Eigenvalues and Eigenvectors

Definition: Matrix

A system of mn numbers(elements) arranged in a rectangular arrangement along m rows and n columns and bounded by the brackets [ ] or ( ) is called an m by n matrix, which is written as $m \times n$ matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & .. & .. & a_{1n} \\ a_{21} & a_{22} & .. & .. & a_{2n} \\ ... & .. & .. & .. & .. \\ .. & .. & .. & .. & .. \\ a_{m1} & a_{m2} & .. & .. & a_{mn} \end{bmatrix}$$

Characteristic polynomial

The determinant $|A - \lambda I|$ when expanded will give a polynomial, which we call as characteristic polynomial of matrix A.

Definition: Eigenvalues

$A = [\, a_{ij} \,]$ be a square matrix.

The characteristic equation of A is $|A - \lambda I| = 0$.

The roots of the characteristic equation are called Eigenvalues of A.

<u>Definition:</u> Eigenvectors

A = [a_{ij}] be a square matrix of order 'n'

If there exist a non zero vector X = $\begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{bmatrix}$

such that AX = λX , then the vector X is called an Eigenvector of A corresponding to the Eigenvalue λ .

Method of finding characteristic equation of a 3x3 matrix and 2x2 matrix

The characteristic equation of a 3x3 matrix is $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

Where, S1= sum of main diagonal elements.

S_2 = sum of minor of main diagonal elements.

S_3 = Det (A) =|A|

The characteristic equation of a 2x2 matrix is $\lambda^2 - S_1\lambda + S_2 = 0$

Where, S_1 = sum of main diagonal elements.

S_2 = Det (A) =|A|

1. Find the characteristic equation of the matrix $\begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$

Solution:

The characteristic equation is $\lambda^2 - S_1\lambda + S_2 = 0$

S_1 = sum of main diagonal elements

= 1+2=3

S_2 = Det (A) =|A|

$= \begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix}$

S_2 = 2-0 = 2

The characteristic equation is $\lambda^2 - 3\lambda + 2 = 0$.

2. Find the characteristic equation of $\begin{pmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{pmatrix}$

Solution:

The characteristic equation is $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

Where, S_1 = sum of the main diagonal elements

= 2+1-4 = -1

S_2 = sum of minor of main diagonal elements

$= \begin{vmatrix} 1 & 3 \\ 2 & -4 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ -5 & -4 \end{vmatrix} + \begin{vmatrix} 2 & -3 \\ 3 & 1 \end{vmatrix}$

= (-4-6)+(-8+5)+(2+9) = -10+(-3)+11 = -2

S_3 = Det (A) =|A|

$= \begin{vmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{vmatrix}$

= 2(-4-6)-(-3)(-12+15)+1(6+5)

= 2(-10) +3(3) +1(11) = -20+9+11= 0

The characteristic equation is $\lambda^3 + \lambda^2 - 2\lambda = 0$