SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
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DEPARTMENT OF BIOMEDICAL ENGINEERING

19BMB302 - BIOMEDICAL SIGNAL PROCESSING

III YEAR/ V SEMESTER

UNIT II FINITE IMPULSE RESPONSE FILTERS

- Introduction to FIR
- Linear phase FIR filter
- FIR filter design using window method
- Low Pass Filter
- Frequency sampling method
- Realization of FIR filter using direct form 1, Direct form 2
- Realization of FIR filter using Cascade structures
- Realization of FIR filter using parallel structures

Example 6.6 Design an ideal highpass filter with a frequency response

$$H_d(e^{j\omega}) = 1 \text{ for } \frac{\pi}{4} \leq |\omega| \leq \pi$$
$$= 0 \text{ for } |\omega| \leq \frac{\pi}{4}$$

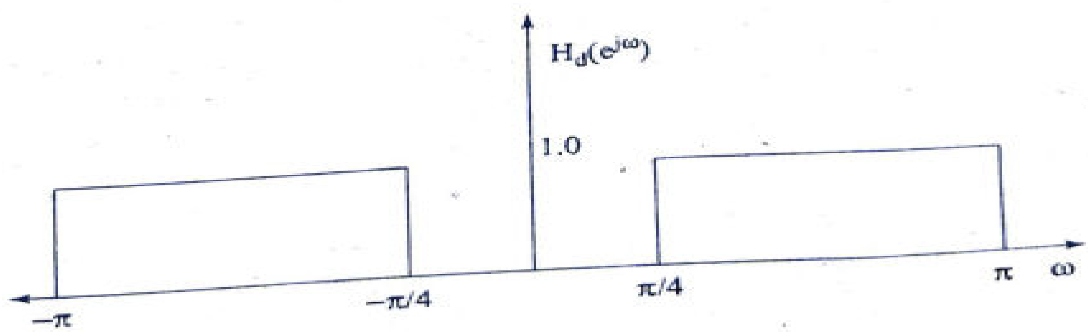
Find the values of $h(n)$ for $N = 11$. Find $H(z)$. Plot the magnitude response.

Solution

The desired frequency response is shown in Fig. 6.10.

We know

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega$$
$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\frac{\pi}{4}} e^{j\omega n}d\omega + \int_{\frac{\pi}{4}}^{\pi} e^{j\omega n}d\omega\right]$$
$$= \frac{1}{2\pi jn}\left[e^{j\omega n}\Big|_{-\pi}^{-\pi/4} + e^{j\omega n}\Big|_{\pi/4}^{\pi}\right]$$



$$= \frac{1}{\pi n(2j)}\left[e^{-j\pi n/4} - e^{-j\pi n} + e^{j\pi n} - e^{j\pi n/4}\right]$$

$$= \frac{1}{\pi n}\left[\sin \pi n - \sin \frac{\pi}{4} n\right] \qquad -\infty \le n \le \infty$$

(6.5

Truncating $h_d(n)$ to 11 samples, we have

$$h(n) = h_d(n) \quad \text{for} \quad |n| \le 5$$
$$= 0 \quad \text{otherwise}$$

For $n = 0$

$$h(0) = \lim_{n\to 0} \frac{\sin \pi n}{\pi n} - \lim_{n \to 0} \frac{\sin \frac{\pi}{4} n}{\pi n}$$
$$= \left((1 - \frac{1}{4}\right)$$

$$\because \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$$
$$\lim_{\theta \to 0} \frac{\sin n\theta}{\theta} = n$$

From the given frequency response we can find that $\alpha = 0$. Therefore, The filt coefficients are symmetrical about $n = 0$ satisfying the condition $h(n) = h(-n)$

For $n = 1$

$$h(1) = h(-1) = \frac{\sin \pi - \sin \frac{\pi}{4}}{\pi} = -0.225$$

Similarly

$$h(2) = h(-2) = \frac{\sin 2\pi - \sin \frac{\pi}{2}}{2\pi} = -0.159$$

$$h(3) = h(-3) = \frac{\sin 3\pi - \sin \frac{3\pi}{4}}{3\pi} = -0.075$$

$$h(4) = h(-4) = \frac{\sin 4\pi - \sin \pi}{4\pi} = 0$$

$$h(5) = h(-5) = \frac{\sin 5\pi - \sin \frac{5\pi}{4}}{5\pi} = 0.045$$

The transfer function of the filter is given by

$$\begin{aligned} H(z) &= h(0) + \sum_{n=1}^{\frac{N-1}{2}} [h(n)(z^n + z^{-n})] \\ &= 0.75 + \sum_{n=1}^{5} [h(n)(z^n + z^{-n})] \\ &= 0.75 - 0.225(z + z^{-1}) - 0.159(z^2 + z^{-2}) - 0.075(z^3 + z^{-3}) \\ &\qquad + 0.045(z^5 + z^{-5}) \end{aligned} \quad (6.$$

The transfer function of the realizable filter is

$$\begin{aligned} H'(z) &= z^{-5} H(z) \\ &= z^{-5}[0.75 - 0.225(z + z^{-1}) - 0.159(z^2 + z^{-2}) - 0.075(z^3 + z^{-3}) \\ &\quad + 0.045(z^5 + z^{-5})] \\ &= 0.045 - 0.075z^{-2} - 0.159z^{-3} - 0.225z^{-4} + 0.75z^{-5} - 0.225z^{-6} \\ &\qquad - 0.159z^{-7} - 0.075z^{-8} + 0.045z^{-10} \end{aligned} \quad (6.$$

From Eq. (6.61) the filter coefficients of causal filter are

$h(0) = h(10) = 0.045; \quad h(1) = h(9) = 0; \quad h(2) = h(8) = -0.075$
$h(3) = h(7) = -0.159; \quad h(4) = h(6) = -0.225; \quad h(5) = 0.75$

$$\overline{H}(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} a(n) \cos \omega n \quad \text{where}$$

$$a(0) = h\left(\frac{N-1}{2}\right) = h(5) = 0.75$$

$$a(n) = 2h\left(\frac{N-1}{2} - n\right)$$

$$a(1) = 2h(5-1) = 2h(4) = -0.45$$

$$a(2) = 2h(5-2) = 2h(3) = -0.318$$

$$a(3) = 2h(5-3) = 2h(2) = -0.15$$

$$a(4) = 2h(5-4) = 2h(1) = 0$$

$$a(5) = 2h(5-5) = 2h(0) = 0.09$$

$$\begin{aligned}\overline{H}(e^{j\omega}) &= a(0) + a(1)\cos\omega + a(2)\cos 2\omega + a(3)\cos 3\omega \\ &\quad + a(4)\cos 4\omega + a(5)\cos 5\omega \\ &= 0.75 - 0.45\cos\omega - 0.318\cos 2\omega - 0.15\cos 3\omega + 0.09\cos 5\omega \qquad \textbf{(6.62)}\end{aligned}$$

ω (in degrees)	0	10	20	30	40	50	60	70	80	90
$\overline{H}(e^{j\omega})$	−0.08	−0.066	−0.0086	0.122	0.34	0.61	0.88	1.05	1.11	1.07
$\|H(e^{j\omega})\|_{dB}$	−22	−23.62	−41.3	−18.2	−9.36	−4.2	−1.1	0.504	0.95	0.587

ω (in degrees)	100	110	120	130	140	150	160	170	180
$\overline{H}(e^{j\omega})$	0.98	0.93	0.94	0.995	1.26	1.05	1.01	0.96	0.94
$\|H(e^{j\omega})\|_{dB}$	−0.132	−0.625	−0.537	−0.037	2	0.48	0.16	−0.31	−0.537

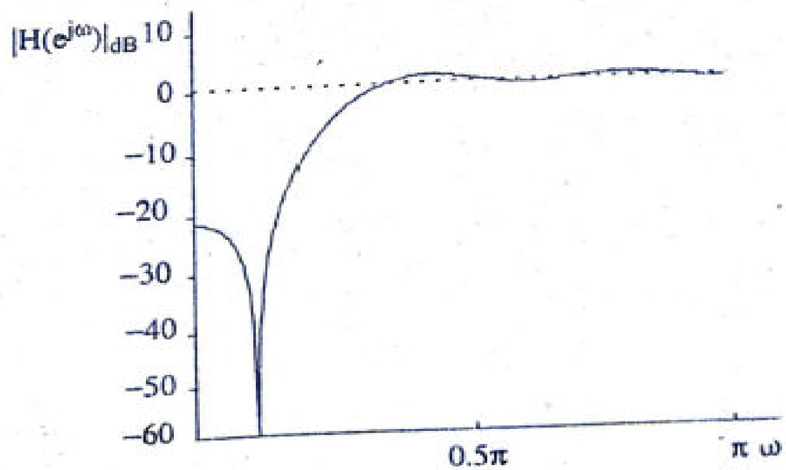


Fig. 6.11 Frequency response of highpass filter of example 6.6

Example 6.8 Design an ideal bandreject filter with a desired frequency response

$$H_d(e^{j\omega}) = 1 \quad \text{for} \quad |\omega| \leq \frac{\pi}{3} \quad \text{and} \quad |\omega| \geq \frac{2\pi}{3}$$
$$= 0 \quad \text{otherwise}$$

Find the value of $h(n)$ for $N = 11$. Find $H(z)$. Plot the magnitude response.

Solution

The desired frequency response is shown in Fig. 6.14.

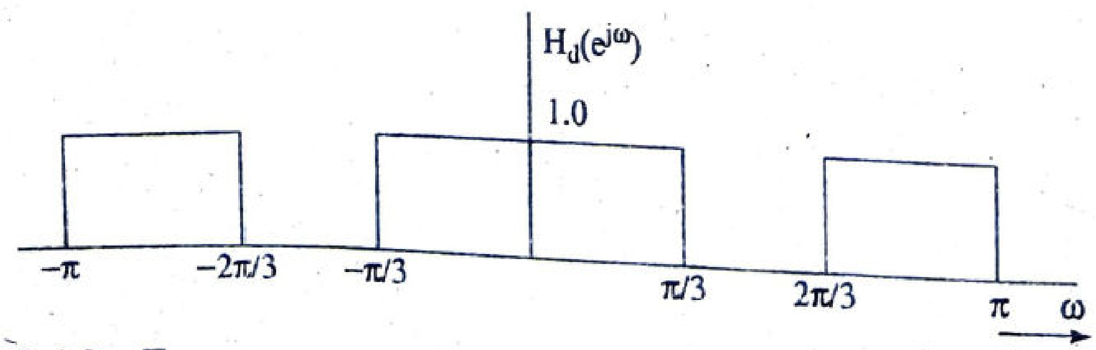


Fig. 6.14 Frequency response of Bandreject filter of example 6.8.

We know

$$\begin{aligned} h_d(n) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega \\ &= \frac{1}{2\pi}\left[\int_{-\pi}^{-2\pi/3} e^{j\omega n}d\omega + \int_{-\pi/3}^{\pi/3} e^{j\omega n}d\omega + \int_{2\pi/3}^{\pi} e^{j\omega n}d\omega\right] \\ &= \frac{1}{2\pi jn}\left[e^{-j2\pi n/3} - e^{-j\pi n} + e^{j\pi n/3} - e^{-j\pi n/3} + e^{j\pi n} - e^{j2\pi n/3}\right] \\ &= \frac{1}{\pi n}\left[\sin\pi n + \sin\frac{\pi}{3}n - \sin\frac{2\pi}{3}n\right] \qquad -\infty \le n \le \infty \end{aligned}$$

Truncating $h_d(n)$ to 11 samples, we have

$$\begin{aligned} h(n) &= h_d(n) \quad \text{for} \quad |n| \le 5 \\ &= 0 \quad \text{otherwise} \end{aligned}$$

The filter coefficients are symmetrical about $n = 0$ satisfying the condition $h(n) = h(-n)$.

For $n = 0$

$$h(0) = \lim_{n \to 0} \left[\frac{\sin \pi n}{\pi n} + \frac{\sin \frac{\pi}{3} n}{\pi n} - \frac{\sin \frac{2\pi}{3} n}{\pi n} \right]$$

$$= \left(1 + \frac{1}{3} - \frac{2}{3} \right) = 0.667$$

$$h(1) = h(-1) = \frac{\sin \pi + \sin \frac{\pi}{3} - \sin \frac{2\pi}{3}}{\pi} = 0$$

$$h(2) = h(-2) = \frac{\sin 2\pi + \sin \frac{2\pi}{3} - \sin \frac{4\pi}{3}}{2\pi} = 0.2757$$

$$h(3) = h(-3) = \frac{\sin 3\pi + \sin \pi - \sin 2\pi}{3\pi} = 0$$

$$h(4) = h(-4) = \frac{\sin 4\pi + \sin \frac{4\pi}{3} - \sin \frac{8\pi}{3}}{4\pi} = -0.1378$$

$$h(5) = h(-5) = \frac{\sin 5\pi + \sin \frac{5\pi}{3} - \sin \frac{10\pi}{3}}{5\pi} = 0$$

The transfer function of the filter is

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} \left[h(n)(z^n + z^{-n}) \right]$$

$$= 0.667 + 0.2757(z^2 + z^{-2}) - 0.1378(z^4 + z^{-4}) \qquad (6.67)$$

The transfer function of the realizable filter

$$H'(z) = z^{-5}H(z)$$
$$= -0.1378z^{-1} + 0.2757z^{-3} + 0.667z^{-5} + 0.2757z^{-7} - 0.1378z^{-9}$$

The filter coefficients of the causal filter are

$$h(0) = h(10) = h(2) = h(8) = h(4) = h(6) = 0$$
$$h(1) = h(9) = -0.1378$$
$$h(3) = h(7) = 0.2757$$
$$h(5) = 0.667$$

$$\overline{H}(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} a(n)\cos\omega n$$

$$a(0) = h\left(\frac{N-1}{2}\right) = h(5) = 0.667$$
$$a(n) = 2h\left(\frac{N-1}{2} - n\right)$$
$$a(1) = 2h(5-1) = 2h(4) = 0$$
$$a(2) = 2h(5-2) = 2h(3) = 0.5514$$
$$a(3) = 2h(5-3) = 2h(2) = 0$$
$$a(4) = 2h(5-4) = 2h(1) = -0.2756$$
$$a(5) = 2h(5-5) = 2h(0) = 0$$

ω (in degrees)	0	15	30	45	60	75	90	105	120	135	150	165	180
$\overline{H}(e^{j\omega})$	0.9428	1.0067	1.08	0.9426	0.529	0.0516	16	0.0516	0.529	0.9426	1.08	1.0067	0.9428
$\lvert H(e^{j\omega})\rvert_{dB}$	−0.5	0.058	0.67	−0.513	−5.53	−25.7	−15.9	−25.7	−5.53	−0.513	0.67	0.058	−0.5

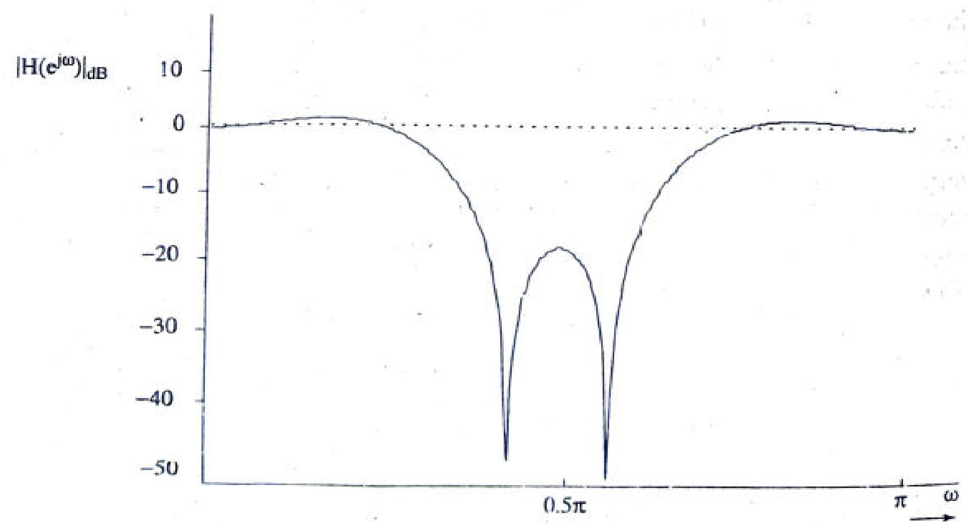


Table 6.2 Filter coefficients of FIR filters.

Type	Coefficients of zero phase filter	Coefficients of linear phase filter with delay $\alpha = \frac{N-1}{2}$
1. Lowpass filter with cutoff frequency ω_c	$h_d(0) = \frac{\omega_c}{\pi}$ $h_d(n) = \frac{\sin \omega_c n}{n\pi} \lvert n \rvert > 0$	$h_d(n) = \frac{\omega_c}{\pi}$ for $n = \alpha$ $= \frac{\sin \omega_c(n-\alpha)}{\pi(n-\alpha)}$ for $n \neq \alpha$
2. Highpass filter with cutoff frequency ω_c	$h_d(0) = 1 - \frac{\omega_c}{\pi}$ $h_d(n) = \frac{-\sin \omega_c n}{n\pi} \lvert n \rvert > 0$	$h_d(n) = 1 - \frac{\omega_c}{\pi}$ for $n = \alpha$ $= \frac{1}{\pi(n-\alpha)}[\sin(n-\alpha)\pi - \sin(n-\alpha)\omega_c] n \neq \alpha$
3. Bandpass filter with cutoff frequencies ω_{c_1} and ω_{c_2}	$h_d(0) = \frac{\omega_{c_2} - \omega_{c_1}}{\pi}$ $h_d(n) = \frac{1}{n\pi}[\sin(\omega_{c_2} n) - \sin(\omega_{c_1} n)] \lvert n \rvert > 0$	$h_d(n) = \frac{\omega_{c_2} - \omega_{c_1}}{\pi}$ for $n = \alpha$ $= \frac{1}{\pi(n-\alpha)}[\sin \omega_{c_2}(n-\alpha) - \sin \omega_{c_1}(n-\alpha)]$
4. Bandreject filter with cutoff frequencies ω_{c_1} and ω_{c_2}	$h_d(0) = 1 - \frac{\omega_{c_2} - \omega_{c_1}}{\pi}$ $= \frac{1}{n\pi}[\sin(\omega_{c_1} n) - \sin(\omega_{c_2} n)]$	$h_d(n) = 1 - \frac{\omega_{c_2} - \omega_{c_1}}{\pi}$ for $n = \alpha$ $= \frac{1}{\pi(n-\alpha)}[\sin \omega_{c_1}(n-\alpha) - \sin \omega_{c_2}(n-\alpha) + \sin(n-\alpha)\pi]$