SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
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DEPARTMENT OF BIOMEDICAL ENGINEERING

19BMB302 - BIOMEDICAL SIGNAL PROCESSING

III YEAR/ V SEMESTER

UNIT III INFINITE IMPULSE RESPONSE FILTERS

INFINITE IMPULSE RESPONSE FILTERS

Characteristics of practical frequency selective filters. characteristics of commonly used analog filters - Butterworth filters, Chebyshev filters. Design of IIR filters from analog filters (LPF, HPF, BPF, BRF) - Approximation of derivatives, Impulse invariance method, Bilinear transformation. Frequency transformation in the analog domain. Structure of IIR filter - direct form I, direct form II, Cascade, parallel realizations.

Continuous-time IIR filters

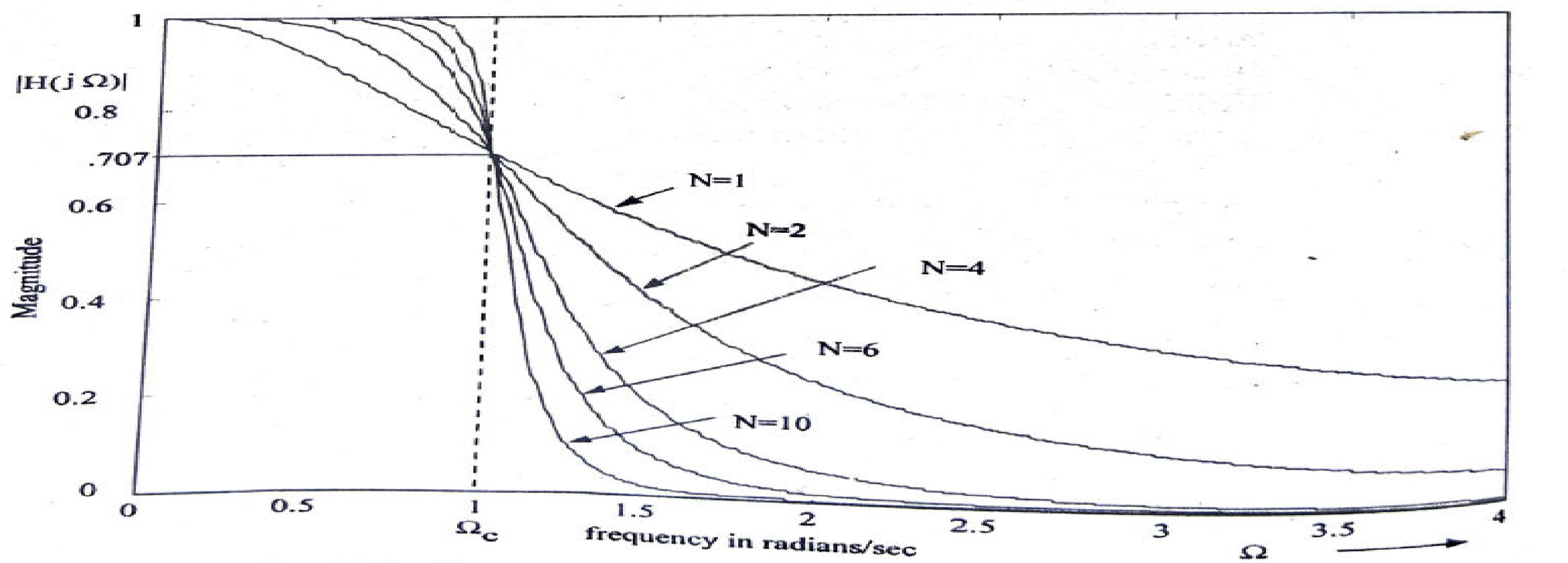
- Butterworth filters
- Chebyshev Type I filters
- Chebyshev Type II filters

Analog Filter Type	**Pass-Band Ripple**	**Stop-Band Ripple**	**Transition Band**
Butterworth	Monotonic (Maximally Flat)	Monotonic	Wide
Chebyshev-I	Equi-ripple	Monotonic	Narrow
Chebyshev-II	Monotonic	Equi-ripple	Narrow

Analog lowpass Butterworth Filter

The magnitude function of the Butterworth lowpass filter is given by

$$|H(j\Omega)| = \frac{1}{\left[1 + (\Omega/\Omega_c)^{2N}\right]^{1/2}} \quad N = 1, 2, 3, \ldots$$



The following table gives Butterworth polynomials for various values of N for $\Omega_c = 1$ rad/sec.

List of Butterworth Polynomials

N	Denominator of $H(s)$
1	$s + 1$
2	$s^2 + \sqrt{2}s + 1$
3	$(s + 1)(s^2 + s + 1)$
4	$(s^2 + 0.76537s + 1)(s^2 + 1.8477s + 1)$
5	$(s + 1)(s^2 + 0.61803s + 1)(s^2 + 1.61803s + 1)$
6	$(s^2 + 1.931855s + 1)(s^2 + \sqrt{2}s + 1)(s^2 + 0.51764s + 1)$
7	$(s + 1)(s^2 + 1.80194s + 1)(s^2 + 1.247s + 1)(s^2 + 0.445s + 1)$

For fourth order Butterworth filter the transfer function for $\Omega_c = 1$ rad/sec is given by

$$H(s) = \frac{1}{(s^2 + 0.76536s + 1)(s^2 + 1.84776s + 1)}$$

Example 5.1 Given the specification $\alpha_p = 1\,\text{dB}$; $\alpha_s = 30\,\text{dB}$; $\Omega_p = 200$ rad/sec; $\Omega_s = 600$ rad/sec. Determine the order of the filter.

Solution

From Eq. (5.25)

$$A = \frac{\lambda}{\varepsilon} = \left(\frac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}\right)^{0.5}$$

$$= \left(\frac{10^3 - 1}{10^{0.1} - 1}\right)^{0.5} = 62.115$$

From Eq. (5.26)

$$k = \frac{\Omega_p}{\Omega_s} = \frac{200}{600} = \frac{1}{3}$$

From Eq. (5.27)

$$N \geq \frac{\log A}{\log 1/k}$$

$$\geq \frac{\log 62.115}{\log 3} = 3.758$$

Rounding off N to the next higher integer we get $N = 4$.

Example 5.2 Determine the order and the poles of lowpass Butterworth filter that has a 3 dB attenuation at 500 Hz and an attenuation of 40 dB at 1000 Hz.

Solution

Given data $\alpha_p = 3$ dB; $\alpha_s = 40$ dB; $\Omega_p = 2 \times \pi \times 500 = 1000\pi$ rad/sec.
$\Omega_s = 2 \times \pi \times 1000 = 2000\pi$ rad/sec.

The order of the filter

$$N \geq \frac{\log \sqrt{\dfrac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}}}{\log \dfrac{\Omega_s}{\Omega_p}}$$

$$\geq \frac{\log \sqrt{\dfrac{10^4 - 1}{10^{0.3} - 1}}}{\log \dfrac{2000\pi}{1000\pi}} = 6.6$$

Rounding 'N' to nearest higher value we get $N = 7$.

The poles of Butterworth filter are given by

$$s_k = \Omega_c e^{j\phi_k} = 1000\pi e^{j\phi_k} \qquad k = 1, 2, \ldots 7$$

where $\phi_k = \dfrac{\pi}{2} + \dfrac{(2k-1)\pi}{2N} \qquad k = 1, 2, \ldots 7.$

Steps to design an analog Butterworth lowpass filter

1. From the given specifications find the order of the filter N.
2. Round off it to the next higher integer.
3. Find the transfer function $H(s)$ for $\Omega_c = 1$ rad/sec for the value of N.
4. Calculate the value of cutoff frequency Ω_c.
5. Find the transfer function $H_a(s)$ for the above value of Ω_c by substituting s $\frac{s}{\Omega_c}$ in $H(s)$.

Example 5.4 Design an analog Butterworth filter that has a – 2 dB passband attenuation at a frequency of 20 rad/sec and atleast – 10 dB stopband attenuation at 30 rad/sec.

Solution

Given $\alpha_p = 2\,\text{dB}; \quad \Omega_p = 20\,\text{rad/sec}$

$\alpha_s = 10\,\text{dB}; \quad \Omega_s = 30\,\text{rad/sec}$

$$N \geq \frac{\log \sqrt{\dfrac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}}}{\log \dfrac{\Omega_s}{\Omega_p}}$$

$$\geq \frac{\log \sqrt{\frac{10 - 1}{10^{0.2} - 1}}}{\log \frac{30}{20}}$$

$$\geq 3.37$$

Rounding off N to the next highest integer we get

$$N = 4$$

The normalized lowpass Butterworth filter for $N = 4$ can be found from table 5.1 as

$$H(s) = \frac{1}{(s^2 + 0.76537s + 1)(s^2 + 1.8477s + 1)}$$

From Eq. (5.31) we have

$$\Omega_c = \frac{\Omega_p}{(10^{0.1\alpha_p} - 1)^{1/2N}} = \frac{20}{(10^{0.2} - 1)^{1/8}} = 21.3868$$

The transfer function for $\Omega_c = 21.3868$ can be obtained by substituting

$$s \rightarrow \frac{s}{21.3868} \text{ in } H(s)$$

$$\text{i.e.,} \quad H(s) = \frac{1}{\left(\frac{s}{21.3868}\right)^2 + 0.76537\left(\frac{s}{21.3868}\right) + 1} \times \frac{1}{\left(\frac{s}{21.3868}\right)^2 + 1.8477\left(\frac{s}{21.3868}\right) + 1}$$

$$= \frac{0.20921 \times 10^6}{(s^2 + 16.3686s + 457.394)(s^2 + 39.5176s + 457.394)}$$

For the given specifications design an analog Butterworth filter. $0.9 \le |H(j\Omega)| \le 1$ for $0 \le \Omega \le 0.2\pi$. $|H(j\Omega)| \le 0.2$ for $0.4\pi \le \Omega \le \pi$.

Solution

From the data we find $\Omega_p = 0.2\pi$; $\Omega_s = 0.4\pi$; $\frac{1}{\sqrt{1+\varepsilon^2}} = 0.9$ and $\frac{1}{\sqrt{1+\lambda^2}} = 0.2$ from which we obtain

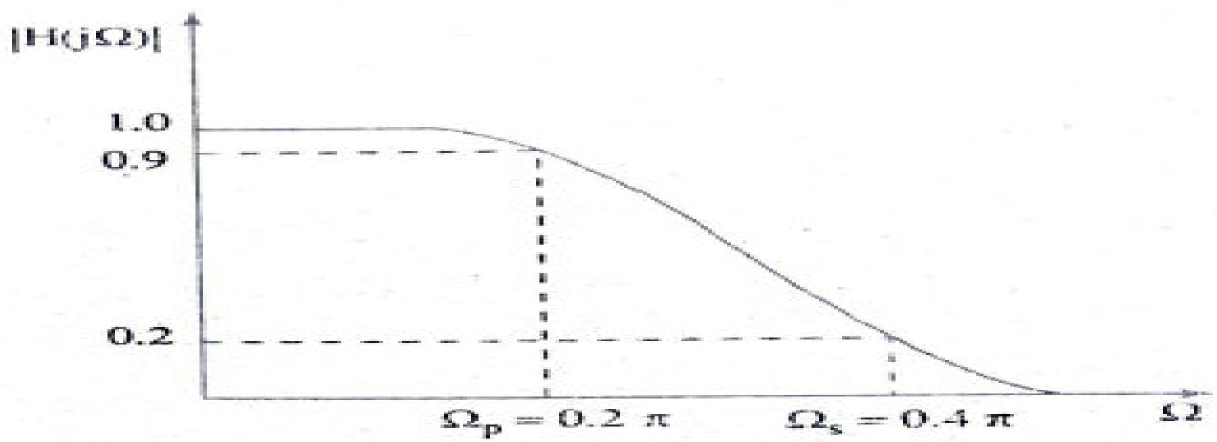


Fig. 5.8 Magnitude response of example 5.5

$\varepsilon = 0.484$ and $\lambda = 4.898$

$$N \ge \frac{\log\left(\frac{\lambda}{\varepsilon}\right)}{\log\frac{\Omega_s}{\Omega_p}} = \frac{\log\frac{4.898}{0.484}}{\log\left(\frac{0.4\pi}{0.2\pi}\right)} = 3.34$$

i.e., $N = 4$

From the table 5.1, for $N = 4$, the transfer function of normalised Butterworth filter is

$$H(s) = \frac{1}{(s^2 + 0.76537s + 1)(s^2 + 1.8477s + 1)}$$

we know $\Omega_c = \dfrac{\Omega_p}{(10^{0.1\alpha_p} - 1)^{1/2N}} = \dfrac{\Omega_p}{\varepsilon^{1/N}} = \dfrac{0.2\pi}{(0.484)^{1/4}} = 0.24\pi.$

$H(s)$ for $\Omega_c = 0.24\pi$ can be obtained by substituting $s \rightarrow \dfrac{s}{0.24\pi}$ in $H(s)$ i.e.,

$$H(s) = \frac{1}{\left\{\left(\frac{s}{0.24\pi}\right)^2 + 0.76537\left(\frac{s}{0.24\pi}\right) + 1\right\}} \times \frac{1}{\left(\frac{s}{0.24\pi}\right)^2 + 1.8477\left(\frac{s}{0.24\pi}\right) + 1}$$

$$= \frac{0.323}{(s^2 + 0.577s + 0.0576\pi^2)(s^2 + 1.393s + 0.0576\pi^2)}$$