SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
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DEPARTMENT OF BIOMEDICAL ENGINEERING

19BMB302 - BIOMEDICAL SIGNAL PROCESSING

III YEAR/ V SEMESTER

UNIT III INFINITE IMPULSE RESPONSE FILTERS

INFINITE IMPULSE RESPONSE FILTERS

Characteristics of practical frequency selective filters. characteristics of commonly used analog filters - Butterworth filters, Chebyshev filters. Design of IIR filters from analog filters (LPF, HPF, BPF, BRF) - Approximation of derivatives, Impulse invariance method, Bilinear transformation. Frequency transformation in the analog domain. Structure of IIR filter - direct form I, direct form II, Cascade, parallel realizations.

Comparison between Butterworth Filter and Chebyshev Filter

1. The magnitude response of Butterworth filter decreases monotonically as the frequency Ω increases from 0 to ∞, whereas the magnitude response of the Chebyshev filter exhibits ripples in the passband or stopband according to the type.
2. The transition band is more in Butterworth filter when compared to Chebyshev filter.
3. The poles of the Butterworth filter lie on a circle, whereas the poles of the Chebyshev filter lie on an ellipse.
4. For the same specifications, the number of poles in Butterworth are more when compared to the Chebyshev filter i.e., the order of the Chebyshev filter is less than that of Butterworth. This is a great advantage because less number of discrete components will be necessary to construct the filter.

5.9 Steps to design an analog Chebyshev lowpass filter

1. From the given specifications find the order of the filter N.
2. Round off it to the next higher integer.
3. Using the following formulas find the values of a and b, which are minor and major axis of the ellipse respectively.

$$a = \Omega_p \frac{[\mu^{1/N} - \mu^{-1/N}]}{2}; \quad b = \Omega_p \left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right]$$

where

$$\mu = \varepsilon^{-1} + \sqrt{\varepsilon^{-2} + 1}$$

$$\varepsilon = \sqrt{10^{0.1\alpha_p} - 1}$$

Ω_p = Passband frequency

α_p = Maximum allowable attenuation in the passband

($\because$ For normalized Chebyshev filter $\Omega p = 1$ rad/sec)

4. Calculate the poles of Chebyshev filter which lie on an ellipse by using the formula.

$$s_k = a\cos\phi_k + jb\sin\phi_k \quad k = 1, 2, \ldots, N$$

$$\text{where } \phi_k = \frac{\pi}{2} + \left(\frac{2k-1}{2N}\right)\pi \quad k = 1, 2, \ldots, N$$

5. Find the denominator polynomial of the transfer function using the above poles.

6. The numerator of the transfer function depends on the value of N.

   (a) For N odd substitute $s = 0$ in the denominator polynomial and find the value. This value is equal to the numerator of the transfer function.
   ($\because$ For N odd the magnitude response $|H(j\Omega)|$ starts at 1.)

   (b) For N even substitute $s = 0$ in the denominator polynomial and divide the result by $\sqrt{1+\varepsilon^2}$. This value is equal to the numerator.

Example 5.6 Given the specifications $\alpha_p = 3\,\text{dB}$; $\alpha_s = 16\,\text{dB}$; $f_p = 1\,\text{KHz}$ and $f_s = 2\,\text{KHz}$. Determine the order of the filter using Chebyshev approximation. Find $H(s)$.

Solution

From the given data we can find

$$\Omega_p = 2\pi \times 1000\,\text{Hz} = 2000\,\pi\ \text{rad/sec}$$
$$\Omega_s = 2\pi \times 2000\,\text{Hz} = 4000\,\pi\ \text{rad/sec}$$

and $\alpha_p = 3\,\text{dB}$; $\alpha_s = 16\,\text{dB}$.

Step 1:

$$N \geq \frac{\cosh^{-1}\sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\cosh^{-1}\dfrac{\Omega_s}{\Omega_p}} = \cosh^{-1}\frac{\sqrt{\dfrac{10^{1.6}-1}{10^{0.3}-1}}}{\cosh^{-1}\dfrac{4000\pi}{2000\pi}}$$
$$= 1.91$$

Step 2: Rounding N to next higher value we get $N = 2$.

For N even, the oscillatory curve starts from $\frac{1}{\sqrt{1+\varepsilon^2}}$.

Step 3: The values of minor axis and major axis can be found as below

$$\varepsilon = (10^{0.1\alpha_p} - 1)^{0.5} = (10^{0.3} - 1)^{0.5} = 1$$

$$\mu = \varepsilon^{-1} + \sqrt{1+\varepsilon^{-2}} = 2.414$$

$$a = \Omega_p \frac{[\mu^{1/N} - \mu^{-1/N}]}{2} = 2000\pi \frac{[(2.414)^{1/2} - (2.414)^{-1/2}]}{2} = 910\pi$$

$$b = \Omega_p \frac{[\mu^{1/N} + \mu^{-1/N}]}{2} = 2000\pi \frac{[(2.414)^{1/2} + (2.414)^{-1/2}]}{2} = 2197\pi$$

Step 4: The poles are given by

$$s_k = a\cos\phi_k + jb\sin\phi_k, \quad k = 1, 2$$

$$\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N} \quad k = 1, 2$$

$$\phi_1 = \frac{\pi}{2} + \frac{\pi}{4} = 135^\circ$$

$$\phi_2 = \frac{\pi}{2} + \frac{3\pi}{4} = 225^\circ$$

$$s_1 = a\cos\phi_1 + j\,b\sin\phi_1 = -643.46\pi + j1554\pi$$

$$s_2 = a\cos\phi_2 + j\,b\sin\phi_2 = -643.46\pi - j1554\pi$$

Step 5: The denominator of $H(s) = (s + 643.46\pi)^2 + (1554\pi)^2$

Step 6: The numerator of $H(s) = \dfrac{(643.46\pi)^2 + (1554\pi)^2}{\sqrt{1+\varepsilon^2}} = (1414.38)^2\pi^2$

The transfer function $H(s) = \dfrac{(1414.38)^2\pi^2}{s^2 + 1287\pi s + (1682)^2\pi^2}$.

Example 5.7 Obtain an analog Chebyshev filter transfer function that satisfies the constraints $\frac{1}{\sqrt{2}} \leq |H(j\Omega)| \leq 1; \quad 0 \leq \Omega \leq 2$

$$|H(j\Omega)| < 0.1; \quad \Omega \geq 4$$

Solution

Step 1: From the given data we can find that

$$\frac{1}{\sqrt{1+\varepsilon^2}} = \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{1+\lambda^2}} = 0.1,$$

$\Omega_p = 2$ and $\Omega_s = 4$, from which we can obtain $\varepsilon = 1$ and $\lambda = 9.95$.

We know

$$N \geq \frac{\cosh^{-1}\frac{\lambda}{\varepsilon}}{\cosh^{-1}\frac{\Omega_s}{\Omega_p}} = \frac{\cosh^{-1} 9.95}{\cosh^{-1} 2} = 2.269$$

Step 2: Rounding N to next higher value we get $N = 3$. For N odd, the oscillatory curve starts from unity as shown in Fig. 5.12.

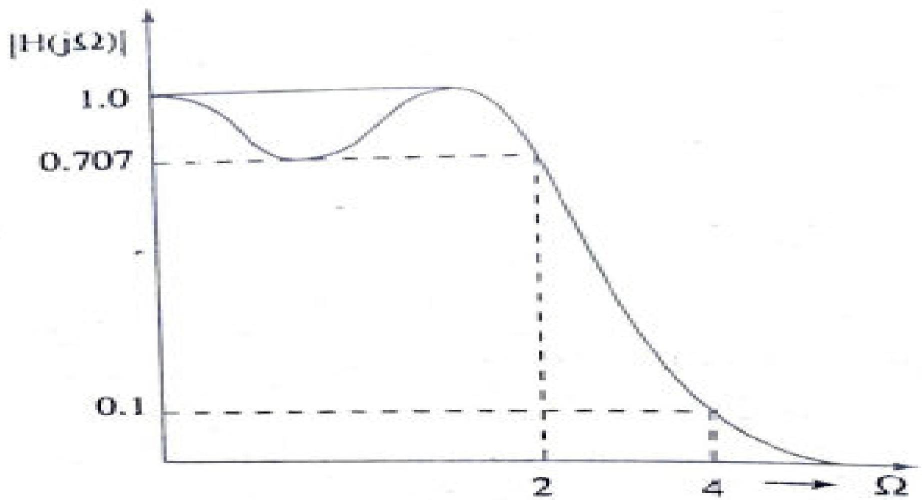

Fig. 5.12 Magnitude response of example 5.7.

Step 3: Finding the values of a and b

$$\mu = \varepsilon^{-1} + \sqrt{1+\varepsilon^{-2}} = 2.414$$

$$a = \Omega_p\left[\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right] = 2\left[\frac{(2.414)^{1/3} - (2.414)^{-1/3}}{2}\right]$$
$$= 0.596$$

$$b = \Omega_p\left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right] = 2\left[\frac{(2.414)^{1/3} + (2.414)^{-1/3}}{2}\right]$$
$$= 2.087$$

Step 4: To calculate the poles of Chebyshev filter

$$\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N} \quad k = 1, 2, 3$$

$$\phi_1 = 120°,\ \phi_2 = 180°,\ \phi_3 = 240°$$

We know $s_k = a\cos\phi_k + jb\sin\phi_k \quad k = 1, 2, 3$ from which we get

$$s_1 = a\cos\phi_1 + jb\sin\phi_1 = 0.596\cos 120° + j2.087\sin 120° = -0.298 + j1.807$$

$$s_2 = a\cos\phi_2 + jb\sin\phi_2 = 0.596\cos 180° + j2.087\sin 180° = -0.596$$

$$s = a\cos\phi_3 + jb\sin\phi_3 = 0.596\cos 240° + j2.087\sin 240° = -0.298 - j1.807$$

Step 5: The denominator polynomial is given by

$$(s + 0.596)\{(s + 0.298) - j1.807\}\{(s + 0.298) + j1.807\}$$

$$= (s + 0.596)[(s + 0.298)^2 + (1.807)^2]$$

$$= (s + 0.596)(s^2 + 0.596s + 3.354)$$

Step 6: The numerator of $H(s)$ can be obtained by substituting $s = 0$ (for N odd) in the denominator.

Therefore the numerator of $H(s) = 2$

The transfer function of Chebyshev filter for the given specifications is given by

$$H(s) = \frac{2}{(s + 0.596)(s^2 + 0.596s + 3.354)}$$

Example 5.8 Determine the order and the poles of a type I lowpass Chebyshev filter that has a 1 dB ripple in the passband and passband frequency $\Omega_p = 1000\pi$, a stopband frequency of 2000π and an attenuation of 40 dB or more.

Solution

Given data $\alpha_p = 1$ dB; $\Omega_p = 1000\pi$ rad/sec; $\alpha_s = 40$ dB
$\Omega_s = 2000\pi$ rad/sec

$$N \geq \frac{\cos h^{-1}\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\cos h^{-1}\frac{\Omega_s}{\Omega_p}} \geq \frac{\cos h^{-1}\sqrt{\frac{10^{4}-1}{10^{0.1}-1}}}{\cos h^{-1}\frac{2000\pi}{1000\pi}} = 4.536$$

i.e., $N = 5$

$$\varepsilon = \sqrt{10^{0.1\alpha_p}-1} = 0.508; \quad \mu = \varepsilon^{-1} + \sqrt{1+\varepsilon^{-2}} = 4.17$$

$$a = \Omega_p\left[\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right] = 289.5\pi; \quad b = \Omega_p\left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right] = 1041\pi$$

$$\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N} \quad k = 1, 2, \ldots 5$$

$$\phi_1 = 180°; \ \phi_2 = 144°; \ \phi_3 = 180°; \ \phi_4 = 216°; \ \phi_5 = 252°$$

$$s_k = a\cos\phi_k + jb\sin\phi_k \quad k = 1, 2, \ldots 5$$

$$s_1 = -89.5\pi + j989\pi; \quad s_2 = -234.2\pi + j612\pi; \quad s_3 = -289.5\pi$$

$$s_4 = -234.2\pi - j612\pi; \quad s_5 = -89.5\pi - j989\pi$$

Example 5.9 Design a Chebyshev filter with a maximum passband attenuation of 2.5 dB; at $\Omega_p = 20$ rad/sec and the stopband attenuation of 30 dB at $\Omega_s = 50$ rad/sec.

Solution

Given

$$\Omega_p = 20 \quad \text{rad/sec}; \quad \alpha_p = 2.5\,\text{dB};$$

$$\Omega_s = 50 \quad \text{rad/sec}; \quad \alpha_s = 30\,\text{dB};$$

$$N = \frac{\cosh^{-1} \lambda/\varepsilon}{\cosh^{-1} 1/k}$$

$$\lambda = \sqrt{10^{0.1\alpha_s} - 1} = 31.607$$

$$\varepsilon = \sqrt{10^{0.1\alpha_p} - 1} = 0.882$$

$$k = \frac{\Omega_p}{\Omega_s} = 0.4$$

Now

$$N \geq \frac{\cosh^{-1} \dfrac{31.607}{0.882}}{\cosh^{-1} \dfrac{1}{0.4}} = 2.726$$

i.e., $N = 3$

$$\mu = \varepsilon^{-1} + \sqrt{1 + \varepsilon^{-2}} = 2.65$$

$$a = \Omega_p \frac{[\mu^{1/N} - \mu^{-1/N}]}{2} = 6.6$$

$$b = \Omega_p \left[\frac{\mu^{1/N} + \mu^{-1/N}}{2} \right] = 21.06$$

$$s_k = a \cos \phi_k + jb \sin \phi_k; \quad k = 1, 2, 3$$

$$\phi_k = \frac{\pi}{2} + \left(\frac{2k - 1}{2N} \right) \pi; \quad k = 1, 2, 3$$

$$\phi_1 = 120^\circ, \phi_2 = 180^\circ, \phi_3 = 240^\circ$$

$$s_1 = -3.3 + j18.23$$

$$s_2 = -6.6$$

$$s_3 = -3.3 - j18.23$$

Denominator of $H(s) = (s + 6.6)(s^2 + 6.6s + 343.2)$

Numerator of $H(s) = (6.6)(343.2) = 2265.27$

Transfer function $H(s) = \dfrac{2265.27}{(s + 6.6)(s^2 + 6.6s + 343.2)}$