

SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

STE INSTITUTIONS

DEPARTMENT OF MATHEMATICS

Problems :

1) Reduce the Quadratic form 6x2+3y2+3z2-4xy-
2yz+4zx into Canonical form by an orthogonal
transformation - Discuss its nature .
$\frac{\sqrt{\text{Solution}}}{Q} = 6x^3 + 3y^3 + 3z^4 - 4xy - 2yz + 4zx$
$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$
CHARLES AND ADDRESS
$\lambda^3 - c_1 \lambda^2 + c_2 \lambda - c_3 = 0 \longrightarrow \bigcirc$
$C_1 = 6 + 3 + 3 = 12$
$C_{2} = \begin{vmatrix} b & -2 \\ -2 & 3 \end{vmatrix} + \begin{vmatrix} b & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 3 & -1 \\ -1 & 3 \end{vmatrix} = 36$
$C_3 = \begin{vmatrix} b & -2 & 2 \\ -2 & 3 & -1 \end{vmatrix} = 32$
12 -1 3]
$\lambda^{3} - 12 \lambda^{2} + 36 \lambda - 32 = 0$
$\left[\lambda=2,2,8\right]$
$(A - \lambda I) X = 0$
$\begin{pmatrix} 6-\lambda & -2 & 2\\ -2 & 3-\lambda & -1\\ 2 & -1 & 3-\lambda \end{pmatrix} \begin{pmatrix} x_1\\ x_2\\ x_3 \end{pmatrix} = \begin{pmatrix} 0\\ 0\\ 0\\ 0 \end{pmatrix}$



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$$\underbrace{Case(i)}_{i} : \underbrace{\lambda = S}_{\left(\begin{array}{c} -2 & -2 & 2 \\ -2 & -5 & -1 \\ 2 & -1 & -5 \end{array}\right)} \begin{pmatrix} \pi_{1} \\ \pi_{2} \\ \pi_{3} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
\frac{\pi_{1}}{12} = \underbrace{\pi_{2}}_{-6} = \underbrace{\pi_{3}}_{-6} \\
X_{1} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

Case (ii) : A = 2

$$\begin{pmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{pmatrix} \begin{pmatrix} 7_1 \\ 7_2 \\ 7_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

All the three rows are equal

Put $X_1 = 0$, $-2X_2 = -2X_3$

$$\frac{\chi_2}{I} = \frac{\chi_3}{I}$$

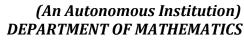
The given matrix is a symmetric matrix. In this

mathix, the third eigen valo vector X3 is orthogonal to x, & x2.

$$X_{3}^{T}X_{i} = 0 \implies (a \ b \ c) \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = 0 \implies (a - b + c = 0 \longrightarrow (i))$$
$$X_{3}^{T}X_{2} = 0 \implies (a \ b \ c) \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0 \implies (a + b + c = 0 \longrightarrow (i))$$



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Solving (i) & (ii), $X_3 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$

The modal mataix is ,

$$M = \begin{pmatrix} 2 & 0 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$$

The normalized matrix is,

$$N = \begin{pmatrix} 2/\sqrt{6} & 0 & 1/\sqrt{3} \\ -1/\sqrt{6} & 1/\sqrt{2} & 1/\sqrt{3} \\ 1/\sqrt{6} & 1/\sqrt{2} & -1/\sqrt{3} \end{pmatrix}$$

$$N^{\mathsf{T}}AN = \begin{pmatrix} 8 & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix} = \mathcal{D}$$

Canonical form :

$$\begin{array}{c} \gamma^{T} \mathfrak{D} \gamma = (y_{1} \quad y_{2} \quad y_{3}) \begin{pmatrix} 8 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} y_{1} \\ y_{2} \\ y_{3} \end{pmatrix} \\ = 8 y_{1}^{2} + 2 y_{2}^{2} + 2 y_{3}^{2} \end{pmatrix}$$

Index = No. of positive servare terms = 3 (3) Rank = No. of hon-zero eigen values = 3 (r) Signature = 28-r = 6-3 = 3

The gruadaatic form is positive definite.