A steel tube of 30 mm external diameter and 20 mm internal diameter encloses a copper rod of 15 mm diameter to which it is rigidly joined at each end. If, at a temperature of 10°C there is no longitudinal stress, calculate the stresses in the rod and tube when the temperature is raised to 200°C. Take E for steel and copper as 2.1×10^5 N/mm² and 1×10^5 N/mm² respectively. The value of co-efficient of linear expansion for steel and copper is given as 11×10^{-6} per °C and 18×10^{-6} per °C respectively.

Sol. Given :

Dia. of copper rod $= 15$ mm

$\therefore$ Area of copper rod, $A_c = \dfrac{\pi}{4} \times 15^2 = 56.25\pi \text{ mm}^2$

Area of steel tube, $A_s = \dfrac{\pi}{4}(30^2 - 20^2) = 125\pi \text{ mm}^2$

Rise of temperature, $T = (200 - 10) = 190°\text{C}$

E for steel, $E_s = 2.1 \times 10^5 \text{ N/mm}^2$

E for copper, $E_c = 1 \times 10^5 \text{ N/mm}^2$

Value of α for steel, $\alpha_s = 11 \times 10^{-6}$ per °C

Value of α for copper, $\alpha_c = 18 \times 10^{-6}$ per °C

As the value of α for copper is more than that of steel, hence the copper rod would expand more than the steel tube if it were free. Since the two are joined together, the copper will be prevented from expanding its full amount and will be put in compression, the steel being put in tension.

Let σ_s = Stress in steel

σ_c = Stress in copper.

For equilibrium of the system,

Compressive load on copper = Tensile load on steel

or $$\sigma_c \cdot A_c = \sigma_s \cdot A_s$$

$\therefore$ $$\sigma_c = \sigma_s \cdot \frac{A_s}{A_c} = \sigma_s \cdot \frac{125\pi}{56.25\pi} = 2.22 \times \sigma_s \quad \ldots(i)$$

We know that the copper rod and the steel tube will actually expand by the same amount.

Now actual expansion of steel = Free expansion of steel + Expansion due to tensile stress

$$= \alpha_s \, . \, T \, . \, L + \frac{\sigma_s}{E_s} \, . \, L$$

and actual expansion of copper = Free expansion of copper

– Contraction due to compressive stress

$$= \alpha_c \, . \, T \, . \, L - \frac{\sigma_c}{E_c} \, . \, L$$

But actual expansion of steel = Actual expansion of copper

$$\alpha_s \, . \, T \, . \, L + \frac{\sigma_s}{E_s} \, . \, L = \alpha_c \, . \, T \, . \, L - \frac{\sigma_c}{E_c} \, . \, L$$

or
$$\alpha_s \, . \, T + \frac{\sigma_s}{E_s} = \alpha_c \, . \, T - \frac{\sigma_c}{E_c}$$

or
$$11 \times 10^{-6} \times 190 + \frac{\sigma_s}{2.1 \times 10^5} = 18 \times 10^{-6} \times 190 - \frac{2.22\,\sigma_s}{1 \times 10^5} \qquad (\because \;\; \sigma_c = 2.22\sigma_s)$$

or
$$\frac{\sigma_s}{2.1 \times 10^5} + \frac{2.22\,\sigma_s}{1 \times 10^5} = 18 \times 10^{-6} \times 190 - 11 \times 10^{-6} \times 190$$

or
$$\frac{\sigma_s + 2.1 \times 2.22\sigma_s}{2.1 \times 10^5} = 5 \times 10^{-6} \times 190$$

or
$$\sigma_s + 4.662\sigma_s = 5 \times 10^{-6} \times 190 \times 2.1 \times 10^5$$

or
$$5.662\sigma_s = 199.5$$

$$\therefore \qquad \sigma_s = \frac{199.5}{5.662} = \mathbf{35.235\ N/mm^2}. \quad \textbf{Ans.}$$

Substituting this value in equation (*i*), we get

$$\sigma_s = 2.22 \times 35.235 = \mathbf{78.22\ N/mm^2}. \quad \textbf{Ans.}$$