Note :-

⇒ A square matrix $A = [a_{ij}]$ is said to be symmetric if $A = A^T$

Eigeon value and Eigeon vector :-

Eigeon values :-

⇒ The values of λ obtained from the characterstic equation determinate of $|A - \lambda I| = 0$ are called Eigeon values of A or latent value of A or characterstic value of A.

Eigeon vectors :-

⇒ Let A be a square matrix of order (3×3) and λ be a scalar (eigeon value). The column matrix $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ which satifies $(A - \lambda I)x = 0$ is called Eigeon vectors or latent vectors or characterstic vectors.

problems on non-symmetric with non-repeated Eigeon values :-

problem : 1

Find the Eigeon values and Eigeon vectors $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$

Sol:

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$$

case (i) :-

The characterstic Equation of (3×3) matrix is

$$\lambda^3 - D_1\lambda^2 + D_2\lambda - D_3 = 0$$

$$D_1 = 0$$

$$D_2 = \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} + \begin{vmatrix} 2 & 0 \\ 7 & -2 \end{vmatrix} + \begin{vmatrix} 2 & 2 \\ 2 & 1 \end{vmatrix}$$

$$= -5 - 6 - 2 \Rightarrow -13$$

$$D_3 = \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{vmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{vmatrix} \end{array}$$

$= 2(-3-2) - 2(-6+7)$

$= 2(-5) - 2(1)$

$= -10 - 2 \Rightarrow -12$

$\therefore$ the characteristic Equation is $\boxed{\lambda^3 - 0\lambda^2 - 13\lambda + 12 = 0}$

$\boxed{\text{case (ii)}}$ To find a value

$$\begin{array}{c|cccc}
1 & 1 & -0 & -13 & 12 \\
 & 0 & 1 & 1 & -12 \\ \hline
3 & 1 & 1 & -12 & \mid 0 \\
 & 0 & 3 & 12 & \\ \hline
-4 & 1 & 4 & \mid 0 & \\
 & 0 & -4 & & \\ \hline
 & 1 & \mid 0 & &
\end{array}$$

$\therefore$ The Eigen values are

$\lambda = 1, 3, -4$

$\boxed{\text{case (iii)}}$ To find Eigen vector

$(A - \lambda I)x = 0$ where $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ $0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$$\begin{bmatrix} 2-\lambda & 2 & 0 \\ 2 & 1-\lambda & 1 \\ -7 & 2 & -3-\lambda \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \longrightarrow ①$$

where $\lambda = 1$ in equation ①

$$\begin{pmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 7 & 2 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{array}{cccc} & \overline{\quad x_2 \quad} & & \\ 2 & 0 & 1 & 2 \\ 0 & 1 & 2 & 0 \\ \multicolumn{2}{c}{\underline{\quad\quad}} & \multicolumn{2}{c}{\underline{\quad\quad}} \\ \multicolumn{2}{c}{x_1} & \multicolumn{2}{c}{x_3} \end{array}$$

$X_1 = \dfrac{x_1}{2}, \dfrac{x_2}{-1}, \dfrac{x_3}{-4}$

$X_1 = \begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix}$

$X_1 = \cancel{\dfrac{x_1}{\begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix}}} = \cancel{\dfrac{x_2}{\begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix}}} = \cancel{\dfrac{x_3}{\begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix}}}$

$x_1 = \dfrac{x_1}{\begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix}}, \dfrac{x_2}{\begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix}}, \dfrac{x_3}{\begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix}}$

where $\lambda = 3$ in equ ①

$$\begin{pmatrix} -1 & 2 & 0 \\ 2 & -2 & 1 \\ -7 & 2 & -6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{array}{cccc} & & x_2 & \\ 2 & 0 & -1 & 2 \\ -2 & 1 & 2 & -2 \\ x_1 & & x_3 & \end{array}$$

$$\cancel{x_2 = \frac{x_1}{\begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix}}, \frac{x_2}{\begin{vmatrix} 2 & 0 \\ -2 & 1 \end{vmatrix}}, \frac{x_3}{\begin{vmatrix} -1 & 2 \\ 2 & -2 \end{vmatrix}}}$$

$$x_2 = \frac{x_1}{2}, \frac{x_2}{1}, \frac{x_3}{-2}$$

$$x_2 = \frac{x_1}{\begin{vmatrix} 2 & 0 \\ -2 & 1 \end{vmatrix}}, \frac{x_2}{\begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix}}, \frac{x_3}{\begin{vmatrix} -1 & 2 \\ 2 & -2 \end{vmatrix}}$$

$$= \frac{x_1}{2}, \frac{x_2}{1}, \frac{x_3}{-2}$$

$$X_2 = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$$

where $\lambda = -4$ in equ ①

$$\begin{pmatrix} 6 & 2 & 0 \\ 2 & 5 & 1 \\ -7 & 2 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{array}{cccc} & & x_2 & \\ 2 & 0 & 6 & 2 \\ 5 & 1 & 2 & 5 \\ x_1 & & x_3 & \end{array}$$

$$\cancel{x_3 = \frac{x_2}{\begin{vmatrix} 0 & 6 \\ 1 & 2 \end{vmatrix}} = \frac{x_1}{\begin{vmatrix} 2 & 0 \\ 5 & 1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 6 & 2 \\ 2 & 5 \end{vmatrix}}}$$

$$= \frac{x_2}{+6} = \frac{x_1}{-2} = \frac{x}{26}$$

$$x_3 = \frac{x_1}{\begin{vmatrix} 2 & 0 \\ 5 & 1 \end{vmatrix}}, \frac{x_2}{\begin{vmatrix} 0 & 6 \\ 1 & 2 \end{vmatrix}}, \frac{x_3}{\begin{vmatrix} 6 & 2 \\ 2 & 5 \end{vmatrix}}$$

$$= \frac{x_1}{2}, \frac{x_2}{-6}, \frac{x_3}{26}$$

$$x_3 = \begin{pmatrix} +6 \\ -2 \\ 26 \end{pmatrix} \begin{pmatrix} 2 \\ -6 \\ 26 \end{pmatrix}$$

$\therefore$ The value of Eigeon vector

$$x_1 = \begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix}, \quad x_2 = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}, \quad x_3 = \begin{pmatrix} 2 \\ -6 \\ 26 \end{pmatrix}$$

Problems:- 2

Problems on symmetric with non repeated Eigeon value. Find the Eigeon value and Eigeon vector $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$

Sol:

$$A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$$

case (i):

The characteristic Equation of (3×3) matrix is

$$\lambda^3 - D_1\lambda^2 + D_2\lambda - D_3 = 0$$

$D_1 = 18$

$$D_2 = \begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix} + \begin{vmatrix} 8 & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 8 & -6 \\ -6 & 7 \end{vmatrix}$$

$$= (21-16) + (24-4) + (56-36)$$

$$= 5 + 20 + 20 \Rightarrow 45$$

$$D_3 = \begin{vmatrix} \overset{+}{8} & \overset{-}{-6} & \overset{+}{2} \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{vmatrix}$$

$$= 8\begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix} + 6\begin{vmatrix} -6 & -4 \\ 2 & 3 \end{vmatrix} + 2\begin{vmatrix} -6 & 7 \\ 2 & -4 \end{vmatrix}$$

$$= 8(21-16) + 6(-18+8) + 2(24-14)$$

$$= 8(5) + 6(-10) + 2(10)$$

$$= 40 - 60 + 20 \Rightarrow 0$$

∴ The characteristic Equation is $\boxed{\lambda^3 - 18\lambda^2 + 45\lambda - 0 = 0}$

case (ii) To find a value

$$\begin{array}{c|cccc}
0 & 1 & -18 & 45 & -0 \\
 & 0 & 0 & 0 & 0 \\ \hline
3 & 1 & -18 & 45 & 0 \\
 & 0 & 3 & -45 & \\ \hline
15 & 1 & -15 & 0 & \\
 & 0 & 15 & & \\ \hline
 & 1 & 0 & &
\end{array}$$

∴ The Eigeon values are

$\lambda = 0, 3, 15$

case (iii) :- To find Eigeon vector

$(A - \lambda I)x = 0$ where $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$, $0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$$\begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \rightarrow (1)$$

where $\lambda = 0$ in equation (1)

$$\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{array}{cccc}
 & \overline{\quad x_2 \quad} & & \\
-6 & 2 \quad 8 & -6 & \\
7 & -4 \quad -6 & 7 & \\
\underline{\quad x_1 \quad} & & \underline{\quad x_3 \quad} &
\end{array}$$

$$x_1 = \frac{x_1}{\begin{vmatrix} -6 & 2 \\ 7 & -4 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 2 & 8 \\ -4 & -6 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 8 & -6 \\ -6 & 7 \end{vmatrix}}$$

$$x_1 = \frac{x_1}{(24-14)} = \frac{x_2}{(-12+32)} = \frac{x_3}{(56-36)}$$

$$x_1 = \begin{pmatrix} 10 \\ 20 \\ 20 \end{pmatrix}$$

$$x_1 = \frac{x_1}{10} = \frac{x_2}{20} = \frac{x_3}{20}$$

Where $\lambda = 3$ in equation ①

$$\begin{pmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{array}{cccc} & \overline{\quad x_2 \quad} & & \\ -6 & 2 & 5 & -6 \\ 4 & -4 & -6 & 4 \\ \underline{\quad x_1 \quad} & & \underline{\quad x_3 \quad} & \end{array}$$

$$x_2 = \frac{x_1}{\begin{vmatrix} -6 & 2 \\ 4 & -4 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 2 & 5 \\ -4 & -6 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 5 & -6 \\ -6 & 4 \end{vmatrix}}$$

$$= \frac{x_1}{(24-8)} = \frac{x_2}{(-12+20)} = \frac{x_3}{(20-36)}$$

$$= \frac{x_1}{16} = \frac{x_2}{8} = \frac{x_3}{-16}$$

$$x_2 = \begin{pmatrix} 16 \\ 8 \\ -16 \end{pmatrix}$$

Where $\lambda = 15$ in equation ④

$$\begin{pmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{array}{cccc} & \overline{\quad x_2 \quad} & & \\ -6 & 2 & -7 & -6 \\ -8 & -4 & -6 & -8 \\ \underline{\quad x_1 \quad} & & \underline{\quad x_3 \quad} & \end{array}$$

$$x_3 = \frac{x_1}{\begin{vmatrix} -6 & 2 \\ -8 & -4 \end{vmatrix}}, \frac{x_2}{\begin{vmatrix} 2 & -7 \\ -4 & -6 \end{vmatrix}}, \frac{x_3}{\begin{vmatrix} -7 & -6 \\ -6 & -8 \end{vmatrix}}$$

$$= \frac{x_1}{(24+16)}, \frac{x_2}{(-12-28)}, \frac{x_3}{(56-36)}$$

$$= \frac{x_1}{40}, \frac{x_2}{-40}, \frac{x_3}{20}$$

$$x_3 = \begin{pmatrix} 40 \\ -40 \\ 20 \end{pmatrix}$$

problems on repeated Eigeon value of non-symmetric matrix

problem : 1

Find Eigeon values and Eigeon vector of A = $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

sol:

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

case (i):

The characterstic Equation is $\lambda^3 - D_1\lambda^2 + D_2\lambda - D_3 = 0$

$D_1 = 2+2+1 = 5$

$$D_2 = \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$$

$= 2+2+3 \Rightarrow 7$

$$D_3 = |A| = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 2(2) - 1(1) + 1(0) \Rightarrow 3$$

∴ The characterstic Equation is $\lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0$

case (ii):

To find Evalue

$$\begin{array}{c|cccc}
1 & 1 & -5 & 7 & -3 \\
 & 0 & 1 & -4 & 3 \\
\hline
1 & 1 & -4 & 3 & 0 \\
 & 0 & 1 & -3 & \\
\hline
3 & 1 & -3 & 0 & \\
 & 0 & 3 & & \\
\hline
 & 1 & 0 & &
\end{array}$$

∴ The Eigeon values are $\lambda = 1, 1, 3$.

$\boxed{\text{case (iii)}}$ To find Eigen vector

$(A - \lambda I)x = 0$

$$\begin{bmatrix} 2-\lambda & 1 & 1 \\ 1 & 2-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \rightarrow ①$$

when $\lambda = 3$ in eqn ①

$$\begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 0 & 0 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{array}{cccccc} & & \overline{\quad x_2 \quad} & & \\ 1 & 1 & & -1 & 1 \\ -1 & 1 & & 1 & -1 \\ \underline{\quad x_1 \quad} & & & \underline{\quad x_3 \quad} & \end{array}$$

$x_1 = \frac{x_1}{2}, \frac{x_2}{2}, \frac{x_3}{0}$

$x_1 = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} \Rightarrow 2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$"

when $\lambda = 1$ in eqn ①

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$x_1 + x_2 + x_3 = 0$

$x_1 + x_2 + x_3 = 0$

$0x_1 + 0x_2 + 0x_3 = 0$

Now, $x_1 + x_2 + x_3 = 0 \rightarrow ②$

put $x_1 = 0$ in ②

$x_2 + x_3 = 0$

$x_2 = -x_3$

$\frac{x_2}{-1} = \frac{x_3}{1}$

$x_2 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

when $\lambda = 1$ in eqn ①

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$x_1 + x_2 + x_3 = 0 \rightarrow (3)$

put $x_2 = 0$

$x_1 + x_3 = 0$

$x_1 = -x_3$

$\frac{x_1}{-1} = \frac{x_3}{1}$

$x_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

$\therefore$ The value of Eigeon vectors are

$x_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad x_2 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}, \quad x_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

problems on repeated Eigeon values of symmetric matrix

problem 1:

Find the value and Eigeon vectors $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

sol: $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

case(i)

The characteristic Equation $\lambda^3 - D_1\lambda^2 + D_2\lambda - D_3 = 0$

$D_1 = 12$

$D_2 = \begin{vmatrix} 3 & -1 \\ -1 & 3 \end{vmatrix} + \begin{vmatrix} 6 & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 6 & -2 \\ -2 & 3 \end{vmatrix}$

$= (9-1) + (18-4) + (18-4)$

$= 8 + 14 + 14 = 36$

$D_3 = \begin{vmatrix} \overset{+}{6} & \overset{-}{-2} & \overset{+}{2} \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{vmatrix} = 6(9-1) + 2(-6+2) + 2(2-6)$

$= 6(8) + 2(-4) + 2(-4)$

$= 48 - 8 - 8 \quad = 48 - 8 - 8$

$= 48 - 16 \Rightarrow 32$

$\therefore$ The characteristic Equation $\boxed{\lambda^3 - 12\lambda^2 + 36\lambda - 32 = 0}$

Case (ii) To find Eigeon value

$$\begin{array}{c|cccc}
2 & 1 & -12 & 36 & -32 \\
 & 0 & 2 & -20 & 32 \\
\hline
2 & 1 & -10 & 16 & |0 \\
 & 0 & 2 & -16 & \\
\hline
8 & 1 & -8 & |0 & \\
 & 0 & 8 & & \\
\hline
 & 1 & |0 & &
\end{array}$$

$\therefore$ The Eigeon values are

$\lambda = (2, 2, 8)$

case (iii) To find Eigeon vectors

$(A - \lambda I)x = 0$

$$\begin{bmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \longrightarrow ①$$

where $\lambda = 8$ in eqn ①

$$\begin{pmatrix} -2 & -2 & 2 \\ -2 & -5 & -1 \\ 2 & -1 & -5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{array}{cccccc}
 & & \multicolumn{2}{c}{x_2} & & \\
-2 & 2 & -2 & -2 \\
-5 & -1 & 2 & -5 \\
\multicolumn{2}{c}{x_1} & \multicolumn{2}{c}{x_3}
\end{array}$$

$$x_3 = \frac{x_1}{\begin{vmatrix} -2 & 2 \\ -5 & -1 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 2 & -2 \\ -1 & -2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -2 & -2 \\ -2 & -5 \end{vmatrix}}$$

$$= \frac{x_1}{(2+10)} = \frac{x_2}{(-4-2)} = \frac{x_3}{(10-4)}$$

$$= \frac{x_1}{12} = \frac{x_2}{-6} = \frac{x_3}{6}$$

$$x_3 = \begin{pmatrix} 12 \\ -6 \\ 6 \end{pmatrix} \Rightarrow 6\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}_{//}$$

where $\lambda = 2$ in eqn ①

$$\begin{pmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$4x^1 - 2x^2 + 2x^3 = 0$

$-2x^1 + x^2 - x^3 = 0$

$2x^1 - x^2 + x = 0$

Now $2x^1 - x^2 + x^3 = 0 \rightarrow$ ②

put $x_1 = 0$ in eqn ②

$-x_2 + x_3 = 0$

$x_3 = x_2$

$\frac{x_3}{1} = \frac{x_2}{1}$

$$x_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

similarly $x_1 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ //

The value of Eigeon vectors are

$$x_1 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad x_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad x_3 = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} //$$

properties of Eigeon value:-

1) Sum of the Eigeon values is equal to sum of the diagonal elements.

2) product of Eigeon values is equal to its determinent value.

3) If λ is the Eigeon values of A then determinent is $\frac{|A|}{\lambda}$ ~~divided~~ by is the eigeon value of adj A

4) Every square matrix and its transpose have the same Eigean value

5) The Eigeon values of real symmetric matrix are real

6) product of Eigeon value is equal to determinant A

7) the Eigeon values of triangular or diagonal Elements

Problem : 1

Find the sum and product of Eigeon values $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 3 \\ -2 & -1 & 3 \end{bmatrix}$

sol:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 3 \\ -2 & -1 & 3 \end{bmatrix}$$

sum of Eigeon values = sum of main diagonal elements

$= 1+0+3$

$= 4$

product of Eigeon values $= |A|$

$= 1(0+3) - 2(3+6) + 3(-1+0)$

$= 3 - 18 - 3$

$= -18$

Problem : 2

~~Let λ' and $\lambda 2$~~ If 2 and 8 are Eigeon values of

$A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ then Find the 3^{rd} Eigeon values and its product of the Eigeon value

sol: $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$

Let λ_1, λ_2 and λ_3 be the Eigeon values.

$\lambda_1 = 2, \lambda_2 = 8, \lambda_3 = ?$

sum of Eigeon values = sum of main diagonal element

$\lambda_1 + \lambda_2 + \lambda_3 = 6 + 3 + 3$

$2 + 8 + \lambda_3 = 12$

$10 + \lambda_3 = 12$

$\lambda_3 = 12 - 10$

$\boxed{\lambda_3 = 2}$

product of Eigeon value $= \lambda_1 \times \lambda_2 \times \lambda_3$

$= 2 \times 8 \times 2$

$= 32$

Problem: 3

Find the Eigeon values of A, A^2, A^{-1}, $5A^2$, ~~and~~ $A^2 - 2I$ and adj(A). when the matrix A is $\begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$

Sol:

$A = \begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$

Eigeon value of A is $\lambda = 3, 2, 5$

Eigeon value of A^2 is $\lambda = 3^2, 2^2, 5^2$

$= 9, 4, 25$

Eigeon value is $A^{-1} = \frac{1}{A} = \frac{1}{3}, \frac{1}{2}, \frac{1}{5}$

Eigeon value is $5A^2 = 45, 20, 125$

Eigeon value of $A^2 - 2I = 9 - 2, 4 - 2, 25 - 2$

$= 7, 2, 23$

Eigeon value of adj $A = \frac{|A|}{\lambda}$

$|A| =$ product of Eigeon value

$= 3 \times 2 \times 5 = 30$

adj $A = \frac{30}{3}, \frac{30}{2}, \frac{30}{5}$

Problem : 4

If 2 is an Eigeon of $A = \begin{pmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & +3 & -1 \end{pmatrix}$ then find the other two.

Sol:

$$A = \begin{pmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$$

sum of Eigeon values = sum of main diagonal Elements

$\boxed{\lambda_1 = 2}$

$$\lambda_1 + \lambda_2 + \lambda_3 = 2 + 1 - 1$$

$$2 + \lambda_2 + \lambda_3 = 2$$

$$\boxed{\lambda_2 = -\lambda_3} \longrightarrow (1)$$

The product of Eigeon values = $|A|$

$$\lambda_1 \times \lambda_2 \times \lambda_3 = 2(-1-3) + 2(-1-1) + 2(3-1)$$

$$= 2(-4) + 2(-2) + 2(2)$$

$$= -8 \cancel{-4} \cancel{+4}$$

$$2\lambda_2\lambda_3 = -8$$

$$\lambda_2\lambda_3 = -\frac{8}{2}$$

$$\boxed{\lambda_2\lambda_3 = -4} \longrightarrow (2)$$

put $\lambda_2 = -\lambda_3$ in (2)

$$-\lambda_3\lambda_3 = -4$$

$$\lambda_3^2 = 2^2$$

$$\boxed{\lambda_3 = 2}$$

$$\therefore \boxed{\lambda_2 = -2}$$

$$\boxed{\lambda_1 = 2, \lambda_2 = -2, \lambda_3 = 2}$$