SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
An Autonomous Institution

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DEPARTMENT OF FOOD TECHNOLOGY

R2019-HEAT AND MASS TRANSFER

II YEAR III SEM

UNIT 1–CONDUCTION

TOPIC –Heat Conduction through Cylindrical systems

Consider a long cylinder of inside radius r_i, outside radius r_o, and length L (Fig. 3.2). We consider the cylinder to be long so that the end losses are negligible. The inside and outside surfaces are kept at constant temperatures T_i and T_o respectively. A steam pipe in a room can be taken as an example of a long hollow cylinder.

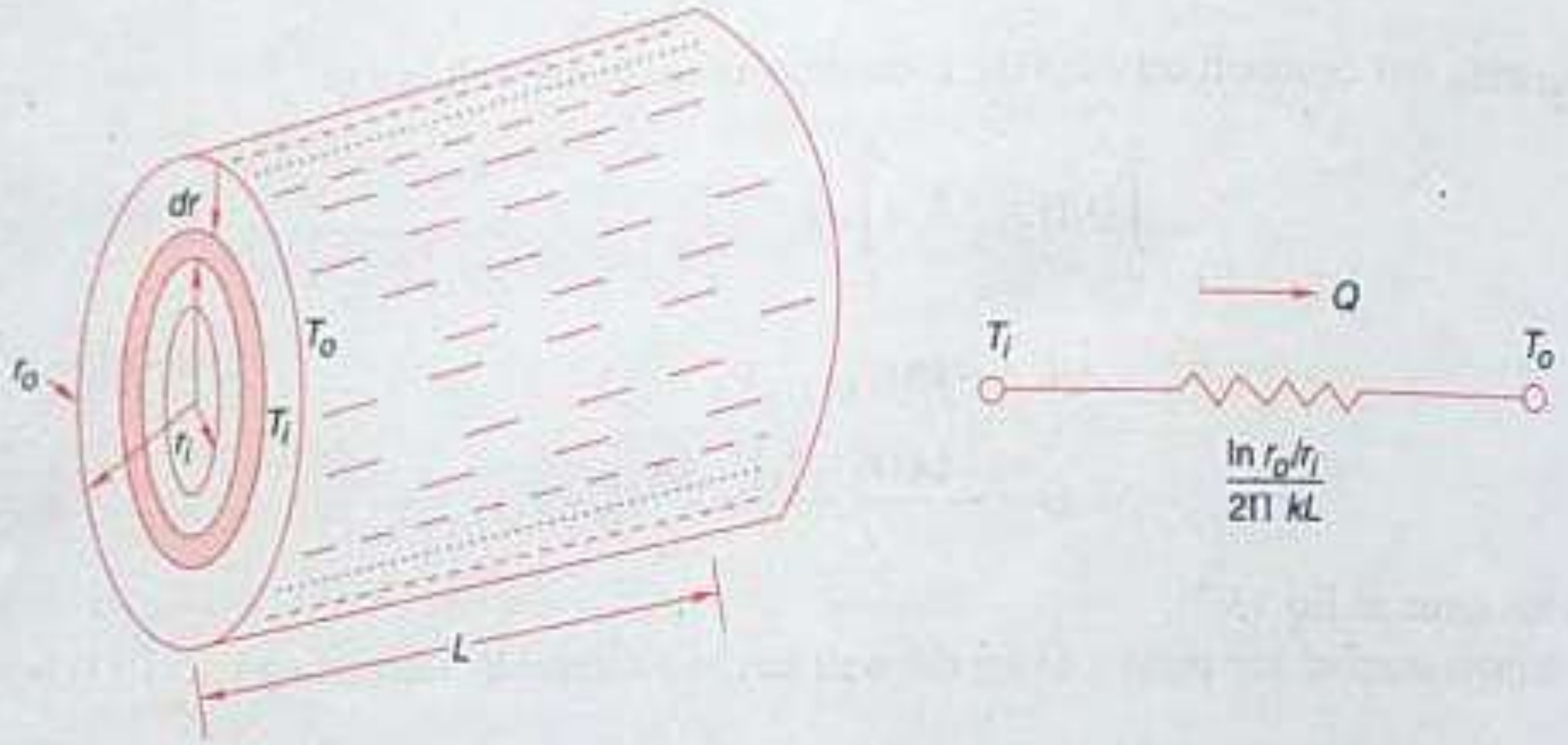

Fig. 3.2 Steady State Conduction through a Hollow Cylinder

The general heat conduction equation in cylindrical coordinates is

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\left(\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (2.21)$$

Assuming that heat flows only in a radial direction, the above equation under steady state (without heat generation) takes the form:

$$\frac{d^2 T}{dr^2} + \frac{1}{r}\left(\frac{dT}{dr}\right) = 0$$

or

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0 \qquad (3.4)$$

Subject to the boundary conditions,

$$T = T_i \quad \text{at} \quad r = r_i$$

$$T = T_0 \quad \text{at} \quad r = r_0$$

Integrating Eq. (3.4) twice we get

$$T = C_1 \ln r + C_2 \tag{3.5}$$

Using the boundary conditions

at $\quad r = r_i, \; T = T_i; \; T_i = C_1 \ln r_i + C_2$

at $\quad r = r_0, \; T = T_0; \; T_0 = C_1 \ln r_0 + C_2$

$$C_1 = \frac{T_i - T_0}{\ln \dfrac{r_i}{r_0}} = \frac{T_0 - T_i}{\ln \dfrac{r_0}{r_i}}$$

$$C_2 = T_i - \frac{T_0 - T_i}{\ln \frac{r_i}{r_0}} \ln r_i = \frac{T_i \ln r_0 - T_0 \ln r_i}{\ln \frac{r_0}{r_i}}$$

Substituting the values of C_1 and C_2 in Eq. (3.5),

$$T = \frac{(T_0 - T_i)}{\ln \frac{r_0}{r_i}} \ln r + \frac{T_i \ln r_0 - T_0 \ln r_i}{\ln \frac{r_0}{r_i}}$$

$$Q = -kA_r \left.\frac{dT}{dr}\right|_{r=r_i} = -k \cdot 2\pi r_i L \cdot \frac{C_1}{r_i}$$

$$= -k \cdot 2\pi r_i L \cdot (T_0 - T_i) \cdot \frac{1}{r_i \ln \frac{r_0}{r_i}} = \frac{2\pi k L (T_i - T_0)}{\ln \frac{r_0}{r_i}} \qquad (3.7)$$

Equation (3.7) can alternatively be derived as follows:

$$Q = -kA\frac{dT}{dr}, \quad \text{where} \quad A = 2\pi rL$$

or

$$Q\frac{dr}{r} = -2\pi kL dT$$

Integration of this equation gives

$$Q\int_{r_i}^{r_0}\frac{dr}{r} = -2\pi kL\int_{T_i}^{T_0} dT$$

$$Q\ln\left(\frac{r_0}{r_i}\right) = -2\pi kL(T_0 - T_i)$$

or

$$Q = \frac{2\pi kL(T_i - T_0)}{\ln\frac{r_0}{r_i}} \quad (3.7)$$

The thermal resistance for the hollow cylinder would be

$$R_{th} = \frac{\ln\left(\frac{r_0}{r_i}\right)}{2\pi kL} \quad (3.8)$$

Example 3.2

A hollow cylinder 5 cm I.D. and 10 cm O.D. has an inner surface temperature of 200°C and an outer surface temperature of 100°C. Determine the temperature of the point halfway between the inner and the outer surfaces. It the thermal conductivity of the cylinder material of 70 W/mK determine the heat flow through the cylinder per linear metre.

Solution.

Equation (3.7) gives

$$Q = \frac{2\pi k L (T_i - T_0)}{\ln \frac{r_0}{r_i}} = \frac{(6.28)\,(70)\,(1)\,(200 - 100)}{\ln \frac{5}{2.5}}$$

$$= 63420.9 \text{ W/m} = 63.42 \text{ kW/m}.$$

At half way between r_i and r_0, radius $r' = \frac{(5+2.5)}{2} = 3.75$ cm. Since Q remains the same,

$$Q = \frac{2\pi kL(T_i - T_0)}{\ln\frac{r_0}{r_i}} = \frac{2\pi kL(T_i - T')}{\ln\frac{r'}{r_i}}$$

$$\therefore \quad \frac{T_i - T_0}{\ln\frac{r_0}{r_i}} = \frac{T_i - T'}{\ln\frac{r'}{r_i}}$$

or

$$T_i - T' = (T_i - T_0)\frac{\ln\frac{r'}{r_i}}{\ln\frac{r_0}{r_i}} = (T_i - T_0)\ln\frac{r'}{r_0} = \frac{(100)\ln\left(\frac{3.75}{2.50}\right)}{\ln\left(\frac{5.00}{2.5}\right)} = 58.5$$

$$\therefore \quad T' = T_i - 58.5 = 141.5°\text{C}$$

Thank You