UNIT 1

1. Define proposition.

   Sol:

   A proposition (or statement) is a declarative sentence which is either true or false but not both.

2. Using truth table verify that the proposition $(P \wedge Q) \wedge \neg(P \vee Q)$ is a contradiction.

   Sol:

P	Q	$P \wedge Q$	$P \vee Q$	$\neg(P \vee Q)$	$(P \wedge Q) \wedge \neg(P \vee Q)$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

$\therefore (P \wedge Q) \wedge \neg(P \vee Q)$ is a contradiction.

3. Give the converse & contra positive of the implication "If it is raining, then I get wet".

   Sol:

   Converse: If I get wet then it is raining.

   Contra positive: If I'm not get wet, then it is not raining.

4. Find the contra positive of inverse of $P \rightarrow Q$.

   Sol:

   Given $P \rightarrow Q$ as the conditional proposition.

   Inverse $P \rightarrow Q$ is $\neg P \rightarrow \neg Q$.

   Therefore the contra positive of $\neg P \rightarrow \neg Q$ is $\neg(\neg Q) \rightarrow \neg(\neg P)$ which is $Q \rightarrow P$

5. Express the statement," The crop will he destroyed if there is a flood", in symbolic form.

Sol:

P: There is a flood
Q: The crop will destroyed
$P \rightarrow Q$

6. Define a Principle Conjunctive normal form of a statement.
   Sol:
   For a given formula an equivalent formula consisting of conjunction of max terms only is known as principle conjunctive normal form or product of sums of canonical form. Consider 2 statements variable P&Q. Then the maxterms are P v Q , P v ¬ Q , ¬ P v Q, ¬ P v ¬Q.

7. Define tautology with example.
   sol:

   A Proposition P is a tautology if it is true under all circumstances. It means it contains only " T " in the final column of the truth table.

   Ex: P v ¬P

P	¬P	P v ¬P
T	F	T
F	T	T

8. Write the rules of inference.
   Sol:
   Rule P:
   A Premise may be introduced at any point in the derivation.
   Rule T:
   A formula S may be introduced in a derivation if S is a tautologically implied by any one or more of the preceding formulas in the derivation.
   Rule CP
   If we can derive S from R and a set of premises then we can derive $R \rightarrow S$ from the set of premises alone.

9. . Define quantifiers

Sol:

There are two types of quantifiers

1. Universal quantifiers

For all x, x is an integer is written as $(\forall x)$ I(x) (or)

(x) I(x). Here “for all x” is called Universal quantifiers

2. Existential quantifiers

“There exists an integer x which is prime “can be written as $(\exists x)$ P(x), where P(x): x is prime.

Here the phrase,”there exists” is called an existential quantifier.

10. List out all the possible maxterms for three variables.

UNIT-2

1. Show that if seven colours are used to paint 50 bicycles, at least 8 bicycles will be of same color.
2. State the Principle of Mathematical Induction.

   Sol:

   Let P(n) be a statement or proposition involving for all positive integers n.

   Step 1: P(1) is true.

   Step2: Assume that P(k) is true.

   Step3: We have to prove P(k+1) is true.

3. State the Principle of Strong induction

   Sol:

   Let P(n) be a statement or proposition involving for all positive integers n.

   Step 1: P(1) is true.

   Step2: Assume that P(n) is true for all integers $1 \le n \le k$.

   Step3: We have to prove P(k + 1) is true.

4. State the Pigeonhole Principle.

   Sol:

   If n pigeons are assigned to m pigeonholes and ,m<n ,than at least one pigeonhole contains two or more pigeons.

5. State the Extended Pigeonhole Principle:

   Sol:

   If n pigeons are assigned to m pigeon holes than one pigeonhole must contains at least $\left[\frac{(n-1)}{m}\right]+1$ pigeons.

6. Show that, among 100 people, atleast 9 of them were born in the same month.

   Sol:

   Here No. of pigeon = m = No. of People =100

   No. of Holes = n = No. of Month = 12

Then by Generalized pigeon hole principle ,

$\left[\frac{(100-1)}{12}\right]+1=9$ were born in the same month.

7. How many permutations of the letters ABCDEFGH contain the string ABC.

Sol:

Because the letter ABC must occur as block ,we can find the answer by finding number of permutations of six objects, namely the block ABC and individual letters D,E,F,G and H

Therefore, there are 6! = 720 permutations of the letter ABCDEFGH In which ABC occurs.

8. Find the recurrence relation for $S(n) = 6(-5)^n$. n≥0.

Sol:

Given $S(n) = 6(-5)^n$

$S(n-1) = 6(-5)^{n-1}$

$= 6(5)^n / -5$

$S(n-1) = S(n) /-5$

$S_n = - 5S (n-1)$, n≥0 with S(0) =6

9. In how many ways can 5 persons be selected from amongst 10 persons?

Sol:

The selection can be done in $10\ C_5$ ways.

=10 X 9 X 8 X 7 X 6 / 1 X 2 X 3 X 4 X 5 =9 X 28 Ways.

either 7 or 11

10. Show that if 25 dictionaries in a library contain a total of 40,325 pages, then one of the dictionaries must have atleast 1614 pages.

Sol :

Here No. of pages = m = No. of pigeon = 40,325

No. of dictionaries = n = No. of Holes = 25

Then by Generalized pigeon hole principle, one dictionaries must contain atleast $\left[\frac{(40325-1)}{25}\right]+1=1614$ Pages.

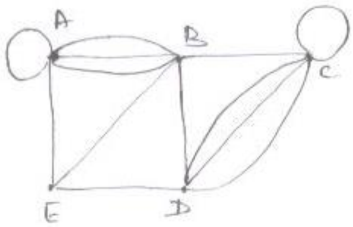
UNIT -3

1. Define graph with an example?

Solution :

A graph G= (V, E) consists of V ,a nonempty set of vertices, and E, a set of unordered pairs of distinct elements of V called edges.

2) Find the number of vertices ,the number of edges and the degree of each vertices in the graph



Solution ,

The number of vertices = 5,

The number of edges= 13

Degree of each vertices:

Deg (A)=6 Deg (B)=6

Deg (C)=6 Deg (D)=5 Deg (E)=3

3). State handshaking theorem

Solution :

Let G = (V,E) be an undirected graph with 'e' edges.

Then $\sum_{v \in V} \deg(v) = 2e$

4). How many edges are there in a graph with ten vertices each of degree six

Solution :

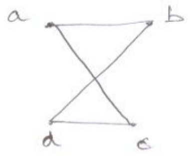
Let e be the number of edges of the graph

2e= Sum of all degrees

= 10 X 6=60

e = 30 There are 30 edges.

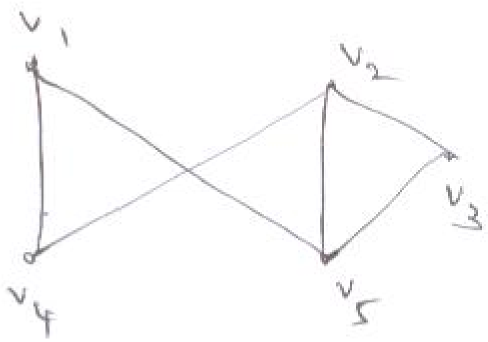
5). Write the adjacency matrix for the graph



Solution :

$$A=[a_{ij}]=\begin{matrix} & V_1 & V_2 & V_3 & V_4 \\ V_1 & 0 & 1 & 1 & 0 \\ V_2 & 1 & 0 & 0 & 1 \\ V_3 & 1 & 0 & 0 & 1 \\ V_4 & 0 & 1 & 1 & 0 \end{matrix}$$

6). Write an incidence matrix for the graph



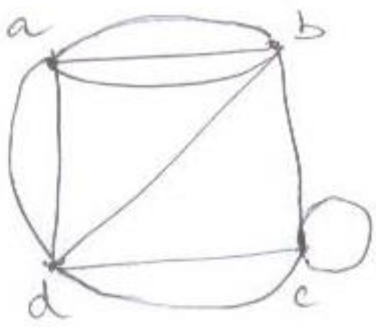
Solutoin:

$$A=[a_{ij}]=\begin{array}{c} \\ V_1\\ V_2\\ V_3\\ V_4\\ V_5\end{array}\begin{array}{c}\begin{array}{ccccc}V_1 & V_2 & V_3 & V_4 & V_5\end{array}\\ \begin{bmatrix}0&0&0&1&1\\0&0&1&1&1\\0&1&0&0&1\\1&1&0&0&0\\1&1&1&0&0\end{bmatrix}\end{array}$$

7). Draw a graph with the given adjacency matrix

$$\begin{bmatrix}0&3&0&2\\3&0&1&1\\0&1&1&2\\2&1&2&0\end{bmatrix}$$

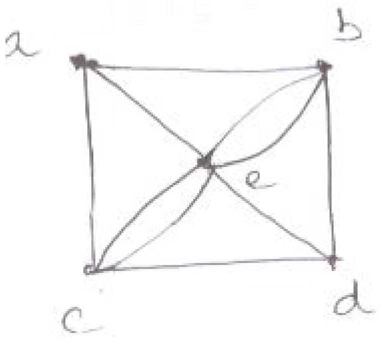
Solutoin:



8) Find the Euler path in the graph

Solutoin:

A path of a graph G is called an Eulerian path,if it contains each edge of the graph exactly once.



9) Define a Regular graph

Solutoin:

If every vertex of a simple graph has the same degree, then the graph is called a regular graph.

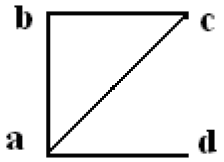
. If every vertex in a regular graph has degree n, then the graph is called n-regular.

10) Define a Simple graph

Solutoin:

A graph is said to be simple graph if it has no loops and parallel edges. Otherwise it is multi graph.

Example:

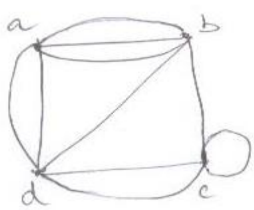


11) Define Pseudo graph

Solutoin:

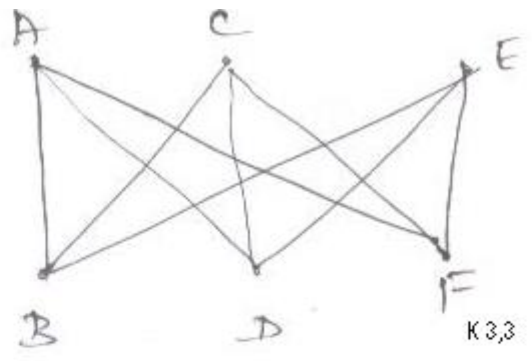
A graph in which loops and parallel edges are allowed is called a Pseudo graph

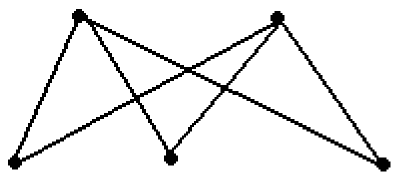
Example :



12) Draw a complete bipartite graph of K 2,3 and K 3,3

Solution





K 2,3

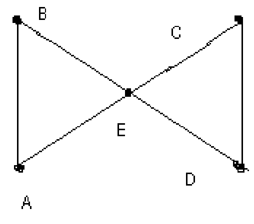
13)Define strongly connected graph.

Solutoin:

A simple digraph is said to be strongly connected, if for any pair of nodes of the graph both the nodes of the pair are reachable from one another.

14) Give an example of an Euler graph.

Solution :



Consider the cycle A – E – C – D – E – B – A

Since. It includes each of the edges exactly once, the above cycle is a Eulerian cycle.Since the graph contains Eulerian cycle, it is a Euler graph

15) Define Hamiltonian graph.

Solutoin:

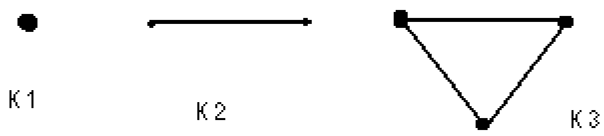
A simple path in a graph G that passes through every vertex exactly once is called a Hamilton path. A circuit in a graph G that passes through every vertex exactly once is called a Hamilton circuit. A graph containing a Hamiltonian circuit.

16) Define complete graph and give an example.

Solutoin:

A simple graph G with n vertices is said to be a complete graph if the degree of every vertex is n-1. The complete graph on n vertices is denoted by K_n

Example



17) Define isomorphism of two graph.

Solutoin:

Two graph G1 and G2 are said to be Isomorphic to each other, if there exist a one -to-one correspondence between the vertex sets which preserves adjacency of the vertices,

18) Define Eulerian graph

Solutoin:

A path of graph G is called an Eulerian path, if it includes each edge of G exactly once. A circuit of a graph G is called an Eulerian circuit, if it includes each edge of G exactly one. A graph containing an Eulerian circuit is called an Eulerian graph.

How many edges are there in a graph with ten vertices

each of degree six?

19) Draw a graph with the given adjacency matrix

$$\begin{bmatrix} 0 & 3 & 0 & 2 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 2 & 0 \end{bmatrix}$$

UNIT 4

1. Define a semi group.

A non empty set together with the following property is called a semi group denoted by (S, *)

(i) closure : $a*b \in G$

(ii) Associative : $(a*b) * c = a*(b * c)$ $a,b,c \in G$

2. When is a group (G, *) is called an abelian group?

If the group G satisfies $(a*b) = (b*a)$ i.e it satisfies commutative law

3. Prove that the identity of a subgroup is the same(unique) as that of the group.

Let (G, *) be the group.

Let $a \in G$ the we have

$a*a = a$

$a = a*e$

$= a*(a*a^{-1})$

$= (a*a)*a^{-1}$

$= a*a^{-1}$

$a = e$

4. Define homomorphism between two algebraic systems.

A mapping $f: X \rightarrow Y$ is said to hmmomorphism if it it satisfies

$f(xy) = f(x).f(y)$

5. Define isomorphism between two algebraic systems.

A mapping $f: X \rightarrow Y$ is said to hmmomorphism if it it satisfies

(i) It satisfies $f(xy) = f(x).f(y)$

(ii) It one to one and onto

6. If a and b are any two elements of a group<G,*> , show that is an abelian group if and only if .(a* b)^2 = a^2*b^2 .

Assume that G is abelian

$\therefore$ a*b = b*a , a,b $\in$ G

a^2*b^2 = (a*a)(b*b)

= a*[a*(b*b)] [* is associative]

= a*[a*b]*b [* is associative]

= (a*b)*(a*b) [* is associative]

= (a*b)^2

a^2*b^2 = (a*b)^2

conversely , assume that

(a*b)^2 = a^2*b^2

(a*b)*(a*b) = (a*a)(b*b)

a*[a*(b*b)] = a*[a*(b*b)] [* is associative]

b*(a*b) = a*(b*b) [left cancellation law]

(b*a)*b = (a*b)*b

b*a = a*b [right cancellation law]

7. State Lagrange's theorem in group theory.

The order of each subgroups of a finite group is a divisor of a order of a group.

8. Define Coset:

If H is a subgroup of a group G under the operation *, then the set aH, where $a \in G$,

define by $\boldsymbol{aH} = \{\boldsymbol{a*h} / \boldsymbol{h} \in \boldsymbol{H}\}$ is called the left coset of H in G generated by the element $\boldsymbol{a} \in \boldsymbol{G}$. Similarly the set Ha is called the right coset of H in G generated by the
element $\boldsymbol{a} \in \boldsymbol{G}$.

9. Define Normal subgroup:

A subgroup H of the group G is said to be normal subgroup under the operation *, if for any $a \in H$ *then* $aH = H.a$

10. Cayley's theorem:

Every finite group of order is isomorphic to a permutation group of degree n .

11. Fundamental theorem of homomorphism

If f is a homomorphism of G on to G^1 with kernel K, then G/K is isomorphic to G^1

12.Show that every cyclic group is abelian.

UNIT-5

1) Define poset and give an example
   A relation R on a set A is called partial order relation,if R is a reflexive,antisymmetric and transitive.The set A together with a partial order relation defined on it is called partially ordered set or poset.
   Eg. The greater than or equal to(≥) a relation is a partial ordering on the set of integers Z.
2) Define a Distribution Lattice
   A lattice (L,*,⊕) is called a distributive lattice if for any a,b,c ∈ L

   a*(b⊕c)=(a*b) ⊕(a*c)

   a⊕(b*c)=(a⊕b)*(a⊕c)

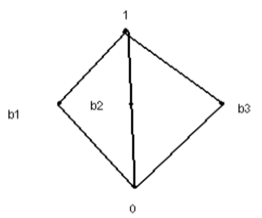
3) Define lattices

   A Lattice is a partially ordered set (L,≤) in which every pair of elements a,b ∈ L has a glb and lub.

4) Give an example of a lattice which is a modular but not a distributive.

   In M_5,aV(a∧ c)=aV0=0 while (aVb) ∧(aVc)= | ∧ |=1

   So M_5 is not distributive.



5) State any two properties of lattices.
   Let (L,≤) be a lattice.For any a,b,c ∈ L we have
   (i) a*a=a and a⊕ a=a [Idempotent law]

   (ii) a*b=b*a and a ⊕b= b ⊕a [Commutative law]

6) Write the condition for the algebraic lattice.
   A lattice (L,*,⊕) is said to be algebraic if it satisfies commutative Law, Associative Law, Absorption Law and Existence of Idempotent element.

7) Define Modular Lattice.

A lattice $(L,*,\oplus)$ is said to be Modular if for any a,b,c $\in$ L

(i)$a \le c \Rightarrow a\oplus(b*c)=(a\oplus b)*c$

(ii) $a\ge b \Rightarrow a*(b\oplus c)=(a*b) \oplus c$

8) Define Boolean Algebra

A Boolean algebra is a lattice which is both complemented and distributive.it is
denoted by $(B,*, \oplus)$.

9) Show that in any Boolean algebra, $(a+b)(a'+c)=ac+a'b+bc$

Sol:

Let $(B,+, \square ,')$ be a Boolean algebra,and $a,b,c \in B$

L.H.S $= (a+b)(a'+c)=(a+b)a'+(a+b)c$

$= aa'+ba' +ac+bc$

$= 0+a' b+ac+bc$

$=ac+ a' b+bc$ =R.H.S

10) Show that absorption laws are valid in a Boolean algebra

Sol:

If a and b are two elements of a Boolean algebra,prove that

$a+(a . b) = a : a . (a+b) = a$

L.H.S $= a+(a . b) = a . 1 + a . b$

$= a . (1+b) = a.1 = a$

$a . (a+b) = a . a + a .b = a + a .b = 1 . a + a . b= a . (1 + b) = a . 1$

$= a$

11) Prove that in a Boolean algebra ,the complement of any element is unique

Sol:

Let b and c be the complement of the element $a \in B'$

Then b+a=1 ; b . a = 0

a+c = 1 ; a.c = 0

b = 1 . b = (a + c) . b = a . b + c . b = 0 + c . b = a . c + b . c = (a + b) . c

= 1.c = c

b = c The complement is unique

12) Write the Properties of lattice

Sol:

Idempotent law

$$\mathbf{a \wedge a = a \quad a \vee a = a}$$

Absorption law

$$\mathbf{a \wedge (a \vee b) = a \quad a \vee (a \wedge b) = a}$$

Commutative law

$$\mathbf{a \wedge a = b \wedge a \quad a \vee b = b \vee a}$$

Associative law

$$\mathbf{(a \wedge b) \wedge c = a \wedge (b \wedge c)}$$
$$\mathbf{(a \vee b) \vee c = a \vee (b \vee c)}$$

13) Prove a.(a+b)=a+(a.b) in Boolean algebra

Sol:

L.H.S = a.a+a.b=a+a.b $[\because a.a = a]$

=a.(1+b) [1+b]=1]

=a

R.H.S = (a+a).(a+b)=a.(a+b)=a.a+a.b $[\because a.a = a]$

= a+a.b

= a.(1+b)

= a

L.H.S = R.H.S

a.(a+b)=a+(a.b)