SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore – 35

Department of Electrical & Electronics Engineering

CONTROLLABILITY & OBSERVABILITY

INTRODUCTION

A linear system is said to be completely controllable if, for all initial times and all initial states , there exists some input function (or sequence for discrete systems) that drives the state vector to any final state at some finite time.

A linear system is said to be completely observable if, for all initial times, the state vector can be determined from the output function (or sequence), defined over a finite time.

CONTROLLABILITY

Controllability Matrix

- Consider a single-input system (u ∈ R):

$$\dot{x} = Ax + Bu, \qquad y = Cx \qquad x \in \mathbb{R}^n$$

- The Controllability Matrix is defined as

$$\mathcal{C}(A,B) = [B \mid AB \mid A^2B \mid \ldots \mid A^{n-1}B]$$

(A,B) Controllable $\Leftrightarrow rank(C) = n$,

$C(A,B) = \begin{bmatrix} B & AB & A^2B & \cdots & A^{n-1}B \end{bmatrix}$

- We say that the above system is controllable if its controllability matrix $C(A,B)$ is invertible.
- As we will see later, if the system is controllable, then we may assign arbitrary closed-loop poles by state feedback of the form $u = -Kx$.
- Whether or not the system is controllable depends on its state-space realization.

CONTROLLABILITY

Observability Matrix

$$(A, C)\text{Observable} \Leftrightarrow rank(V) = n \quad \Leftrightarrow \det(V) \neq 0 \quad \text{if } y \in R$$

$$\text{Observability Matrix } V = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

CONTROLLABILITY

Example

- Plant:

$$\dot{x} = Ax + Bu, x \in R^n$$
$$y = Cx + Du$$

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

Controllability Matrix $V = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Obervability Matrix $N = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$rank(V) = rank(N) = 2$$

- Hence the system is both controllable and observable.