

## COMBINATION OF SOLIDS

Whenever two or more solids combine, a definite curve is seen at their intersection. This curve is called the curve of intersection (COI). Lines of intersection are a common feature in engineering applications or products. Figure 1 shows few examples of intersection lines frequently observed in chemical plants, domestic appliances, pipe joints, etc. Curves of intersections are important from the point of view of production of components for engineering applications.



The two solids may intersect in different ways. The axes of the solids may be parallel, inclined or perpendicular to each other. The axes may be intersecting, offset or coinciding. Therefore, the following sub-cases exist:

- (i) Axes perpendicular and intersecting
- (ii) Axes perpendicular and offset
- (iii) Axes inclined and intersecting
- (iv) Axes inclined and offset
- (v) Axes parallel and coinciding
- (vi) Axes parallel and offset

The type of intersection created depends on the types of geometric forms, which can be two- or three- dimensional. Intersections must be represented on multiview drawings correctly and clearly. For example, when a conical and a cylindrical shape intersect, the type of intersection that occurs depends on their sizes and on the angle of intersection relative to their axes as well as relative position of their axes.

The line of intersection is determined using auxiliary views and cutting planes.

Methods – (1) Line and (2) Cutting-plane methods

**Line method:** A number of lines are drawn on the lateral surface of one of the solids and in the region of the line of intersection. Points of intersection of these lines with the surface of the other solid are then located. These points will lie on the required line of intersection. They are more easily located from the view in which the lateral surface of the second solid appears edgewise (i.e. as a line). The curve drawn through these points will be the line of intersection.

**Cutting-plane method:** The two solids are assumed to be cut by a series of cutting planes. The cutting planes may be vertical (i.e. perpendicular to the H.P.), edgewise (i.e. perpendicular to the V.P.) or oblique. The cutting planes are so selected as to cut the surface of one of the solids in straight lines and that of the other in straight lines or circles.

### Intersection of two prisms

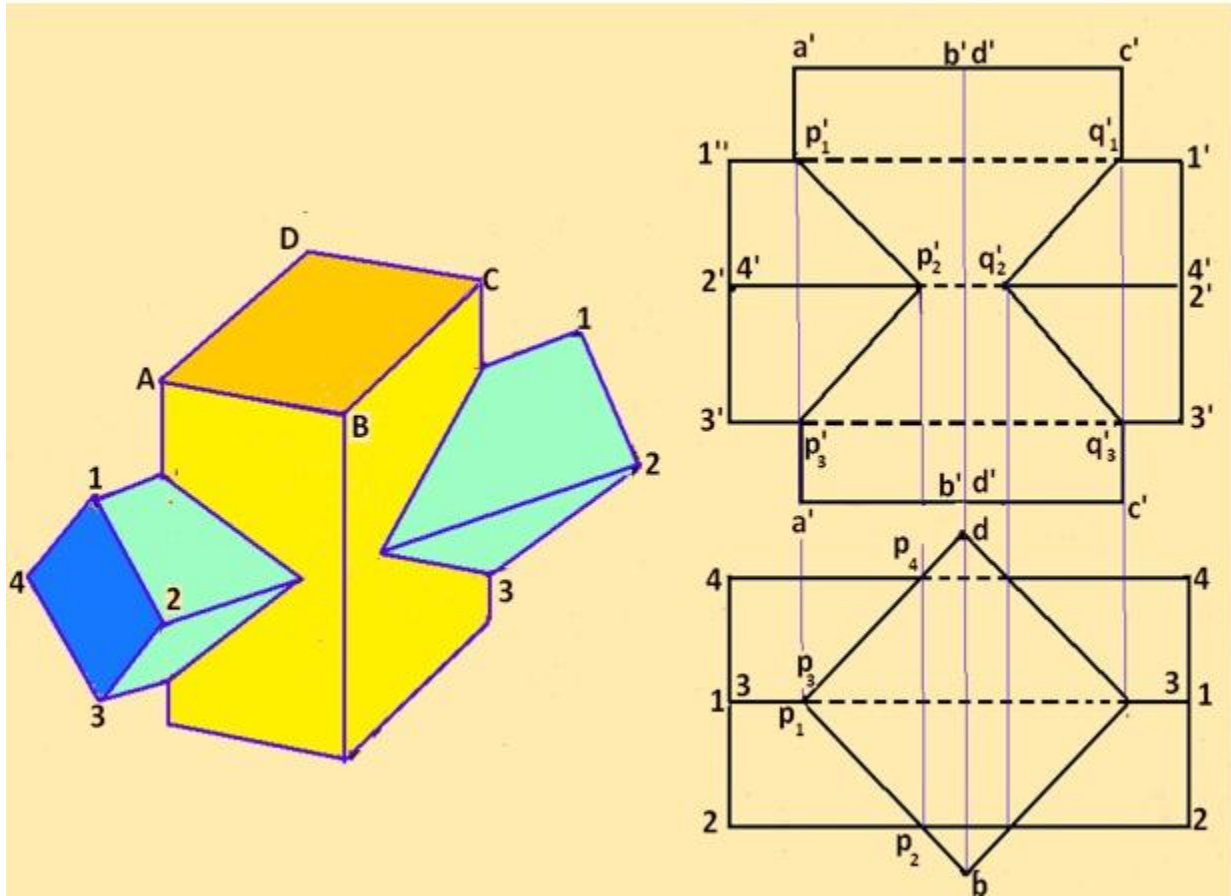
Prisms have plane surfaces as their faces. The line of intersection between two plane surfaces is obtained by locating the positions of points at which the edges of one surface intersect the other surface and then joining the points by a straight line. These points are called vertices. The line of intersection between two prisms is therefore a closed figure composed of a number of such lines meeting at the vertices. The method of obtaining intersection lines are shown by means of problem.

**Problem 1.** A vertical square prism, base 50 mm side, is completely penetrated by a horizontal square prism, base 35 mm side, so that their axes intersect. The axis of the horizontal prism is parallel to the prism, while the faces of the two prisms are equally inclined to the prism. Draw the projections of the solids, showing lines of intersection. (Assume suitable lengths for the prisms.)

#### Solution

The intersection obtained is shown in figure 2. Draw the projections of the prisms in the required position. The faces of the vertical prism are seen as lines in the top view. The points of intersection in this view can be located by the following method.

Lines 1-1 and 3-3 intersect the edge of the vertical prism at points  $p_1$  and  $p_3$  (coinciding with  $a$ ). Lines 2-2 and 4-4 intersect the faces at  $p_2$  and  $p_4$  respectively. The exact positions of these points along the length of the prism may now be determined by projecting them on corresponding lines in the front view. For example,  $p_2$  is projected to  $p_2'$  on the line 2'2'. Note that  $p_4'$  coincides with  $p_2'$ . Similarly other points are obtained. Draw lines  $p_1'p_2'$  and  $p_2'p_3'$ . Lines  $p_1'p_4'$  and  $p_3'p_4'$  coincide with the front lines. These lines show the line of intersection. Lines  $q_1'q_2'$  and  $q_2'q_3'$  on the other side are obtained in the same manner. Note that the lines for the hidden portion of the edges are shown as dashed lines. The green lines in the figure represents construction lines. The portions  $p_1'p_3'$  and  $q_1'q_3'$  of vertical edges  $a'a'$  and  $c'c'$  do not exist and hence, must be removed or kept fainter.



### Intersection of Cylinder and Cylinder

The line of intersection between cylinders will be curved since the lateral surfaces of cylinders are curved. Points on the intersection line can be located by either line method or cutting plane method. For plotting an accurate curve, the points at which the curve changes direction must also be located. These points lie on the outermost or extreme lines of each cylinder pierce the surface of the other cylinder. For obtaining the intersection of cylinder and cylinder, cutting plane method is more useful. In this technique, a series of horizontal cutting planes are assumed to be passing through the lines of the horizontal cylinder and these planes cut both cylinders as shown in figure 3. After each sectioning, the top view of the section of the horizontal cylinder will be a rectangle with its width depending on the position of the cutting plane, whereas the top view of the vertical cylinder will be a circle with diameter equal to the diameter of the vertical cylinder. The points at which the sides of the rectangles intersect the circle will lie on the curve of intersection. The procedure is illustrated through problem 2.

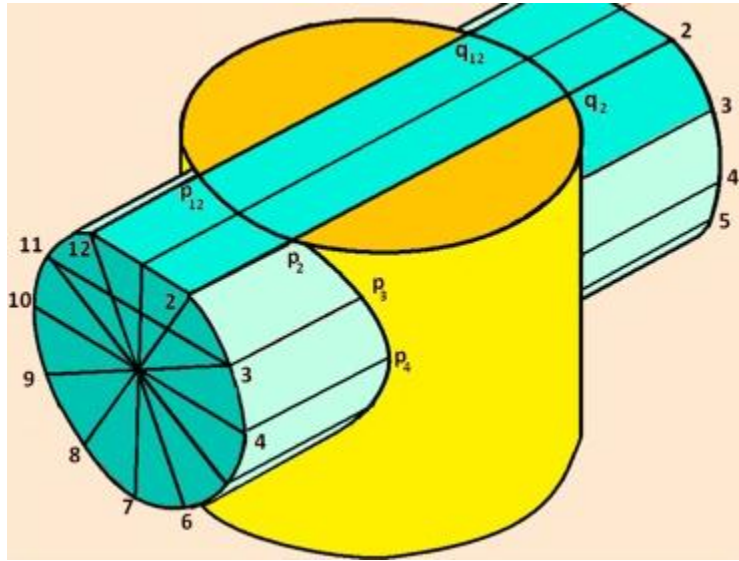


Figure 3 illustrating the cutting plane method for obtaining the curve of intersection.

### Problem 2

A vertical cylinder of 80 mm diameter is completely penetrated by another cylinder of 60 mm diameter, their axes bisecting each other at right angles. Draw their projections showing curves of penetration, assuming the axis of the penetrating cylinder to be parallel to the VP.

### Solution:

The solution is shown in figure 3. The Front view, top view and side view of the two cylinders are drawn with out the intersection lines. Divide the circumference of the circle in the side view in to 12 equal parts. Draw horizontal projectors from these points on to the front view. Project the same points from the side view on to the top view and obtain lines 1-1, 2-2, 3-3 , etc. Let us now consider a horizontal section plane passing through points 2-2 and 12-12 (shown in figure 3).

In the front view, it will be seen as a line coinciding with line 2' 2'. In the top view, the section of the horizontal cylinder will be a rectangle of width (i.e. the line 2-12). The section of the vertical cylinder will be a circle. Points  $p_2$  and  $p_{12}$  at which the sides (2-2 and 12-12) of the rectangle cuts the circle will lie on the curve of intersection. This point is first obtained in the top view by the intersecting point of line 12-12 and 2-2 with the circle. Vertical projector lines are drawn from these points to the front view so as to intersect with the horizontal projectors drawn through points 2 and 12 in the side view to obtain  $P_2'$  and  $P_{12}'$  in the front view.

Other cutting planes are also assumed passing through 3-11, 4-10, etc and the procedure repeated to obtain other points  $p_3'$ ,  $p_4'$ ,  $p_5'$ , etc. Similar procedure is adopted to obtain points  $q_1'$ ,  $q_2'$ , etc on the right hand side. Since the axis to the two cylinders intersect, points  $p_2'$  and  $p_{12}'$  will coincide and hence cannot be shown in the figure.

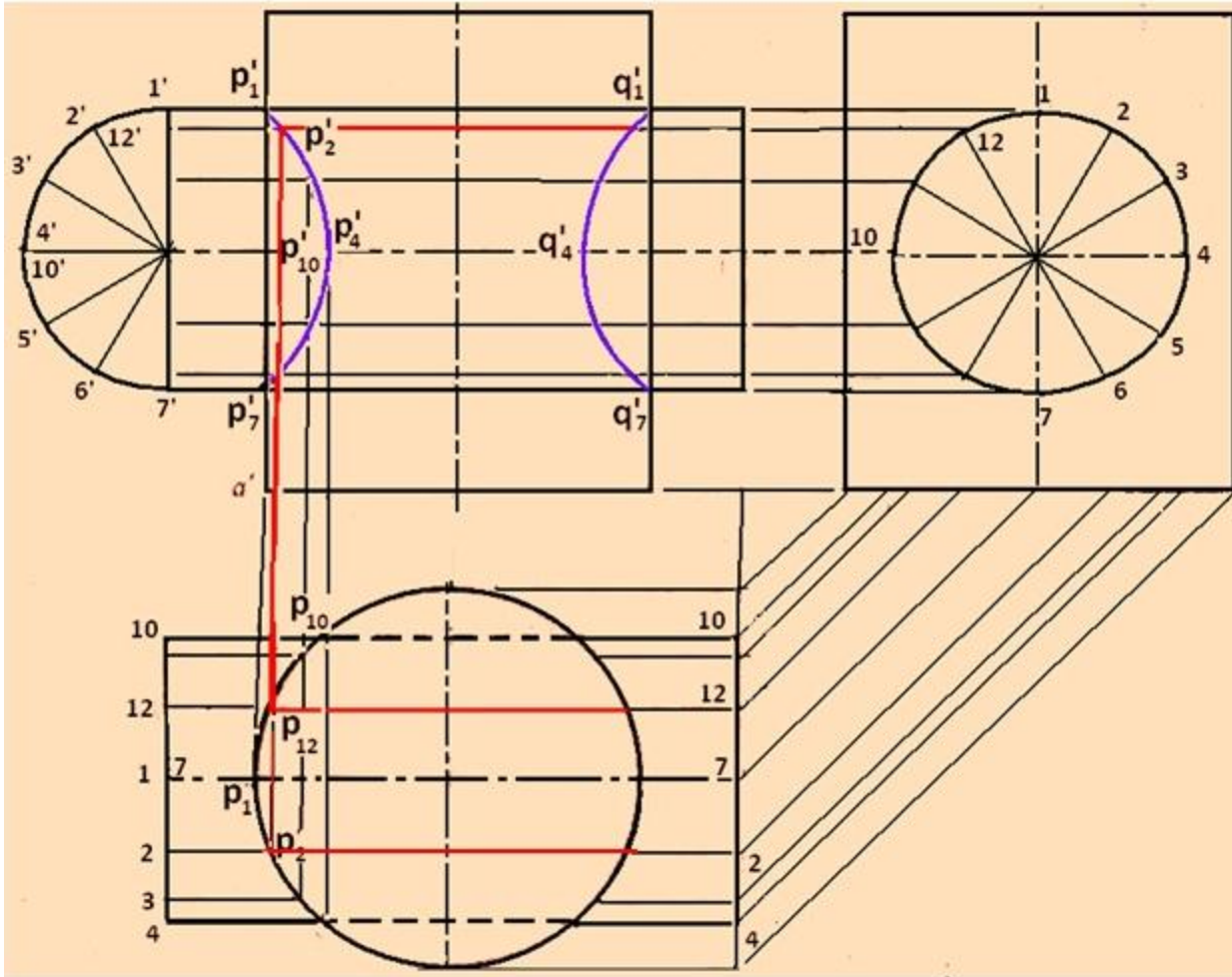


Figure 4. solution to problem 2.

### Intersection of Cone and Cylinder

Figure 5 illustrate the cutting plane method for obtaining the curve of intersection. In this the top view of the cone after sectioning will be a circle and that of the cylinder will be a rectangle with width  $W$ . The diameter of the circle and the width of the rectangle in the top view will depend on the height of the section from the cone base. At each height, the intersection points  $P_2, P_3, P_4 \dots$  are obtained and finally these points are joined by a smooth curve in the front view.



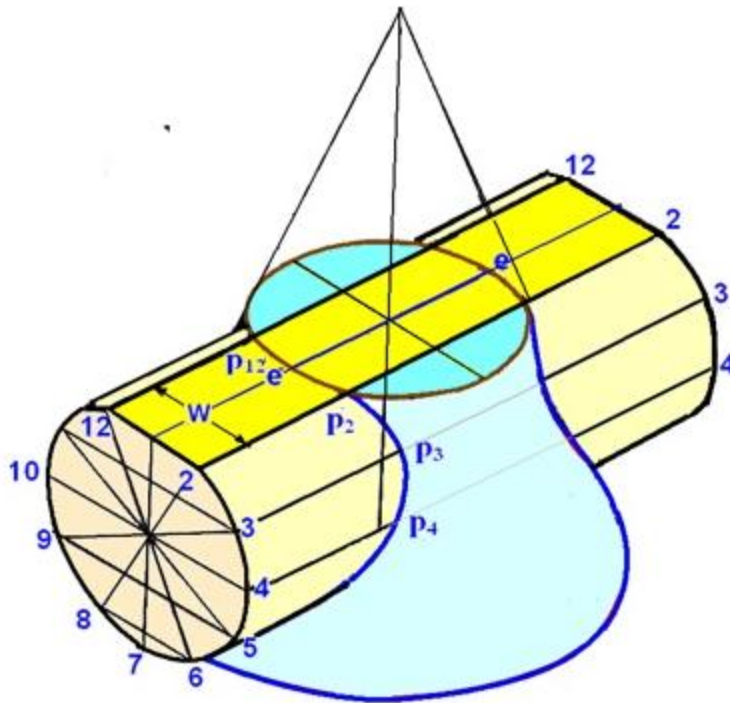


Figure 5 illustrating the cutting plane method for obtaining the curve of intersection.

### Problem 3

Example - A vertical cone, diameter of base 75 mm and axis 100 mm long, is completely penetrated by a cylinder of 45 mm diameter. The axis of the cylinder is parallel to HP and VP and intersects the axis of the cone at a point 22 mm above the base. Draw the projections of the solids showing curves of intersection.

### Solution

Draw lines dividing the surface of the cylinder into twelve equal parts.

Assume a horizontal cutting plane passing through say, point 2. The section of the cylinder will be a rectangle of width  $w$  (i.e. the line 2-12), while that of the cone will be a circle of diameter  $ee$ . These two sections intersect at points  $p_2$  and  $p_{12}$ . These sections are clearly indicated in the top view by the rectangle 2-2-12-12 and the circle of diameter  $ee$ . In the front view, the cutting plane is seen as a line coinciding with  $2' 2'$ . Points  $p_2$  and  $p_{12}$  when projected on the line  $2' 2'$  (with which the line  $12'-12'$  coincides) will give a point  $p_2'$  (with which  $p_{12}'$  will coincide). Then  $p_2'$  and  $p_{12}'$  are the points on the curve of intersection. To obtain the points systematically, draw circles with centre  $O$  and diameters  $dd, ee, ff,$  etc. cutting lines through 1, 2 and 12, 3 and 11 etc. at points  $p_1, p_2$  and  $p_{12}, p_3$  and  $p_{11}$  etc. Project these points to the corresponding lines in the front view.

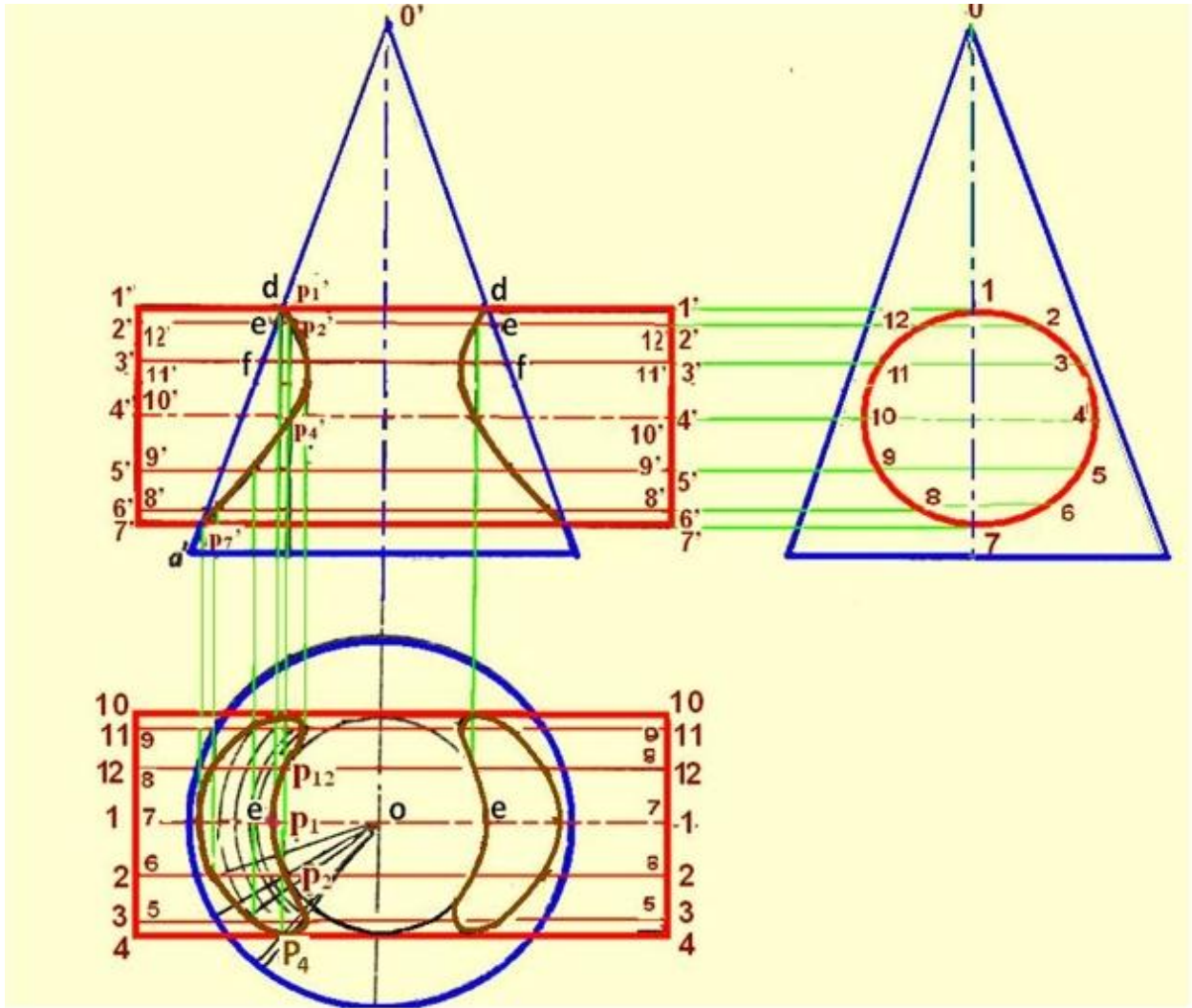


Figure 6. solution to problem 3.