(5) The Leslie Model describes age specified grow
Let the oldest age attained by the females in some animal population be 6 years. Dividing the population into three age classes 2 years each. we have the Leslie Matrix as

$$L = \begin{bmatrix} 0 & 2.3 & 0.4 \\ 0.6 & 0 & 0 \\ 0 & 0.3 & 0 \end{bmatrix}.$$

(i) What is the no. of females in each class after 2, 4 and years. if each class initially contains 500 females.

9 3 3 6 and 9 600

(ii). For what initial distribution will the no. of female in each class change by the same proportion what is the rate of change.

Solution:

Initially $X_0 = \begin{bmatrix} 500 \\ 500 \\ 500 \end{bmatrix}$

After two years $X_1 = L X_0$

$$= \begin{bmatrix} 0 & 2.3 & 0.4 \\ 0.6 & 0 & 0 \\ 0 & 0.3 & 0 \end{bmatrix} \begin{bmatrix} 500 \\ 500 \\ 500 \end{bmatrix}$$

$$= \begin{bmatrix} 1350 \\ 300 \\ 150 \end{bmatrix}$$

Similarly after 4 years the no. of females in each class is given by

$$X_2 = L X_1$$

$$= \begin{bmatrix} 0 & 2.3 & 0.4 \\ 0.6 & 0 & 0 \\ 0 & 0.3 & 0 \end{bmatrix} \begin{bmatrix} 1350 \\ 300 \\ 150 \end{bmatrix}$$

$$= \begin{bmatrix} 750 \\ 810 \\ 90 \end{bmatrix}$$

After 6 years we have

$$X_3 = L X_2 = \begin{bmatrix} 0 & 2.3 & 0.4 \\ 0.6 & 0 & 0 \\ 0 & 0.3 & 0 \end{bmatrix} \begin{bmatrix} 750 \\ 810 \\ 90 \end{bmatrix}$$

$$= \begin{bmatrix} 1899 \\ 450 \\ 243 \end{bmatrix}$$

(ii) The distribution vector $LX = \lambda X$ where λ is the rate of change.

The characteristic eqn is given by

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$$

S_1 = sum of the diagonals $= 0+0+0 = 0$

S_2 = sum of the minors of the diagonals

$= 0+0+(0-(2.3)(0.6))$

$= -1.38$

$$\begin{array}{r} 2.3 \\ 6 \\ \hline 1.38 \end{array}$$

$S_3 = |A| = -0.6\left[-(0.4)(0.3)\right]$

$= (0.6)(0.4)(0.3)$

$= 0.072$

0.072

The characteristic eqn is given by

$$\lambda^3 - 0\lambda^2 + (-1.38)\lambda - 0.072 = 0$$

$$\lambda^3 - 1.38\lambda - 0.072 = 0$$

$$\cancel{\lambda(\lambda^2 - 1.388) = 0.072}$$

$$\cancel{\lambda = 0}$$

$\lambda = 0$ $\quad \neq 0 = -0.072$

$\lambda = 1$ $\quad 1 - 1.38 - 0.072 = -0.452$

$\lambda = 2$ $\quad 8 - 2.76 - 0.072 = 5.168$

$\lambda = 1.1$ $\quad (1.1)^3 - 1.38(1.1) - 0.072 = -0.072$

$\lambda = 1.2$ $\quad (1.2)^3 - 1.38(1.2) - 0.072 = 0$

$\lambda = 1.2$ is a root of the equation

$$\begin{array}{c|cccc} & \lambda^3 & \lambda^2 & \lambda & \text{cons} \\ & 1 & 0 & -1.38 & -0.072 \\ 1.2 & 0 & 1.2 & +1.44 & 0.072 \\ \hline & 1 & +1.2 & \cancel{+0.06}\ 0.06 & \boxed{0} \end{array}$$

$$\lambda^2 + 1.2\lambda + 0.06 = 0$$

$$\lambda^2 + 1.2\lambda + 0.06 = 0$$

$a = 1$

$b = +1.2$

$c = 0.06$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1.2 \pm \sqrt{(1.2)^2 - 4(1)(0.06)}}{2}$$

$$= \frac{-1.2 \pm \sqrt{1.44 - 0.24}}{2} = \frac{1.2 \pm \sqrt{1.20}}{2}$$

$$\lambda = \frac{-1.2 \pm \sqrt{1.2}}{2}$$

$$\sqrt{1.2} = 1.095$$

$$\lambda = \frac{-1.2 \pm 1.095}{2}$$

$$= \frac{-1.2 + 1.095}{2}, \frac{-1.2 - 1.095}{2}$$

$$\lambda = \frac{-2.295}{2}, \frac{-0.105}{2}$$

$$\lambda = 1.1475 \ ; \ 0.052$$

$$\boxed{\therefore \lambda = 1.2 ; -1.148 ; -0.052}$$

$\therefore$ The Eigen Values are 1.2, ~~1.148~~ and 0.052

+ve values are taken

To find Eigen Vector,

$\lambda = 1.2$

$[L - \lambda I]X = 0$, where $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$\begin{bmatrix} 0-\lambda & 2.3 & 0.4 \\ 0.6 & 0-\lambda & 0 \\ 0 & 0.3 & 0-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} -1.2 & 2.3 & 0.4 \\ 0.6 & -1.2 & 0 \\ 0 & 0.3 & -1.2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$-1.2x_1 + 2.3x_2 + 0.4x_3 = 0$

$+0.6x_1 - 1.2x_2 + 0x_3 = 0$

$0x_1 + 0.3x_2 - 1.2x_3 = 0$

	x_1		x_2		x_3	
2.3		0.4		-1.2		2.3
$\cancel{1}$ -1.2		0		+0.6		-1.2

$$\frac{x_1}{\begin{vmatrix} 2.3 & 0.4 \\ -1.2 & 0 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 0.4 & -1.2 \\ 0 & +0.6 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -1.2 & 2.3 \\ +0.6 & -1.2 \end{vmatrix}}$$

$$\frac{x_1}{0.48} = \frac{x_2}{0.24} = \frac{x_3}{0.05}$$ 0.06

$\div$ by 0.48

$$\frac{x_1}{1} = \frac{x_2}{0.5} = \frac{x_3}{0.125}$$

To find the initial distribution with the no. of females in each class change by the same proportion, consider the equation

$$x + 0.5x + 0.125x = 500 + 500 + 500$$

$$1.625x = 1500$$

$$x = 923$$

age class 1 : $x = 923$

age class 2 : $0.5x = 0.5(923) = 461.5 \simeq 461$

age class 3 : $0.125x = 0.125(923) = 115.37 \simeq 115$

Elastic Deformations:

Stretching of an Elastic Membrane:

Problem:

An Elastic Membrane in the $x_1 x_2$-plane with boundary circle $x_1^2 + x_2^2 = 1$ is stretched so that a point $P: (x_1, x_2)$ goes over into the point $Q: (y_1, y_2)$ given by

$$Y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = AX = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

In components

$$y_1 = 5x_1 + 3x_2$$

$$y_2 = 3x_1 + 5x_2$$

Find the principal directions, i.e., the directions of the position vector x of P for which the direction of the position vector y of Q is the same or exactly opposite. What shape does the boundary circle take under this deformation