Macaulay's Method:

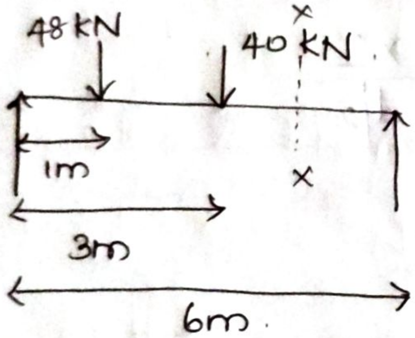
1) A Beam of Length 6m is Simply Supported at its ends & Carries 2 point Loads of 48 kN & 40kN at a distance of 1m & 3m respectively from the Left Support. Find

i) Deflection Under Each Loads

ii) Max. Deflection &

iii) The Point at which Max. Deflection Occurs

$E = 2 \times 10^5 \, N/mm^2$ & $I = 85 \times 10^6 \, mm^4$.



$I = 85 \times 10^5 \, mm^4$

$E = 2 \times 10^5 \, N/mm^2$

$R_A + R_B = 48 + 40 = 88 \text{ kN}.$

$R_B \times 6 - 40 \times 3 - 48 \times 1 = 0$

$6 R_B = 120 + 48$

$R_B = \frac{168}{6} = 28 \text{ kN}$

$\boxed{R_B = 28 \text{ kN}}$ $\boxed{R_A = 60 \text{ kN}}$

Consider Section X at distance x from Left Support A,

$$EI \cdot \frac{d^2y}{dx^2} = R_A x \; \vdots \; -48(x-1) \; \vdots \; -40(x-3)$$

$$= 60x \; \vdots \; -48(x-1) \; \vdots \; -40(x-3)$$

Integrating the above Eqn,

$$EI \cdot \frac{dy}{dx} = \frac{60x^2}{2} + C_1 \; \vdots \; -48\frac{(x-1)^2}{2} \; \vdots \; -40\frac{(x-3)^2}{2}$$

$$= 30x^2 + C_1 \; \vdots \; -24(x-1)^2 \; \vdots \; -20(x-3)^2$$

Integrating the above Eqn, We get

$$EI \cdot y = \frac{30x^3}{3} + C_1 x + C_2 \; \vdots \; -\frac{24(x-1)^3}{3} \; \vdots \; -\frac{20(x-3)^3}{3}$$

$$= 10x^3 + C_1 x + C_2 \; \vdots \; -8(x-1)^3 \; \vdots \; -\frac{20}{3}(x-3)^3$$

To find C_1 & C_2, Boundary conditions

i) $x=0, y=0$

ii) $x=6, y=0$

Sub $x=0, y=0$ in Eqn in first part as $x=0$ lies in first Part

$$0 = 0 + 0 + C_2$$

$$\boxed{C_2 = 0}$$

Sub $x=6, y=0$ in Eqn, as it lies in Last Part

$$0 = 10 \times 6^3 + C_1 \times 6 + 0 - 8(6-1)^3 - \frac{20}{3}(6-3)^3$$

$$0 = 2160 + 6C_1 - 8(5^3) - \frac{20}{3} \times 3^3$$

$$= 2160 + 6C_1 - 1000 - 180$$

$$0 = 980 + 6C_1$$

$$C_1 = \frac{-980}{6} = -163.33$$

Now, Substituting C_1 & C_2

$$EI.y = 10x^3 - 163.33x \;\vdots\; -8(x-1)^3 \;\vdots\; -\frac{20}{3}(x-3)^3$$

i) Deflection Under first Load at Point C

$$EI.y_c = 10 \times 1^3 - 163.33(1)$$

$$= -153.33 \text{ kNm}^3$$

$$= -153.33 \times 10^{12} \text{ Nmm}^3$$

$$y_c = \frac{-153.33 \times 10^{12}}{EI} = \frac{-153.33 \times 10^{12}}{2 \times 10^5 \times 85 \times 10^6} \text{ mm}$$

$$\boxed{y_c = -9.019 \text{ mm}}$$

Deflection Under Second Load, at pt D

$$EI.y_B = 10 \times 3^3 - 163.33 \times 3 - 8(x-1)^3$$

$$= -283.99 \text{ kN m}^3$$

$$= -283.99 \times 10^{12} \text{ Nmm}^3$$

$$y_D = \frac{-283.99 \times 10^{12}}{2 \times 10^5 \times 85 \times 10^6} = -16.7 \text{ mm}$$

ii) Max. Deflections, $\frac{dy}{dx} = 0$,

$$30x^2 + C_1 - 24(x-1)^2 = 0$$

$$30x^2 - 163.33 - 24(x^2+1-2x) = 0$$

$$6x^2 + 48x - 187.33 = 0$$

$$\boxed{x = 2.87\ m}$$

$x = 2.87\ m$, finding $EI \cdot y_{max}$

$$EI \cdot y_{max} = 10 \times 2.87^3 - 163.33(2.87) - 8(2.87-1)^3$$

$$= -284.67\ KNm^3$$

$$= -284.67 \times 10^{12}\ Nmm^3.$$

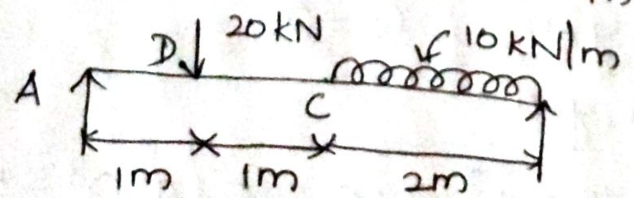
$$y_{max} = \frac{-284.67 \times 10^{12}}{2 \times 10^5 \times 85 \times 10^6}$$

$$\boxed{y_{max} = -16.745\ mm.}$$

2) A Beam of 4m span is Simply Supported at the ends & is Loaded as shown in fig. Determine

1) Deflection at C
2) Max. Deflection
3) Slope at the End A

$E = 200 \times 10^6\ KN/m^2$ & $I = 20 \times 10^{-6}\ m^4$



$L = 4m$

$E = 200 \times 10^6 \; KN/m^2$

$I = 20 \times 10^{-6} \; m^4$.

$R_A + R_B = 20 + 20 = 40 \; KN$

Moment about A,

$$R_B \times 4 - 20 \times 3 - 20 \times 1 = 0$$

$$4R_B - 60 - 20 = 0$$

$$4R_B - 80 = 0$$

$$R_B = \frac{80}{4} = 20 \; kN$$

$$\boxed{R_B = 20 \; KN} \quad \boxed{R_A = 20 \; KN}$$

Consider any section XX, at x from A,

$$M_x = EI \frac{d^2y}{dx^2} = 20x \;\vert\; -20(x-1) \;\vert\; -10(x-2)\frac{(x-2)}{2}$$

Integrating,

$$EI \frac{dy}{dx} = 10x^2 + C_1 \;\vert\; -10(x-1)^2 \;\vert\; -\frac{5}{3}(x-2)^3$$

Integrating,

$$EI\,y = \frac{10}{3}x^3 + C_1 x + C_2 \;\vert\; -\frac{10}{3}(x-1)^3 \;\vert\; -\frac{5}{12}(x-2)^4$$

$x = 0, y = 0 \quad \therefore C_2 = 0$

$x = 4, y = 0$,

$$0 = \frac{10}{3} \times 4^3 + 4C_1 - \frac{10}{3}(4-1)^3 - \frac{5}{12}(4-2)^4$$

$= 213.33 + 4C_1 - 90 - 6.67$

$$\boxed{C_1 = -29.16}$$

Slope Eqn

$$EI\frac{dy}{dx} = 10x^2 - 29.16 \;\vdots\; -10(x-1)^2 \;\vdots\; -\frac{5}{3}(x-2)^3$$

$$EIy = \frac{10}{3}x^3 - 29.16x \;\vdots\; -\frac{10}{3}(x-1)^3 \;\vdots\; -\frac{5}{12}(x-2)^4$$

Deflection at C, y_c

$$\boxed{x = 2m}$$

$$EI \cdot y_c = \frac{10}{3}(2)^3 - 29.16 \times 2 - \frac{10}{3}(2-1)^3$$

$$= 26.67 - 58.32 - 3.33$$

$$= -34.98$$

$$y_c = \frac{-34.98}{EI} = -\frac{34.98}{200 \times 10^6 \times 20 \times 10^{-6}} \times 10^3 \, mm$$

$$\boxed{y_c = -8.74 \, mm}$$

Max. Deflection, y_{max}

$$EI \cdot \frac{dy}{dx} = 10x^2 - 29.16 - 10(x-1^2) = 0$$

$$= 10x^2 - 29.16 - 10x^2 + 20x - 10 = 0$$

$$x = \frac{39.16}{20} = 1.95 \, m$$

$$\boxed{x = 1.95 \, m}$$

$$EI . y_{max} = \frac{10}{3}(1.958)^3 - 29.16 \times 1.958 - \frac{10}{3} (1.958-1)^3$$

$$= 25.02 - 57.09 - 2.93$$

$$= -35$$

$$y_{max} = \frac{-35}{200 \times 10^6 \times 20 \times 10^{-6}} \times 10^3 = -8.75 \text{ mm}$$

$y_{max} = 8.75$ mm (Downward)

Putting $\boxed{x=0}$ in slope Eqn:

$$EI . \frac{dy}{dx} = -29.16$$

$$\theta_A = \frac{-29.16}{200 \times 10^6 \times 20 \times -6}$$

$$= -0.00729 \text{ radians.}$$

$$= -0.00729 \times \frac{180}{\pi} = -0.0417^\circ$$

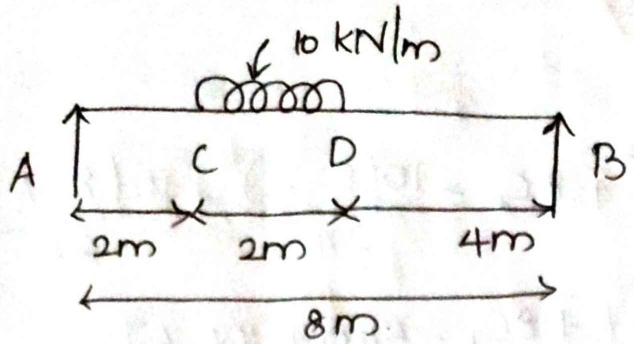
$$\boxed{\theta_A = -0.417^\circ}$$

3) A Beam AB of span 8m is simply supported at the ends A & B & is Loaded as shown in fig $E = 200 \times 10^6$ kN/m² & $I = 120 \times 10^{-6}$ m⁴

Determine i) Deflection at Mid span.

ii) Max. Deflection

iii) Slope at the end A.



$L = 8\,m$

$E = 200 \times 10^6\ kN/m^2$

$I = 120 \times 10^{-6}\ m^4$

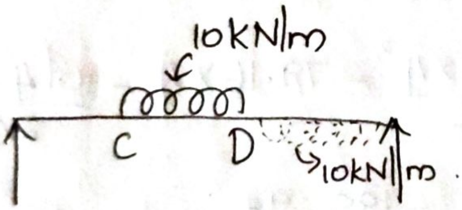
$R_A + R_B = 20\ KN$

$R_B \times 8 - 20 \times 3 = 0$

$8R_B = 60$

$R_B = \frac{60}{8} = 7.5\ KN$

$\boxed{R_B = 7.5\ KN}$ $\boxed{R_A = 12.5\ KN}$



Consider Section XX from end A,

$$M_x = EI.\frac{d^2y}{dx^2} = 12.5x \;\vdots\; -\frac{10(x-2)^2}{2} \;\vdots\; +\frac{10(x-4)^2}{2}$$

Integrating,

$$EI.\frac{dy}{dx} = \frac{12.5x^2}{2} + C_1 \;\vdots\; -\frac{10(x-2)^3}{6} \;\vdots\; +\frac{10(x-4)^3}{6}$$

$$EI.y = \frac{12.5x^3}{6} + C_1x + C_2 \;\vdots\; -\frac{10(x-2)^4}{24} \;\vdots\; +\frac{10(x-4)^4}{24}$$

$x = 0,\ y = 0$ $\boxed{C_2 = 0}$

$x = 8m, \; y = 0$

$$0 = \frac{12.5 \times 8^3}{6} + 8C_1 - \frac{10(8-2)^4}{24} + \frac{10(8-4)^4}{24}$$

$$= 1066.67 + 8C_1 - 540 + 106.67$$

$$0 = 8C_1 + 633.34$$

$$\boxed{C_1 = -79.16}$$

$$EI.\frac{dy}{dx} = \frac{12.5x^2}{2} - 79.16 \;\vdots\; -\frac{10(x-2)^3}{6} \;\vdots\; +\frac{10(x-4)^3}{6}$$

$$EI.y = \frac{12.5x^3}{6} - 79.16x \;\vdots\; -\frac{10(x-2)^4}{24} \;\vdots\; +\frac{10(x-4)^4}{24}$$

i) Deflection at Mid span y_D.

$$\boxed{x = 4m}$$

$$EI.y_D = \frac{12.5 \times 4^3}{6} - 79.16 \times 4 - \frac{10(4-2)^4}{24}$$

$$= -189.98$$

$$y_D = \frac{-189.98}{200 \times 10^6 \times 120 \times 10^{-6}} \times 10^3$$

$$= -7.195 \text{ mm}$$

Def at Mid Span = 7.19 mm (Downward)

ii) Max. Deflection y_{max}:

$$\frac{dy}{dx} = 0$$

$$EI.\frac{dy}{dx} = \frac{12.5x^2}{2} - 79.16 - \frac{10(x-2)^3}{6} = 0$$

$$6.25x^2 - 79.16 - \frac{10(x-2)^3}{6} = 0$$

$$\boxed{x = 3.75\ m}$$

$$EI \cdot y_{max} = \frac{12.5 \times 3.75^3}{6} - 79.16 \times 3.75 - \frac{10(3.75-2)^4}{24}$$

$$= 109.86 - 296.85 - 3.91$$

$$= -190.9$$

$$y_{max} = \frac{190.9}{200 \times 10^6 \times 120 \times 10^{-6}} \times 10^3$$

$$= -7.95\ mm$$

Max. Def = 7.954 mm (Downwards)

Slope at end A, θ_A :-

$$\boxed{x = 0}$$

$$EI \cdot \frac{dy}{dx} = -79.16 \quad \text{or} \quad EI \cdot \theta_A = -79.16$$

$$\theta_A = -\frac{79.16}{200 \times 10^6 \times 120 \times 10^{-6}}$$

$$= -0.00329\ rad$$

$$= -0.00329 \times \frac{180}{\pi} = -0.188^\circ$$

$$\boxed{\theta_A = -0.188^\circ}$$