Properties of Eigen values & Eigen vectors.

1. The sum of Eigen value of matrix is the sum of Principal diagonal elements.
2. The product of Eigen value of matrix is equal to its determinant.
3. If $\lambda_1, \lambda_2, \dots \lambda_n$ are Eigen value of matrix A then $\frac{1}{\lambda_1}, \frac{1}{\lambda_2} \dots \frac{1}{\lambda_n}$ are eigen values of A^{-1}
4. If $\lambda_1, \lambda_2 \dots \lambda_n$ are eigen value of matrix A then $k\lambda_1, k\lambda_2 \dots k\lambda_n$ are eigen value of kA, where k is scalar
5. If $\lambda_1, \lambda_2 \dots \lambda_n$ are eigen value of A then $\lambda_1^m, \lambda_2^m, \dots \lambda_n^m$ are eigen value of A^m
6. If $\lambda_1, \lambda_2 \dots \lambda_n$ are eigen value of A then $\lambda_1 - k, \lambda_2 - k, \dots \lambda_n - k$ are eigen value of $(A - kI)$
7. Eigen values of matrix A & A^T are same.
8. Eigen value of real matrix is real.

<u>Example 1</u>

Show that if $(\lambda_1 \lambda_2 \dots \lambda_n)$ are eigen values of the matrix A then A^3 has the eigen values $\lambda_1^3, \lambda_2^3 \dots \lambda_n^3$.

<u>Sol</u>

Let λ be a root of matrix A, then there exist non-zero vector X s.t $AX = \lambda X$ $\rightarrow$ ①

$$A^2(AX) = A^2(\lambda X) \Rightarrow A^3 X = \lambda(A^2 X) \rightarrow ②$$

$$\begin{aligned} A^2 X &= A(AX) \\ &= A(\lambda X) \\ &= \lambda(AX) \\ &= \lambda(\lambda X) \\ &= \lambda^2 X \end{aligned}$$

② ⇒

$$\begin{aligned} A^3 X &= \lambda(\lambda^2 X) \\ &= \lambda^3 X \end{aligned}$$

$\therefore$ λ^3 is eigen value of A^3