SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
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DEPARTMENT OF FOOD TECHNOLOGY

19FTT201 – HEAT AND MASS TRANSFER

UNIT 1 - CONDUCTION

II YEAR IV SEM

Topic 5 1D SS Conduction equation for wall

Conduction heat transfer:

Objectives of conduction analysis:

1) Primary objective is to determine the temperature field $T(x)$ in a body [i.e how temperature varies with position within the body].

2) $T(x)$ depends on boundary condition, initial condition, material properties (ρ, k, C_p), and geometry.

3) Why we need temperature $T(x)$.

a) Compute heat flow at any point (using Fourier eqn).

b) Compute thermal stresses, expansion, deflection due to temp etc,

c) Design products in application such as insulation thickness, chip temperature calculations (electronics), Heat treatment of metals.

Boundary and Initial conditions:

1) Heat equation is second order in spatial coordinate. Hence, 2 BC's needed for each coordinate.

* 1D problem : 2 BC in x-direction.

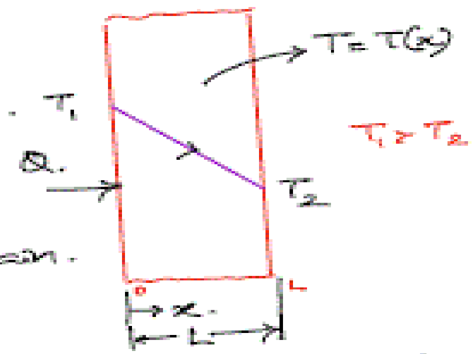
2) Heat equation is first order in time. Hence 1 IC is needed.

1D SS equation for wall

One-dimensional steady state heat conduction without heat generation: $\dot{q}'' = 0$, $\frac{\partial T}{\partial t} = 0$.

$$\frac{\partial}{\partial x}\left(k \cdot \frac{\partial T}{\partial x}\right) = 0 \rightarrow k \cdot \frac{d^2T}{dx^2} = 0 \longrightarrow (1)$$

$$\frac{d^2T}{dx^2} = 0.$$



Integrating the equation once.

$$\frac{dT}{dx} = C_1 \longrightarrow (2)$$

Integrating the equation again.

$$T = C_1 x + C_2 \longrightarrow (3)$$

To determine the constants C_1 & C_2, apply the boundary conditions:

i.e @ $x = 0$, $T = T_1 \rightarrow T_1 = C_1 0 + C_2 \rightarrow C_2 = T_1 \rightarrow (4)$

& @ $x = L$, $T = T_2 \rightarrow T_2 = C_1 L + C_2$

$$T_2 = C_1 L + T_1 \quad \because C_2 = T_1$$

$$\therefore C_1 = \frac{(T_2 - T_1)}{L} \rightarrow (5)$$

$\therefore$ (3) becomes,

$$T = \frac{(T_2 - T_1)}{L} x + T_1 \rightarrow \frac{T - T_1}{T_2 - T_1} = \frac{x}{L} \longrightarrow (6)$$

($T \rightarrow T(x)$)

Temperature distribution across the plane wall.

Rate of heat transfer through the plane wall:

$$Q = -KA\frac{dT}{dx} \longrightarrow (7) \quad \text{[Fourier conduction equation]}$$

$$Q = -K.A\,\frac{T_2 - T_1}{L} \qquad \because C_1 = \frac{T_2 - T_1}{L}.$$

$$Q = K.A_{c/s}\frac{(T_1 - T_2)}{L} \longrightarrow \Delta T = (T_1 - T_2)$$

$L \longrightarrow$ thickness of the wall.

$$Q = \frac{(T_1 - T_2)}{\left[\frac{L}{KA}\right]} \longrightarrow (8)$$

Thermal resistance concept using Ohm's law:

$V_1 \xrightarrow{R} V_2$.

$$V = I.R. \rightarrow I = \frac{V}{R} \rightarrow (9).$$

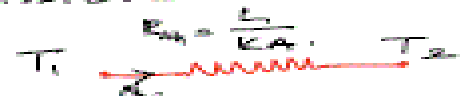
Comparing equations (8) and (9), we have.

$$Q = I; \quad (T_1 - T_2) = V \quad \text{and} \quad \frac{L}{KA} = R.$$

$\therefore$ Thermal resistance for the plane wall.

$$R_{th} = \frac{L}{KA} \longrightarrow (10) \quad \left[\frac{m}{W/m.K \cdot m^2} = \frac{m^2.K.}{W.m^2} = \frac{K}{W}\right].$$

Thermal resistance circuit for plane wall.



unit $\frac{K}{W}$ (or) $°C/W$.

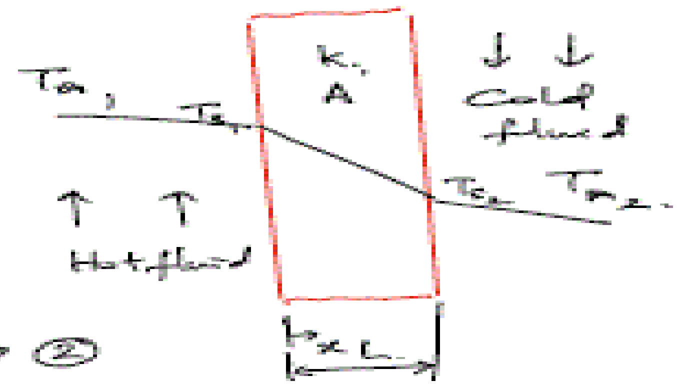
Thermal resistance for convection from newton's law of cooling $Q = hA_s(T_s - T_\infty) \rightarrow Q = \frac{(T_s - T_\infty)}{1/hA_s}$.

$$\therefore R_{th_{conv}} = \frac{1}{hA_s}$$

Plane wall with convective boundary conditions:

The differential equation is:

$$\frac{d}{dx}\left(K \cdot \frac{dT}{dx}\right) = 0 \longrightarrow (1)$$



Integrating twice,

$$T(x) = C_1 x + C_2 .$$

Boundary conditions are -

@ $x = 0$, $T = T_{s_1}$ $T(0) = T_{s_1} \longrightarrow (2)$

@ $x = L$, $T = T_{s_2}$ $T(L) = C_1 L + T_{s_1}$

$$\therefore C_1 = \frac{T(L) - T_{s_1}}{L} . \longrightarrow (3)$$

$$\therefore T(x) = (T(L) - T_{s_1})\frac{x}{L} + T_{s_1} \longrightarrow (4)$$

Heat flux across the wall is given by

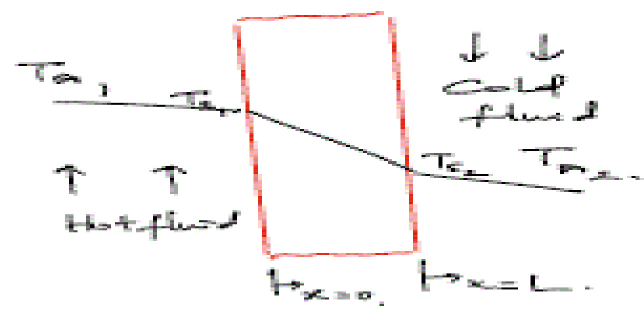
$$q = -KA\frac{dT}{dx} = \frac{K \cdot A}{L}(T_{s_1} - T_{s_2}) = \frac{T_{s_1} - T_{s_2}}{\left(\frac{L}{KA}\right)} \longrightarrow (5)$$

Thermal resistance concept for convection:

$$q = hA(T_{\infty_1} - T_{s_1}) \longrightarrow q = \frac{(T_{\infty_1} - T_{s_1})}{1/hA} .$$

$$\therefore R_{th_{conv}} = \frac{T_{\infty_1} - T_{s_1}}{q} = \frac{1}{hA}$$

Applying thermal resistance concept for the plane wall with convection boundary conditions.



Heat transfer rate may be determined by considering each element of the resistance network, as.

$$q = \frac{T_{\infty 1} - T_{s1}}{\frac{1}{h_1 A}} = \frac{T_{s1} - T_{s2}}{\frac{L}{KA}} = \frac{T_{s2} - T_{\infty 2}}{\frac{1}{h_2 A}} \longrightarrow (1)$$

Since resistances are in series,

$$R_{total} = \Sigma R_{th} = \frac{1}{h_1 A} + \frac{L}{KA} + \frac{1}{h_2 A} \longrightarrow (2)$$

$$= \frac{T_{\infty 1} - T_{s1}}{q} + \frac{T_{s1} - T_{s2}}{q} + \frac{T_{s2} - T_{\infty 2}}{q}.$$

1) Heat transfer deals with the rate of

a. work transfer
b. temperature transfer
c. energy transfer
d. none of the above

Answer :
c. energy transfer

MCQ

2. The amount of heat required to raise the temperature of a substance by 1°C is called:

A. work capacity

B. heat capacity

C. Energy capacity

D. none of the above

Ans: B

MCQ

3 Heat bring ……………… change

A. Physical

B. chemical

C. reversible

D. periodic

Ans: B   Heat bring chemical change

4. The process of transfer of heat in liquids & gases is called:

A. Conduction

B. Radiation

C. Convection

D. Absorption

Ans: C It is the process of transfer of heat in liquids & gases

5) Solids are not heated by convection because:

A. solid are not free to move from one place to another

B. molecules only vibrate about a fixed position

C. both A and B

D. none of the above

Ans: C Solids are not heated by convection because the molecules of a solid are not free to move from one place to another; they can only vibrate about a fixed position

References

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