Unit - I

Cayley Hamilton theorem

Statement: Every square matrix satisfies its own characteristic equation.

Application cayley Hamilton theorem.

i) This method is practical method for computation of inverse of the large matrices.

ii) This method is useful to find the inverse of square matrix A in terms of (n-1) powers of matrix A.

iii) The higer positive integral powers of A can be also calculated using this theorem.

Example 1:

Verify cayley Hamilton theorem of the matrix $A = \begin{bmatrix} 3 & -1 \\ -1 & 5 \end{bmatrix}$

Sol:

The characteristic equation is $\lambda^2 - c_1\lambda + c_2 = 0$

$c_1 = 3+5 = 8$

$c_2 = \begin{vmatrix} 3 & -1 \\ -1 & 5 \end{vmatrix} = 15 - 1 = 14$

The C.E is $\lambda^2 - 8\lambda + 14 = 0$

To verify cayley Hamilton theorem

Every square matrix satisfies its own CE

$A^2 - 8A + 14I = 0$

$$A^2 = \begin{bmatrix} 3 & -1 \\ -1 & 5 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ -1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 9+1 & -3-5 \\ -3-5 & 1+25 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -8 \\ -8 & 26 \end{bmatrix}$$

$$A^2 - 8A + 14I = \begin{bmatrix} 10 & -8 \\ -8 & 26 \end{bmatrix} - \begin{bmatrix} 24 & -8 \\ -8 & 40 \end{bmatrix} + \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 24 & -8 \\ -8 & 40 \end{bmatrix} - \begin{bmatrix} 24 & -8 \\ -8 & 40 \end{bmatrix}$$

$$= 0$$

Hence verified.

Example 2:

verify Cayley Hamilton theorem of the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -5 \end{bmatrix}$

Sol

The characteristic eqn is $\lambda^3 - c_1\lambda^2 + c_2\lambda - c_3 = 0$

$$c_1 = 1+5-5 = 1$$

$$c_2 = \begin{vmatrix} 5 & -4 \\ 7 & -5 \end{vmatrix} + \begin{vmatrix} 1 & -2 \\ 3 & -5 \end{vmatrix} + \begin{vmatrix} 2 & 5 \\ 3 & 7 \end{vmatrix}$$

$$= (-25+28) + (-5+6) - (14-15)$$

$$= 3+1+1 = 5$$

$$c_3 = 1(-25+28) - 2(-10+12) - 2(14-15)$$

$$= 1(3) - 2(2) - 2(-1)$$

$$= 3-4+2$$

$$= 1$$

The C.E is

$\lambda^3 - \lambda^2 + 5\lambda - 1 = 0$

To verify Cayley Hamilton theorem

$$A^3 - A^2 + 5A - I = 0$$

$$A^2 = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -5 \end{bmatrix} \begin{bmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4-6 & 2+10-14 & -2-8+10 \\ 2+10-12 & 4+25-28 & -4-20+20 \\ 3+14-15 & 6+35-35 & -6-28+25 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -2 & 0 \\ 0 & 1 & -4 \\ 2 & 6 & -9 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} -1 & -2 & 0 \\ 0 & 1 & -4 \\ 2 & 6 & -9 \end{bmatrix} \begin{bmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -12 & 10 \\ -10 & -23 & 16 \\ -13 & -29 & 17 \end{bmatrix}$$

Hence

$$A^3 - A^2 + 5A + I = 0$$

$$\Rightarrow \begin{bmatrix} -5 & -12 & 10 \\ -10 & -23 & 16 \\ -13 & -29 & 17 \end{bmatrix} - \begin{bmatrix} -1 & -2 & 0 \\ 0 & 1 & -4 \\ 2 & 6 & -9 \end{bmatrix} + \begin{bmatrix} 5 & 10 & -10 \\ 10 & 25 & -20 \\ 15 & 35 & -25 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 & 0 \\ 0 & 2 & -4 \\ 2 & 6 & -8 \end{bmatrix} - \begin{bmatrix} 0 & -2 & 0 \\ 0 & 2 & -4 \\ 2 & 6 & -8 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$= 0$