Defn: g quadratic form:-

A homogeneous poly. g degree 2 in any no. g variables is called as quadratic form.

General form is $q = \sum_{j=1}^{n} \sum_{i=1}^{n} a_{ij} \, x_i \, x_j = x^T A x$

where $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$ with $a_{ij} = a_{ji}$ and $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ n \end{bmatrix}$

Canonical Form:-

If a quadratic form $q = x^T A x$ can be reduced by a non-singular linear transformation $x = Ny$ to $q = y^T D y$

where $y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$ then $q = y^T D y = \lambda_1 y_1^2 + \lambda_2 y_2^2 + \lambda_3 y_3^2 + \cdots + \lambda_n y_n^2$

is known as canonical form.

Matrix g quadratic form:-

The symmetric matrix A g quadratic form obtained by placing the coeff g x_i^2 in a_{ii} & placing $\frac{1}{2}$ (coeff. g $x_i x_j$) is remaining a_{ij} & a_{ji} position.

Write the matrix of quadratic form.

1) $x_1^2 + 2x_2^2 + 2x_1 x_2$ is $\begin{array}{c} x_1 \quad x_2 \\ \begin{array}{c} x_1 \\ x_2 \end{array}\begin{bmatrix} 1 & \frac{1}{2}(2) \\ \frac{1}{2}(0) & 2 \end{bmatrix} \end{array} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

2) $x_1^2 + x_3^2$ is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3] $x_1^2 + 4x_2^2 - 3x_1 x_2$ is $\begin{array}{c} x_1 \quad x_2 \\ \begin{array}{c} x_1 \\ x_2 \end{array}\begin{bmatrix} 1 & \frac{1}{2}(-3) \\ \frac{1}{2}(-3) & 4 \end{bmatrix} \end{array} = \begin{bmatrix} 1 & -3/2 \\ -3/2 & 4 \end{bmatrix}$

Write the following matrix is quadratic form

1) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is $0x_1^2 + x_1 x_2 + x_1 + x_2 + 0x_3^2 = 2x_1 x_2$

2) $\begin{bmatrix} a & -b \\ -b & a \end{bmatrix}$ is $ax_1^2 - bx_1 x_2 - bx_2 x_1 + ax_2^2 = ax_1^2 - 2bx_1 x_2 + a$

3) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix}$ is $x_1^2 + x_2^2 + 0x_3^2 + 0x_1 x_2 + x_1 x_3 - x_2 x_3 + x_3 x_1 - x_3$

$= x_1^2 + x_2^2 + 2x_1 x_3 - 2x_2 x_3$