07/1/23 Logical equivalences and implication - demorgan's law

Tantology :-

The proposition P and q are called logically equivalent if $P \leftrightarrow Q$

Tantology :-

The truth values are true, for any truth value of variable is called tantology.

Contradection :-

The truth values are false, for any truth value of variables is called contradection.

Contigency :-

Some truth values are true and some truth values are false, then it is called contigency.

Definition :- logical equivalent :

The component proposition P and q are called logically equivalent. If ~~P~~ $P \leftrightarrow Q$ is a tantology. It is denoted by $P \equiv Q$

Truth table :-

P	~P	Tantology (P V ~P)	Contredection (P ∧ ~P)
T	F	T	F
F	T	T	F

Demorgan's law :-

1) $\sim (P \wedge Q) \equiv \sim P \vee \sim Q$

2) $\sim (P \vee Q) \equiv \sim P \wedge \sim Q$

$\boxed{\text{Example 1:-}}$

Show that Negation $\sim (P \vee Q)$ and $(\sim P \wedge \sim Q)$ are logically equivalent

P	Q	~P	~Q	(PVQ)	(~PVQ)	~P∧~Q
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

∴ logically equivalent $\sim(P \vee Q) \cong (\sim P \wedge \sim Q)$

Example:2

P is equivalent to following formula

1) $(P \wedge Q) \vee (P \wedge \sim Q) = t$

2) $(P \vee Q) \wedge (P \vee \sim Q) = S$

P	Q	~q	PVq	P∧q	P∧~q	PV~q	t	S
T	T	F	T	T	F	T	T	T
T	F	T	T	F	T	T	T	T
F	T	F	T	F	F	F	F	F
F	F	T	F	F	F	T	F	F

∴ logically equivalent $(P \wedge Q) \vee (P \wedge \sim Q) \cong P$, $(P \vee Q) \wedge (P \vee \sim Q) \cong P$

Table logic equivalence

Equivalence	Name
$P \wedge T \Leftrightarrow P$, $\sim P \vee T = T$ $P \vee F \Leftrightarrow P$, $\sim P \wedge T = F$	Identity laws
$P \vee T \Leftrightarrow T$ $P \wedge F \Leftrightarrow F$	Domination laws
$P \vee P \Leftrightarrow P$ $P \wedge P \Leftrightarrow P$	Idempotent laws
$\neg(\neg P) \Leftrightarrow P$ (~) (~)	Double negation laws

$p \vee q \Leftrightarrow q \vee p$ $p \wedge q \Leftrightarrow q \wedge p$	commutative laws
$(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$ $(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$	Associative laws $(A \cup B) \cup C = A \cup (B \cup C)$
$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$ $(p \vee q) \wedge r \Leftrightarrow (p \wedge r) \vee (q \wedge r)$ $(p \wedge q) \vee r \Leftrightarrow (p \vee r) \wedge (q \vee r)$	Distributive laws
$\sim(p \wedge q) \Leftrightarrow \sim p \vee \sim q$ $\sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q$	Demorgan's law
$p \vee (p \wedge q) \Leftrightarrow p$ $p \wedge (p \vee q) \Leftrightarrow p$	Adsorption laws
$p \vee \sim p \Leftrightarrow T$ (or) $\sim p \vee p \Leftrightarrow T$ $p \wedge \sim p \Leftrightarrow F$ (or) $\sim p \wedge p \Leftrightarrow F$	Negation laws

Tautological Implication

A statement A is said to tautologically imply a statement B if an only if $A \to B$ is a tautology. In this case we write $A \Rightarrow B$, read an "A implies B."

Table Implications

1. $p \wedge q \Rightarrow p$
2. $p \wedge q \Rightarrow q$
3. $p \Rightarrow p \vee q$
4. $\sim p \Rightarrow p \to q$
5. $q \Rightarrow p \to q$
6. $\sim(p \to q) \Rightarrow p$

(3) $P \rightarrow Q \Leftrightarrow \sim P \vee Q$

7. $\sim(P \rightarrow q) \Rightarrow \sim q$

8. $P \wedge (P \rightarrow q) \Rightarrow q$

9. $\sim q \wedge (P \rightarrow q) \Rightarrow \sim P$

10. $\sim P \wedge (P \vee q) \Rightarrow q$

11. $(P \rightarrow q) \wedge (q \rightarrow R) \Rightarrow P \rightarrow R$

12. $(P \vee q) \wedge (P \rightarrow R) \wedge (q \rightarrow R) \Rightarrow R$

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Example: 1

Show that $\sim(P \vee (\sim P \wedge q))$ and $\sim P \wedge \sim q$ are logically equivalence:

Sol:

$\sim(P \vee (\sim P \wedge Q))$	Reasons
$\Leftrightarrow \cancel{\sim[P \vee (\sim P \wedge Q)]}$	~~Demorgan's law~~
$\Leftrightarrow \sim P \wedge [\sim(\sim P) \vee \sim Q]$	Demorgan's law
$\Leftrightarrow \sim P \wedge (P \vee \sim Q)$	Demorgan's law ~~Double Negation law~~
$\Leftrightarrow (\sim P \wedge P) \vee (\sim P \wedge \sim Q)$	Distributive law
$\Leftrightarrow F \vee (\sim P \wedge \sim Q)$	Negation law $\sim P \wedge P \Leftrightarrow F$
$\Leftrightarrow \cancel{(\sim P \wedge \sim Q) \vee F}$	~~Commutative law~~
$\Leftrightarrow \sim P \wedge \sim Q$	Identity law

Consequently $\sim(P \vee (\sim P \wedge Q))$ and $\sim P \wedge \sim Q$ are logically equivalent.

Example: 2

Show that $\sim(P \wedge Q) \rightarrow (\sim P \vee (\sim P \vee Q)) \Leftrightarrow (\sim P \sim q)$

(use only the laws)

i) $\sim p \vee (\sim p \vee q)$	
$\Leftrightarrow (\sim p \vee \sim p) \vee q$	Associative law
$\Leftrightarrow \sim p \vee q$	Idempotent law

$\sim(p \wedge q) \to (\sim p \vee (\sim p \vee q))$	Given
$\Leftrightarrow \sim(p \wedge q) \to \sim p \vee q)$	by (i)
$\Leftrightarrow (p \wedge q) \vee (\sim p \vee q)$	$p \to q \Leftrightarrow \sim p \vee q$
$\Leftrightarrow (p \vee (\sim p \vee q)) \wedge (q \vee (\sim p \vee q))$	Distributive law $(p \wedge q) \vee R \Leftrightarrow (p \vee R) \wedge (q \vee R)$
$\Leftrightarrow (p \vee \sim p) \vee q) \wedge (q \vee (q \vee \sim p))$	Associative law and commutative law
$\Leftrightarrow (T \vee q) \wedge ((q \vee q) \vee \sim p)$	Negation law and associative law
$\Leftrightarrow T \wedge (q \vee \sim p)$	Negation law and Associative law Domination law and Idempotent law
$\Leftrightarrow q \vee \sim p$	Identity law
$\Leftrightarrow \sim p \vee q$	commutative law

Consequently $\sim(p \wedge q) \Rightarrow (\sim p \vee (\sim p \vee q) \Leftrightarrow (\sim p \vee q)$ are logically equivalent.

Example : 3

Show that $p \to (q \to p) \Leftrightarrow \sim p \Rightarrow (p \to q)$

i) $p \to (q \to p)$	Reason
$\Leftrightarrow p \to (\sim q \vee p)$	$q \to p \Leftrightarrow \sim q \vee p$
$\Leftrightarrow \sim p \vee [\sim q \vee p]$	$p \to q \Leftrightarrow \sim p \vee q$
$\Leftrightarrow \sim p \vee [p \vee \sim q]$	commutative law

$\Leftrightarrow [\sim p \vee p] \vee \sim q$	Associative law
$\Leftrightarrow T \vee \sim q$	Negation law
$\Leftrightarrow T$	$T \vee \sim q = T$

ii) $\sim p \to (p \to q)$	Reason
$\Leftrightarrow \sim p \to (\sim p \vee q)$	$p \to q \Leftrightarrow \sim p \vee q$
$\Leftrightarrow \sim(\sim p) \vee (\sim p \vee q)$	$p \to q \Leftrightarrow \sim p \vee q$
$\Leftrightarrow p \vee (\sim p \vee q)$	Double Negation
$\Leftrightarrow (p \vee \sim p) \sim q$	Associative law
$\Leftrightarrow T \vee q$	$p \vee \sim p \Leftrightarrow T$
$\Leftrightarrow T$	$T \vee q = T$

$\therefore$ From (i) and (ii)

$$p \to (q \to p) \Leftrightarrow \sim p \to (p \to q)$$

11/1/23 **Note :-**

1) $\rightarrow$ means connective conditional
2) $\leftrightarrow$ means, the connective, biconditional
3) $\Leftrightarrow$ means equivalent
4) $\Rightarrow$ means tautological implication

Example 1:

Show that the following implication without constructing the truth table

i) $p \wedge q \Rightarrow p$
ii) $q \Rightarrow (p \to q)$
iii) $p \wedge (p \to q) \Rightarrow q$

i) $p \wedge q \Rightarrow p$

Given $p \wedge q \Rightarrow p$

To prove that, $(p \wedge q) \rightarrow q$ is tautology

$p \wedge q \rightarrow p$	Reasons
$\sim (p \wedge q) \vee p$	$p \rightarrow q \Leftrightarrow \sim p \vee q$
$(\sim p \vee \sim q) \vee p$	Demorgan's law
$(\sim q \vee \sim p) \vee p$	Commutative law
$\sim q \vee (\sim p \vee p)$	Associative law
$\sim q \vee T$	Negation law
T	Identity law

ii) $q \Rightarrow (p \rightarrow q)$

Given $q \Rightarrow (p \rightarrow q)$

To prove that, $q \Rightarrow (p \rightarrow q)$ is tautology

$q \rightarrow (p \rightarrow q)$	Reason
$\sim q \vee (p \rightarrow q)$	$p \rightarrow q \Leftrightarrow \sim p \vee q$
$\sim q \vee (\sim p \vee q]$	$p \rightarrow q \Leftrightarrow \sim p \vee q$
$\sim q \vee (q \vee \sim p)$	Commutative law
$[\sim q \vee q) \vee \sim p$	Associative law
$T \vee \sim p$	Negation law
T	Identity law

iii) $p \wedge (p \rightarrow q) \Rightarrow q$

Given $p \wedge (p \rightarrow q) \Rightarrow q$

To prove that, $p \wedge (p \rightarrow q) \Rightarrow q$ is tautology

$(P \wedge (p \to q))] \to q$	Reasons
$\sim [P \wedge (p \to q)] \vee q$	$p \to q \Leftrightarrow \sim p \vee q$
$\sim (p \wedge (\sim p \vee q)) \vee q$	$p \to q \Leftrightarrow \sim p \vee q$
$(\sim p \vee \sim (\sim p \vee q)] \vee q$	Demorgan's law
$\sim p \vee (p \wedge \sim q) \vee q$	Demorgan's law
$(\sim p \vee p) \wedge (\sim p \vee \sim q) \vee q$	Distributive law
$T \wedge [\sim p \vee \sim q) \vee q$	Negation law
$T \wedge (\sim p \vee (\sim q \vee q))$ $T \wedge [\sim p \vee T]$ $T \wedge T$	Associative law Negation law Identity law
T	

12/1/23 Example: 2

Show that $P \to (q \to R) \Leftrightarrow (p \wedge Q) \to R$

$p \to (q \to R)$	Reason
$\sim p \vee (q \to R)$	$P \to Q \Leftrightarrow \sim P \wedge Q$ $P \to Q \Leftrightarrow \sim P \vee Q$
$\sim P \vee (\sim Q \vee R)$	$P \to Q \Leftrightarrow \sim P \vee Q$
$(\sim p \vee \sim Q) \vee R$	Associative law
$\sim (P \wedge Q) \vee R$	Demorgan's law $\sim (P \vee Q) \Rightarrow \sim P \wedge \sim q$
$(P \wedge Q) \to R$	

Example: 3

Show that $(P \wedge q) \to (P \vee q)$

$(p \wedge q) \rightarrow (p \vee q)$	Reason
$\sim(p \wedge q) \vee (p \vee q)$	$p \rightarrow q \Leftrightarrow \sim p \vee q$
$(\sim p \vee \sim q) \vee (p \vee q)$	De morgan's law
$(\sim q \vee \sim p) \vee (p \vee q)$	commutative law
$\sim q \vee (\sim p \vee p) \vee q$	Associative law
$(\sim q \vee T) \vee q$	Identity law
$\sim q \vee q$	Negation law
T	