UNIT-1

1. Prove the following implications by constructing truth tables

   a) $P \to Q \Rightarrow P \to (P \wedge Q)$ b) $(P \to Q) \to Q \Rightarrow P \vee Q$

2. By constructing truth table or otherwise obtain the principal conjunctive normal form of $(\neg P \to R) \wedge (Q \leftrightarrow P)$
3. Using truth table or otherwise obtain the principal disjunctive normal form of $P \to (P \wedge (Q \to P))$

4. Show that the premises, " One student in this class knows how to write programs in JAVA" and 'Everyone who knows how to write programs in JAVA can get a high-paying job" imply the conclusion " Someone in this class can get a high paying job"

5. Show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \to R) \wedge (Q \to S)$

6. Derive $P \to (Q \to S)$ using CP rule(if necessary) from the premises $P \to (Q \to S)$ and $Q \to (R \to S)$

7. Verify that validating of the following inference.

   If one person is more successful than another, then he has worked harder to deserve success. Ram has not worked harder than Siva. Therefore, Ram is not more successful than Siva.
8. Prove that $\forall x\ (P(x) \to Q(x)),\ \forall x\ (R(x) \to \neg Q(x)) \Rightarrow \forall x\ (R(x) \to \neg P(x))$
9. Show that $(\forall x)(P(x) \to Q(x)),\ (\exists y)p(y) \Rightarrow (\exists x)Q(x)$
10. Show that $\forall x(P(x) \vee Q(x)) \Rightarrow (\forall x\, P(x)) \vee (\exists x\ Q(x))$ indirect proof of method.
11. Show that the following premises are inconsistent.
    (i) If Jack misses many classes due to illness then he fails in high school.
    (ii) If Jack fails in high school, then he is uneducated.
    (iii) If Jack reads a lot of books, then he is not uneducated
    (iv) Jack misses many classes due to illness and reads a lot of books.
12. Obtain the PCNF of $(\neg P \to R) \wedge (Q \leftrightarrow P)$ and also find PDNF

13. Determine whether the compound preposition $7(Q \to R) \wedge R \wedge (P \to Q)$ is contradiction or tautology
14. Show that $R \vee S$ follows logically from the premises $C \vee D$, $(C \vee D) \to \neg H$, $\neg H \to (A \wedge \neg B)$, and $(A \wedge \neg B) \to (R \vee S)$
15. Prove that $A \to (B \to C), D \to (B \wedge 7C)$ and $A \wedge D$ are inconsistent.

UNIT-2

1. Determine the number of integer between 1 and 250 that are not divisible by 2,3 or 5
2. Find the number of distinct permutations that can be formed from all the letters of each word (1) RADAR (2) UNUSUAL
3. The password for a computer system consists of eight distinct alphabetic characters. Find the number of passwords possible that

(i) End in the string MATH
(ii) Begin with the string CREAM
(iii)Contain the word COMPUTER as a string.

4. Solve the recurrence relation. $a_{n+1} - a_n = 3n^2 - n, n \geq 0, a_0 = 3.$
5. Solve the recurrence relation $a_n - 7a_{n-1} + 10a_{n-2} = 0, n \geq 2$ with $a_0 = 4, a_1 = 17$.

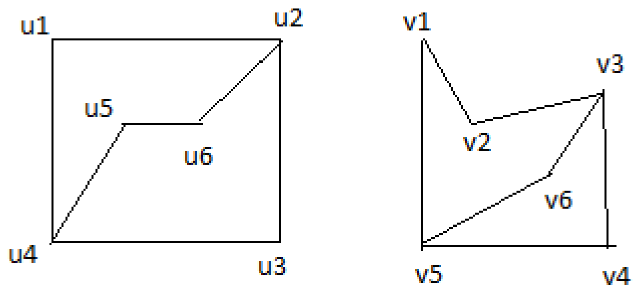
6. Solve the recurrence equation by using the method of generating function $a_n = 3a_{n-1} + 1,\ n \geq 1$ given $a_0 = 1$.

7. Show that if n is a positive integer, then $1 + 2 + 3 + \ldots\ldots + n = \frac{n(n+1)}{2}$ by mathematical induction.
8. Prove by mathematical induction, $8^n - 3^n$ is a multiple of 5.
9. Find the number of integers between 1 and 250 both inclusive that are divisible by any of the integers 2,3,5,7.
10. Determine the number of positive integers n, $1 \leq n \leq 1000$ that are not divisible by 2,3,5 and 7 respectively.
11. In a survey of 100 students, it was found that 30 studied mathematics, 54 studied statistics, 25 studied operation research, 1 studied all the 3 subjects. 20 studied mathematics and statistics, 3 studied mathematics and operation research and 15 studied statistics and operation research.

(i)How many students studied none of these subjects?
(ii) How many students studied only mathematics?

<u>UNIT-3</u>

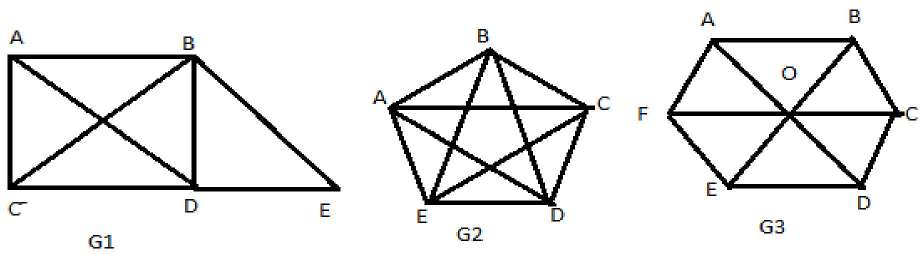
1. Determine whether the graphs G and H given below are isomorphic.



2. Prove that an undirected graph has an even number of vertices of odd degree.
3. Prove that the maximum number of edges in a simple disconnected graph G with n vertices and k components (n-k)(n-k+1)/2
4. Draw a graph that is both Eulerian and Hamiltonian.
5. Prove that a connected graph G is Eulerian iff all the vertices are of even degree.

6. Give an example of a graph which is
   1). Eulerian but not Hamiltonian
   2). Hamiltonian but not Eulerian
   3). Both Eulerian and Hamiltonian
   4). Non-Eulerian and Non-Hamiltonian

7.Find an Euler path or an Euler circuit, if it exist in each of the three graphs below.If it does not exist, explain why?



8. Draw the graph with 5 vertices, ***A*,*B*,*C*,*D*, *E*** such that **deg(*A*)** $=$ **3** , ***B*** is an odd vertex,**deg(*C*)** $=$ **2** and ***D*** and ***E*** are adjacent.

9. If all the vertices of an undirected graph are each of degree ***k*** , show that the number of edges of the graph is a multiple of ***k*** .
10. Draw the complete graph K_5 with vertices ***A,B,C,D, E*** . Draw all complete sub graph of K_5 with 4 vertices.

UNIT-4

1. State and prove Lagrange's theorem.
2. If (G,*) is an group ,show that $(a*b)^{-1}= b^{-1}*a^{-1}$
3. Show that the Kernel of a homomorphism of a group $(G,*)$ into an another group (H',Δ) is a subgroup of G.
4. Let $\mathbf{f : G \to G^1}$ be a onto homomorphism of groups with kernel K. Then $\mathbf{\frac{G}{K} \cong G^1}$ (Fundamental theorem of Homomorphism)
5. If * is the operation defined S=QxQ, the set of ordered pairs of rational numbers given by (a,b)*(x,y)=(ax,ay+b). Show that (S,*) is semi group. Is it commutative? Also find the identity element of S.

6. Prove that every finite group of order "n" is isomorphic to a permutation group of degree n. (Cayley's theorem on permutation group)

7. The intersection of two subgroups of a group is also a subgroup of the group

8. If G is a finite group, then prove that a $O(G)$ = e for any element $a \in G$.

9. Prove the necessary and sufficient condition for a non – empty sub set to be a sub group of a group.
10. If a and b are any two elements of a group<G,*> , show that is an abelian group if and only if .(a* b)^2 = a^2*b^2 .
11. Let $(G,*)$ and (H',Δ) be groups and f is homomorphism from G to H , then prove that the kernel of f is a normal subgroup.
12. If * is a binary operation on the set of real number defined by x*y=x+y+2xy (1)Find (R,*) is a semi group (2) Find the identity element if it exist. (3) Which element has inverse and what are they?

13. Prove that the union of two subgroups of a group G is a subgroup iff one is contained in the other
14. Prove that any two right (or left) cosets of H in G are either disjoint or identical.

UNIT-5

1. Prove that every distributive latice is modular. Is the converse true? Justify your claim.
2. Show tha in a complemented distributive lattice,
   a≤ b ⇔a * b′ = 0 ⇔ a ′⊕ b = 1 ⇔ b ′≤ a ′
3. For any Boolean algebra Prove the following
   (i) x∨y= x∨z and x′∨y= x′∨z⇒y=z
   (ii) x∨y= 0⇔x=0 and y=0

   (iii) x≤y′⇔x∧y=0
   (iv) x∧y=1⇔x=1 and y=1
4. In a lattice (L,≤) prove that

   (i) $\mathbf{a \vee (b \wedge c) \le (a \vee b) \wedge (a \vee c)}$
   ii) $\mathbf{a \wedge (b \vee c) \le (a \wedge b) \vee (a \wedge c)}$
5. In any Boolean algebra ,show that $ab^1 + a^1b = 0$ if and only if a=b
6. Show that Demorgon’s law are true in a complemented and distributive lattice.
7. If L is a distributive lattice with 0 and 1, show that each element has at most one complement.
8. Prove that every chain is a distributive lattice.

9. Let $D_{30} = \{1,2,3,5,6,10,15,30\}$ with the relation $x \le y$ iff x divides y. Find
   (a) all the lower bounds of 10 and 15.
   (b) GLB of 10 and 15.
   (c) all the upper bounds of 10 and 15.
   (d) LUB of 10 and 15.
   (e) draw the Hasse diagram for D_{30}.
10. State and prove the absorption law in Boolean Algebra.

11. Prove the following Boolean Identities:

   (a) $a + (a'.b) = a + b$

(b) $a.(a' + b) = a.b$

(c) $(a.b) + (a.b') = a$