5.96}\begin{bmatrix} 2.83 & -0.42 \\ -0.42 & 2.17 \end{bmatrix}$$

$$= EI\begin{bmatrix} 0.47 & -0.07 \\ -0.07 & 0.36 \end{bmatrix}$$

Step 9:- Internal member force.

$$[Q] = [b_R][R] + [b_X][X] + FEM.$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 50 \\ 33.3 \\ -47.8 \\ -71.1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Step 9 :- Redundant Force. Matrix.

$$[X] = -[F_{XX}]^{-1}[F_{XR}][R].$$

$$[F_{XR}] = [b_X]^T[F][b_R]$$

$$= \begin{bmatrix} 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix} \frac{1}{EI} \begin{bmatrix} 1.34 & -0.67 & 0 & 0 & 0 & 0 \\ -0.67 & 1.34 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.83 & -0.42 & 0 & 0 \\ 0 & 0 & -0.42 & 0.83 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}_{(6\times6)} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{(6\times4)}.$$

$$= \frac{1}{EI}\underset{(2\times4)}{\begin{bmatrix} 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix}} \begin{bmatrix} 1.34 & -0.67 & 0 & 0 \\ -0.67 & 1.34 & 0 & 0 \\ 0 & 0 & -0.42 & 0 \\ 0 & 0 & 0.83 & 0. \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 2 \end{bmatrix}_{(6\times4)}$$

$$= \frac{1}{EI}\begin{bmatrix} 0.67 & -1.34 & -0.42 & 0 \\ 0 & 0 & -0.83 & -1 \end{bmatrix}.$$

$$[x] = -[F_{xx}]^{-1}[F_{xR}][R].$$

$$\begin{bmatrix} 50 \\ 33.3 \\ -47.8 \\ -71.1 \end{bmatrix} (4\times1)$$

$$= -\frac{\cancel{EI}}{\cancel{EI}}\begin{bmatrix} 0.47 & -0.07 \\ -0.07 & 0.36 \end{bmatrix} \times \frac{1}{\cancel{EI}}\underset{(2\times4)}{\begin{bmatrix} 0.67 & -1.34 & -0.42 & 0 \\ 0 & 0 & -0.83 & -1 \end{bmatrix}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

16X

$$= 1\begin{bmatrix} 0.47 & -0.07 \\ -0.07 & 0.36 \end{bmatrix} \begin{bmatrix} 33.5 - 44.62 + 20.07 + 0 \\ 39.67 + 71.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.47 & -0.07 \\ -0.07 & 0.36 \end{bmatrix} \cdot \begin{bmatrix} 8.95 \\ 110.77 \end{bmatrix}.$$

$$= \begin{bmatrix} 4.206 - 7.753 \\ -0.6265 + 39.877 \end{bmatrix}$$

16.91

-13.57.

$$= \begin{bmatrix} 3.55 \\ -39.25 \end{bmatrix}.$$

Step 10 :- Internal member force

$$[Q] = [b_R][R] + [b_x][x] + FEM.$$

$$= \underset{(6\times4)}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \underset{(4\times1)}{\begin{bmatrix} 50 \\ 33.3 \\ -47.8 \\ -71.1 \end{bmatrix}} + \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} +3.55 \\ -39.25 \end{bmatrix} + \begin{bmatrix} -50 \\ 50 \\ -83.3 \\ 83.3 \\ -35.56 \\ 71.1 \end{bmatrix}.$$

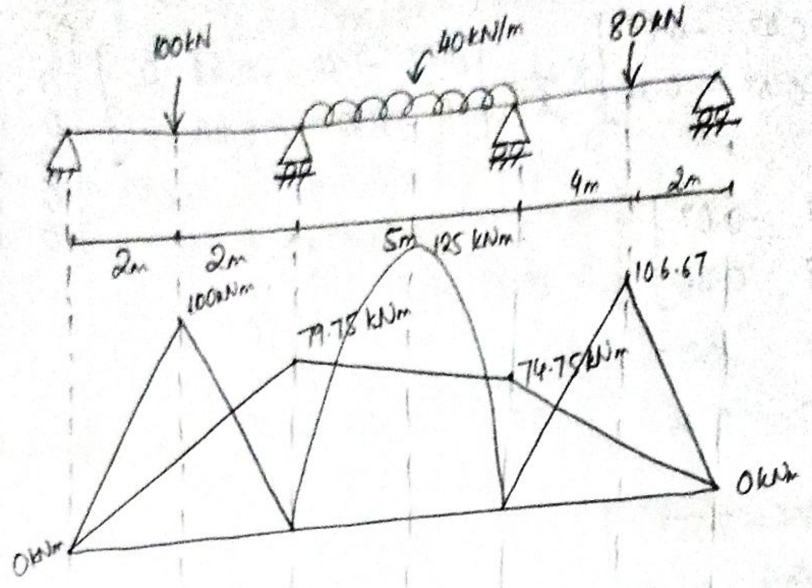
$$= \begin{bmatrix} 50 \\ 33.3 \\ 0 \\ -47.8 \\ 0 \\ -71.1 \end{bmatrix} + \begin{bmatrix} 0 \\ -3.55 \\ 3.55 \\ 39.25 \\ -39.25 \\ 0 \end{bmatrix} + \begin{bmatrix} -50 \\ 50 \\ -83.3 \\ 83.3 \\ -35.56 \\ 71.1 \end{bmatrix} = \begin{bmatrix} 0 \\ 79.75 \\ -79.75 \\ -74.75 \\ -74.81 \\ 0 \end{bmatrix}$$

Step 11:- Final Moment.

$$\begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \\ M_{CD} \\ M_{DC} \end{bmatrix} = \begin{bmatrix} \cancel{0} \\ \cancel{86.88} \\ \cancel{-86.85} \\ \cancel{3.75} \\ \cancel{3.69} \\ \cancel{0} \end{bmatrix} \begin{bmatrix} 0 \\ 79.75 \\ -79.75 \\ 74.75 \\ -74.81 \\ 0 \end{bmatrix}$$

Date:- 13/04/2022.

Flexibility Method :-



Bending Moment

$$M_{AB} = \frac{wl}{4} = \frac{100 \times 4}{4} = 100\,kNm.$$

$$M_{BC} = \frac{wl^2}{8} = \frac{40 \times 5^2}{8} = 125\,kNm.$$

$$M_{CD} = \frac{wab}{l} = \frac{80 \times 4 \times 2}{6} = 106.67\,kNm.$$