Eigen Vectors of a Real Matrix:

Corresponding to each root of the characteristic Equation $|A-\lambda I|=0$, the homogeneous system $(A-\lambda I)X=0$ has a non-zero solution $X=\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ which is called an Eigen Vector or Latent Vector of A.

i.e., the column Matrix $X=\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ satisfying the ~~ac~~ equation $(A-\lambda I)X=0$ is called the Eigen Vector or characteristic Vector.

Note:

1) The Eigen Vectors corresponding to distinct Eigen Values of a real symmetric matrix are orthogonal.

Ex: 1 If the Eigen Values of $A=\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ are $-1, -1, 2$ and if two of the Eigen Vectors of A are $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ then find the third eigenvector.

Solution:

Given A is a symmetric Matrix.

Hence the Eigen Vectors of A are orthogonal

[By Note 1]

Let $X_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$; $X_2 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$.

Let $X_3 = \begin{pmatrix} l \\ m \\ n \end{pmatrix}$ as X_3 is orthogonal to X_1 + X_2

$\therefore X_1^T X_3 = 0 \Rightarrow [1 \quad 1 \quad 1] \begin{pmatrix} l \\ m \\ n \end{pmatrix} = l+m+n = 0$ — ①

$\therefore X_2^T X_3 = 0 \Rightarrow [0 \quad 1 \quad -1] \begin{pmatrix} l \\ m \\ n \end{pmatrix} = 0l+m-n=0$ — ②

from ① and ② $\Rightarrow m = n$

$$\frac{l}{-1-1} = \frac{-m}{0-1} = \frac{n}{0+1}$$ (since it is orthogonal)

① ⇒ $l = -m-n$

② ⇒ $-m = 0l - n$

③ ⇒ $n = 0l + m$

$$\frac{l}{-2} = \frac{-m}{-1} = \frac{n}{1}$$

$$\frac{l}{2} = \frac{m}{1} = \frac{n}{-1}$$

$\therefore$ The third Eigen Vector is $X_3 = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$

Ex: 2 If -1 is the Eigen Value of $\begin{pmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{pmatrix}$ write the corresponding eigen vector.

Solution:

Given $\lambda = -1$

The Eigen Vector corresponding to the Eigen value λ is $(A - \lambda I)(X) = 0$.

$$(A - (-1)I)\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0.$$

$$\left[\begin{pmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right]\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} =$$

$$\begin{pmatrix} 4 & -4 & 4 \\ 1 & -1 & 4 \\ 1 & -1 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\Rightarrow \quad -4x_1 - 4x_2 + 4x_3 = 0$

$(\div -4) \quad +x_1 + x_2 - x_3 = 0$ ——— (1)

~~and~~ $\left.\begin{aligned} x_1 - x_2 + 4x_3 &= 0 \\ x_1 - x_2 + 4x_3 &= 0 \end{aligned}\right\}$ ——— (2)

By cross multiplication rule we get

$$\begin{matrix} 1 & -1 & \cancel{4} & 1 \\ 1 & -1 & 4 & 1 \end{matrix}$$

$$\frac{x_1}{\begin{vmatrix} 1 & -1 \\ 1 & -1 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} -1 & 1 \\ -1 & 4 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 1 \\ 4 & 1 \end{vmatrix}}$$

$$\frac{x_1}{-1+1} = \frac{x_2}{-4+1} = \frac{x_3}{1-4}$$

$$\frac{x_1}{0} = \frac{x_2}{-3} = \frac{x_3}{-3}$$

$$X = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\frac{x_1}{0} = \frac{x_2}{1} = \frac{x_3}{1}$$

The Eigen Vector is $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

Eigen Vectors for Symmetric or Non-Symmetric matrices with non-repeated Eigen Values.

Ex: 1 Find the Eigen Values and Eigen Vectors of the matrix $\begin{pmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{pmatrix}$

Solution:

Let $A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{pmatrix}$ above

The characteristic equation is given by

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$$

S_1 = sum of the diagonal elements

$= 2+1+1 = 4$

S_2 = sum of the minors of the diagonal elts

$$= \begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}$$

$$= -3 + 1 + 1$$

$S_2 = -1$

$$S_3 = |A| = 2\begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & -2 \\ -1 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ -1 & -2 \end{vmatrix}$$

$$= 2(1-4) - 1(1-2) - 1(-2+1)$$

$$= 2(-3) - 1(-1) - 1(-1)$$

$$= -6 + 2 + 2$$

$S_3 = -4$

$\therefore$ The characteristic equation is

$$\lambda^3 - 4\lambda^2 - 1\lambda + 4 = 0 \quad \text{(1)}$$

To find the Eigen Values : (use coe. of λ)

Synthetic division

$$\begin{array}{c|cccc} & \lambda^3 & \lambda^2 & \lambda & con \\ & 1 & -4 & -1 & 4 \\ 1 & 0 & 1 & -3 & -4 \\ \hline & 1 & -3 & -4 & \boxed{0} \end{array}$$

R.W

$\lambda = 1$

$1-4-1+4=0$

$\lambda=0$ in C.E

C.E $=0$ or,

$\lambda=1$

$\lambda=-1$

$$\lambda^2 - 3\lambda - 4 = 0$$

$$(\lambda-4)(\lambda+1)=0$$

$$\Rightarrow \lambda = 4, -1$$

-4

$-4 \quad +1$

$\therefore$ The Eigen values are $\boxed{4, -1, 1}$.

To find the Eigen Vectors : $-1, 1, 4$

The Eigen vectors X of A corresponding to any eigen values λ is given by

$\boxed{(A-\lambda I)X = 0}$ (ie find $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$)

$$\Rightarrow \left(\begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\left[\begin{pmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \right] \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 2-\lambda & 1 & -1 \\ 1 & 1-\lambda & -2 \\ -1 & -2 & 1-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

(1)

~~Case (i) When $\lambda = -1$~~

from ②

$$\therefore \left.\begin{aligned} (2-\lambda)x_1 + x_2 - x_3 &= 0 \\ x_1 + (1-\lambda)x_2 - 2x_3 &= 0 \\ -x_1 - 2x_2 + (1-\lambda)x_3 &= 0 \end{aligned}\right\} \text{— ②}$$

Case (i):

When $\lambda = -1$

$$3x_1 + x_2 - x_3 = 0$$

$$x_1 + 2x_2 - 2x_3 = 0$$

$$-x_1 - 2x_2 + 2x_3 = 0$$

consider any two of the above eqns.

$$\begin{array}{cccccc} x_1 & & x_2 & & x_3 & \\ 1 & -1 & & 3 & & 1 \\ 2 & -2 & & 1 & & 2 \end{array}$$

$$\frac{x_1}{\begin{vmatrix} 1 & -1 \\ 2 & -2 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} -1 & 3 \\ -2 & 1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}}$$

$$\frac{x_1}{-2+2} = \frac{x_2}{-1+6} = \frac{x_3}{6-1}$$

$$\frac{x_1}{0} = \frac{x_2}{5} = \frac{x_3}{5}$$

$$\frac{x_1}{0} = \frac{x_2}{1} = \frac{x_3}{1}$$

$$\Rightarrow X_1 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

Case (ii) when $\lambda = 1$.

Using ② $\quad x_1 + x_2 - x_3 = 0$

$$x_1 + 0x_2 - 2x_3 = 0$$

$$-x_1 - 2x_2 + 0x_3 = 0.$$

$$\begin{matrix} 1 & 1 & -1 \\ 1 & 0 & -2 \end{matrix}$$

Consider first two equations

$$\begin{array}{cccccc} x_1 & & x_2 & & x_3 & \\ 1 & -1 & & 1 & & 1 \\ 0 & -2 & & 1 & & 0 \end{array}$$

$$\frac{x_1}{\begin{vmatrix} 1 & -1 \\ 0 & -2 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} -1 & 1 \\ -2 & 1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}}$$

$$\frac{x_1}{-2} = \frac{x_2}{1} = \frac{x_3}{-1}$$

$$\Rightarrow \frac{x_1}{2} = \frac{x_2}{-1} = \frac{x_3}{1}$$

$$\Rightarrow X_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

Case (iii) when $\lambda = 4$

Using ② $\quad -2x_1 + x_2 - x_3 = 0$

$$x_1 - 3x_2 - 2x_3 = 0$$

$$-x_1 - 2x_2 - 3x_3 = 0.$$

Consider first two equations

$$\begin{array}{cccc} & x_1 & x_2 & x_3 \\ 1 & -1 & -2 & 1 \\ -3 & -2 & 1 & -3 \end{array}$$

$$\frac{x_1}{\begin{vmatrix} 1 & -1 \\ -3 & -2 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} -1 & -2 \\ -2 & 1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -2 & 1 \\ 1 & -3 \end{vmatrix}}$$

$$\frac{x_1}{-5} = \frac{x_2}{-5} = \frac{x_3}{+5}$$

$$\Rightarrow \frac{x_1}{1} = \frac{x_2}{1} = \frac{x_3}{-1}$$

$$\Rightarrow X_3 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$$\boxed{\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}}$$

vectors

$$\begin{pmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{pmatrix}$$

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1) ✓ $\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$

2) ✓ $\begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$

3) ✓ $\begin{pmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{pmatrix}$

Characteristic Eqn	Eigen Value	Eigen Vector
$\lambda^3 - 4\lambda^2 - \lambda + 4 = 0$	$\lambda = -1$	$X_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$
	$\lambda = 1$	$X_2 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$
	$\lambda = 4$	$X_3 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

Non-Symmetric Matrix with repeated roots

Ex: 1 Find the Eigen values and Eigenvectors of the matrix $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

Soln: Let $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

The characteristic equation is

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$$

S_1 = sum of the diagonal elements

$S_1 = 0$

S_2 = Sum of the minors of the diagonal elements

$$= \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$$

$S_2 = -1 - 1 - 1 = -3$

$S_3 = |A| = 0 - 1(-1) + 1(1)$

$S_3 = 1 + 1 = 2$

$\therefore$ The charac. eqn is $\lambda^3 - 0\lambda^2 - 3\lambda - 2 = 0$.

$\lambda = -1$ is a root of the eqn.

R.W

$\lambda = 1$

$1 - 3 - 2 \neq 0$

$\lambda = -1$

$-1 + 3 - 2 = 0$

$$\begin{array}{r|rrrr} & 1 & 0 & -3 & -2 \\ -1 & 0 & -1 & 1 & 2 \\ \hline & 1 & -1 & -2 & \boxed{0} \end{array}$$

$$\lambda^2 - \lambda - 2 = 0.$$

$$(\lambda - 2)(\lambda + 1) = 0$$

$$\Rightarrow \lambda = 2, -1$$

-2

$-2 \quad 1$

∴ The Eigen values of A are $-1, -1$ and 2

To find the Eigen Vectors:

$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ satisfies the eqn $(A - \lambda I)X = 0$.

$$\left(\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0.$$

$$\begin{pmatrix} 0-\lambda & 1 & 1 \\ 1 & 0-\lambda & 1 \\ 1 & 1 & 0-\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0.$$

$$\left.\begin{aligned} -\lambda x_1 + x_2 + x_3 &= 0 \\ x_1 - \lambda x_2 + x_3 &= 0 \\ x_1 + x_2 - \lambda x_3 &= 0 \end{aligned}\right\} \text{——— } (2)$$

Note: If the roots are repeated check whether the matrix is symmetric or non-symmetric and use the property.

Case (i) when $\lambda = 2$

Using (2)

$$\begin{aligned} -2x_1 + x_2 + x_3 &= 0 \\ x_1 - 2x_2 + x_3 &= 0 \\ x_1 + x_2 - 2x_3 &= 0 \end{aligned}$$

Consider first two equations

$$\begin{array}{cccc} & x_1 & x_2 & x_3 \\ 1 & 1 & -2 & 1 \\ -2 & 1 & 1 & -2 \end{array}$$

$$\frac{x_1}{\begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix}}$$

$$\frac{x_1}{3} = \frac{x_2}{3} = \frac{x_3}{3}$$

$$\frac{x_1}{1} = \frac{x_2}{1} = \frac{x_3}{1}$$

$\therefore X_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is the E.V corresponding to $\lambda = 2$

Case (ii): $\lambda = -1$

$$x_1 + x_2 + x_3 = 0$$
$$x_1 + x_2 + x_3 = 0$$
$$x_1 + x_2 + x_3 = 0.$$

Taking the first row alone

put $x_1 = 0 \Rightarrow x_2 + x_3 = 0$

$$x_2 = -x_3$$

i.e., $\frac{x_2}{1} = \frac{x_3}{-1}$

$\therefore X_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

Now, put $x_2 = 0 \Rightarrow x_1 + x_3 = 0.$

$$x_1 = -x_3$$

i.e., $\frac{x_1}{1} = \frac{x_3}{-1}$

$\therefore X_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$\therefore$ The three Eigen Vectors are $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

Example : 2 :

Find the Eigenvalues and Eigenvectors of the matrix $\begin{pmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$.

Soln: Let $A = \begin{pmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$

To find Eigen Values:

The characteristic eqn of A is given by

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0 \quad \text{—— (1)}$$

S_1 = sum of the diagonal elements

$S_1 = 2+1-1 = 2$

S_2 = sum of the minors of the diagonal elements

$$= \begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix} + \begin{vmatrix} 2 & 2 \\ 1 & -1 \end{vmatrix} + \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix}$$

$$= -4 - 4 + 4 = -4$$

$$S_3 = |A| = 2(-1-3) + 2(-1-1) + 2(3-1)$$

$$= 2(-4) + 2(-2) + 2(2)$$

$$= -8 - 4 + 4 = -8$$

$S_3 = -8$

$\therefore$ The characteristic eqn. is given by

$$\lambda^3 - 2\lambda^2 - 4\lambda + 8 = 0 \quad \text{—— (1)}$$

$\lambda = 2$ is a root of the equation

put $\lambda = 1$
$1 - 2 - 4 + 8 \neq 0$
$\lambda = -1$
$-1 - 2 + 4 + 8 \neq 0$
$\lambda = 2$
$8 - 8 - 8 + 8 = 0$

$$\begin{array}{c|cccc} 2 & 1 & -2 & -4 & 8 \\ & 0 & 2 & 0 & -8 \\ \hline & 1 & 0 & -4 & \boxed{0} \end{array}$$

$\Rightarrow \lambda^2 - 0\lambda - 4 = 0$

$\Rightarrow \lambda^2 = 4$

$\boxed{\lambda = \pm 2}$

$\therefore$ The Eigen Values are $\lambda = -2, 2, 2$.

$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ satisfies the equation $(A - \lambda I)X = 0$.

$$\begin{bmatrix} 2-\lambda & -2 & 2 \\ 1 & 1-\lambda & 1 \\ 1 & 3 & -1-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0.$$

$$\left.\begin{aligned} (2-\lambda)x_1 - 2x_2 + 2x_3 &= 0 \\ x_1 + (1-\lambda)x_2 + x_3 &= 0 \\ x_1 + 3x_2 + (-1-\lambda)x_3 &= 0. \end{aligned}\right\} \text{———} (2)$$

Case (i) $\lambda = -2$.

$4x_1 - 2x_2 + 2x_3 = 0$

$x_1 + 3x_2 + x_3 = 0$

$x_1 + 3x_2 + x_3 = 0.$

Using 1st two equations

$$\begin{array}{cccc} & x_1 & x_2 & x_3 \\ -2 & 2 & 4 & -2 \\ 3 & 1 & 1 & 3 \end{array}$$

$$\frac{x_1}{-8} = \frac{x_2}{-2} = \frac{x_3}{14}$$

$$\Rightarrow \frac{x_1}{4} = \frac{x_2}{1} = \frac{x_3}{-7}$$

$X_1 = \begin{bmatrix} 4 \\ 1 \\ -7 \end{bmatrix}$ is the Eigen vector for $\lambda = -2$

Case (ii) $\lambda = 2$.

$$0x_1 - 2x_2 + 2x_3 = 0$$

$$x_1 - x_2 + x_3 = 0$$

$$x_1 + 3x_2 - 3x_3 = 0$$

consider first two equations

$$\begin{array}{ccccccc} & x_1 & & x_2 & & x_3 & \\ -2 & & 2 & & 0 & & -2 \\ -1 & & 1 & & 1 & & -1 \end{array}$$

$$\frac{x_1}{0} = \frac{x_2}{2} = \frac{x_3}{2}$$

$$\frac{x_1}{0} = \frac{x_2}{1} = \frac{x_3}{1}$$

$$X_2 = X_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

cha. eqn	Eigen Value	Eigen Vector
$\lambda^3 - 2\lambda^2 - 4\lambda + 8 = 0$	$\lambda = -2$	$X_1 = \begin{bmatrix} 4 \\ 1 \\ -7 \end{bmatrix}$
	$\lambda = 2$, $\lambda = 2$	$X_2 = X_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$