SNS COLLEGE OF TECHNOLOGY

DEPARTMENT OF MECHANICAL ENGINEERING

19MET201Engineering Thermodynamics

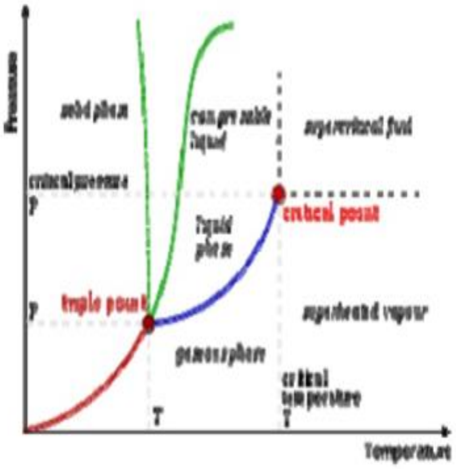
CLAUSIUS–CLAPEYRON RELATION

The **Clausius–Clapeyron relation**, named after Rudolf Clausius and Émile Clapeyron, who defined it sometime after 1834, is a way of characterizing the phase transition between two phases of matter, such as solid and liquid. On a pressure–temperature (P–T) diagram, the line separating the two phases is known as the coexistence curve. The Clausius–Clapeyron relation gives the slope of this curve. Mathematically,

$$\frac{\mathrm{d}P}{\mathrm{d}T} = \frac{L}{T\,\Delta V}$$

where dP / dT is the slope of the coexistence curve, L is the latent heat, T is the temperature, Visthevolumeandchange of the phase transition.

Pressure Temperature Relations



Derivation

Using the state postulate, take the specific entropy, s, for a homogeneous substance to be a function of specific volume, v, and temperature, T.

$$ds = \frac{\partial s}{\partial v} dv + \frac{\partial s}{\partial T} dT.$$

During a phase change, the temperature is constant, so

$$ds = \frac{\partial s}{\partial v} dv.$$

Using the appropriate Maxwell relation gives

$$ds = \frac{\partial P}{\partial T} dv.$$

Since temperature and pressure are constant *during a phase change*, the derivative of pressure with respect to temperature is not a function of the specific volume. Thus the partial derivative may be changed into a total derivative and be factored out when taking an integral from one phase to another,

$$s_2 - s_1 = \frac{dP}{dT}(v_2 - v_1),$$

$$\frac{dP}{dT} = \frac{s_2 - s_1}{v_2 - v_1} = \frac{\Delta s}{\Delta v}.$$

Del is used as an operator to represent— final (2) minus initial (1) For a closed system undergoing an internally reversible process, the first law is

$$du = \delta q + \delta w = T ds - P dv.$$

This leads to a version of the Clausius–Clapeyron equation that is simpler to integrate:

$$\frac{dP}{P} = \frac{\Delta h}{R} \frac{dT}{T^2},$$

$$\ln P = -\frac{\Delta h}{R} \frac{1}{T} + C,$$

$$\ln \frac{P_2}{P_1} = \frac{\Delta h}{R} \left(\frac{1}{T_1} - \frac{1}{T_2} \right).$$

C is a constant of integration

These last equations are useful because they relate saturation pressure and saturation temperature to the enthalpy of phase change, *without* requiring specific volume data. Note that in this last equation, the subscripts 1 and 2 correspond to different locations on the pressure versus temperature phase lines. In earlier equations, they corresponded to different specific volumes and entropies at the same saturation pressure and temperature.