1) Determine z-transform of $x(n) = \{1, 2, 3, 4, 5, 0, 7\}$

$x(0) = 1$ $\quad x(4) = 5$

$x(1) = 2$ $\quad x(5) = 0$

$x(2) = 3$ $\quad x(6) = 7$

$x(3) = 4$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=0}^{6} x(n) z^{-n}$$

$$= x(0) z^0 + x(1) z^{-1} + x(2) z^{-2} + x(3) z^{-3} + x(4) z^{-4} + x(5) z^{-5} + x(6) z^{-6}$$

$$= 1 \cdot 1 + 2z^{-1} + 3z^{-2} + 4z^{-3} + 5z^{-4} + 7z^{-6}$$

$$X(z) = 1 + 2/z + 3/z^2 + 4/z^3 + 5/z^4 + 7/z^6$$

ROC is the entire z-plane except at $z=0$

2) Find the z-transform of $u(n)$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

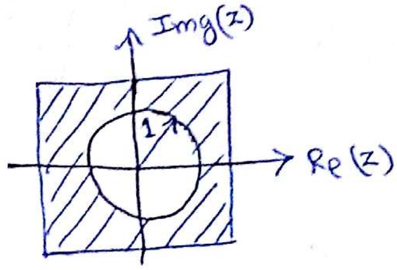
$$= \sum_{n=0}^{\infty} 1 \; z^{-n}$$

$$= 1 + z^{-1} + z^{-2} + z^{-3} + \dots z^{\infty}$$

$$= 1 + 1/z + 1/z^2 + 1/z^3 + \dots$$

$$= \left(1 - \frac{1}{z}\right)^{-1}$$

$$= \left(\frac{z-1}{z}\right)^{-1}$$



$$X(z) = \frac{z}{z-1}, \quad |z| > 1$$

$|z|$ represent a circle in z-plane

$|z| = 1$ represent a circle of radius 1

$|z| > 1$ indicates area outside the circle.

3) Find the z-transform of $\delta(n)$

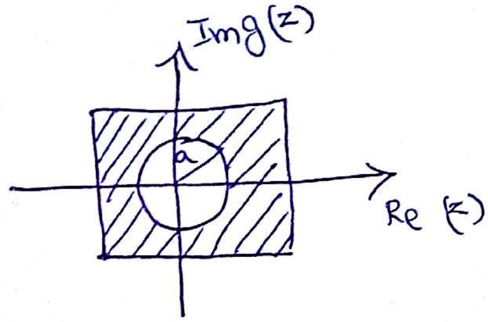
$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

$$= \sum_{n=0}^{\infty} \delta(n) z^{-n} \Rightarrow 1$$

4) Find the z-transform of right hand sided sequence.

$$x(n) = a^n u(n)$$



$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} a^n u(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} (a z^{-1})^n$$

$$= 1 + (az^{-1})^1 + (az^{-1})^2 + (az^{-1})^3 + \dots (az^{-1})^{\infty}$$

$$= \frac{1}{1-az^{-1}} = \frac{1}{1-a/z} = \frac{1}{\frac{z-a}{z}} = \frac{z}{z-a}, \quad z-a > 0$$

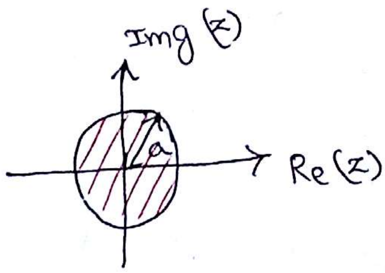
$$z > a$$

5) Find z-transform of $x(n) = a^n u(-n-1)$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} a^n u(-n-1) z^{-n}$$

$$= \sum_{n=-\infty}^{-1} a^n z^{-n}$$



$$= \sum_{n=1}^{\infty} a^{-n} z^{n}$$

$$= \sum_{n=1}^{\infty} (a^{-1} z)^n$$

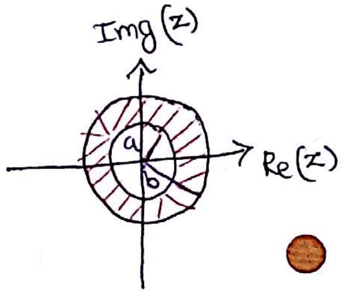
$$= (a^{-1}z)^1 + (a^{-1}z)^2 + \dots (a^{-1}z)^{\infty}$$

$$= z a^{-1} \left[1 + (za^{-1}) + (za^{-1})^2 + \dots \right]$$

$$= \frac{za^{-1}}{1 - za^{-1}} \Rightarrow \frac{z/a}{1 - z/a} \Rightarrow \frac{z/a}{\frac{a-z}{z}} = \frac{z}{a-z}, \ |z| < a$$

6) Find z-transform of both sided sequence.

$$x(n) = a^n u(n) + b^n u(-n-1)$$



$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$a^n u(n) = \frac{1}{1 - az^{-1}} \quad , \quad b^n u(n) = \frac{1}{1 - b^{-1} z}$$

$$X(z) = \frac{1}{1 - az^{-1}} + \frac{1}{1 - b^{-1} z} \qquad a < |z| < b$$

$$|z| > a \qquad |z| < b$$

7) Find z-transform of $x(n) = \cos \omega_0 n \ u(n)$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \cos \omega_0 n \ u(n) z^{-n} \Rightarrow \sum_{n=0}^{\infty} \cos \omega_0 n \ z^{-n}$$

$$z\left[(\cos\omega_0 n)\, u(n)\right] = z\left[\frac{e^{j\omega_0 n} + e^{-j\omega_0 n}}{2}\right] u(n)$$

$$= \frac{1}{2}\, z\left[e^{j\omega_0 n}\, u(n)\right] + \frac{1}{2}\, z\left[e^{-j\omega_0 n}\, u(n)\right]$$

$$z\left[a^n u(n)\right] = \frac{1}{1-az^{-1}}, \quad |z| > |a|$$

$$= \frac{1}{2}\left[\frac{1}{1-e^{j\omega_0}z^{-1}}\right] + \frac{1}{2}\left[\frac{1}{1-e^{-j\omega_0}z^{-1}}\right]$$

Cross multiply

$$= \frac{1}{2}\left[\frac{1-e^{-j\omega_0}z^{-1} + 1 - e^{j\omega_0}z^{-1}}{(1-e^{j\omega_0}z^{-1})(1-e^{-j\omega_0}z^{-1})}\right]$$

$$= \frac{1}{2}\left[\frac{2-2z^{-1}\left(e^{j\omega_0}+e^{-j\omega_0}\right)/2}{1-2z^{-1}\left(\left(e^{j\omega_0}+e^{-j\omega_0}\right)/2\right)+z^{-2}}\right]$$

$$= \frac{1}{2}\left[\frac{2-2z^{-1}\cos\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}\right]$$

$$X(z) = \frac{1-z^{-1}\cos\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}$$

8) Find the z-transform of $x(n) = \sin\omega_0 n\; u(n)$

$$X(z) = \frac{z^{-1}\sin\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}$$

9) Determine z-transform of $x(n) = \{1, 2, 3, \underset{\uparrow}{4}, 5, 0, 7\}$

$$X(z) = z^3 + 2z^2 + 3z + 4 + 5z^{-1} + 7z^{-3}$$