Characteristic Impedance.

consider plane wave propogating in x direction. The wave for free space is

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

The solution for the differential equation is in the form

$$E = f_1(x - v_0 t) + f_2(x + v_0 t)$$

f_1 & f_2 are functions of $(x - v_0 t)$ & $(x + v_0 t)$

The solution of wave eq consisting of two waves one at positive direction and one at negative direction.

Let us consider the wave propogating at positive direction

$$\boxed{f_2(x - v_0 t) = 0}$$

General eq becomes $\boxed{E = f(x - v_0 t)}$

$$\nabla \times E = \begin{vmatrix} \vec{x} & \vec{y} & \vec{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix}$$

$$= \vec{x}\left[\frac{\partial E_z}{\partial z} - \frac{\partial E_y}{\partial z}\right] - \vec{y}\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right) + \vec{z}\left[\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right]$$

Since wave is in x direction E & H in x are zero.

$$\nabla \times E = -y\left(\frac{\partial E_z}{\partial x}\right) + \frac{\partial E_y}{\partial x}\vec{z}$$

IIIly

$$\nabla \times H = -y\frac{\partial H_z}{\partial x} + \frac{\partial H_y}{\partial x}z$$

$$\nabla \times H = \varepsilon\frac{\partial E}{\partial t}$$

$$\varepsilon\frac{\partial E}{\partial t} = -y\frac{\partial H_z}{\partial x} + \frac{\partial H_y}{\partial x}z$$

$$\varepsilon\left(-\frac{\partial E_z}{\partial x}\vec{y} + \frac{\partial E_y}{\partial x}\vec{z}\right) = -y\frac{\partial H_z}{\partial x} + \frac{\partial H_y}{\partial x}z$$

eq. y & z terms.

$$\boxed{-\varepsilon\frac{\partial E_z}{\partial x} = -\frac{\partial H_z}{\partial x}}$$

$$\boxed{\varepsilon\frac{\partial E_y}{\partial x} = \frac{\partial H_y}{\partial x}}$$

From Maxwell's equation.

$$\nabla \times E = -\mu\frac{\partial H}{\partial t}$$

$$-\mu\left(-y\frac{\partial H_z}{\partial x} + \frac{\partial H_y}{\partial x}z\right) = -y\frac{\partial E_z}{\partial x} + \frac{\partial E_y}{\partial x}\vec{z}$$

comparing y & z terms

$$-\mu\frac{\partial H_z}{\partial x} = -\frac{\partial E_z}{\partial x}$$

$$\frac{\partial H_y}{\partial x} = \frac{\partial E_y}{\partial x}.$$

The solution of the eq is given by

$$E_y = f(x - V_0 t)$$

diff $\frac{\partial E_y}{\partial t} = f'(x - V_0 t)(-V_0)$

$$\frac{\partial E_y}{\partial t} = -V_0 f'(x - V_0 t)$$

$$\frac{\partial E_y}{\partial t} = -V_0 f'$$

$$\boxed{-\frac{\partial H_z}{\partial x} = \varepsilon \frac{\partial E_y}{\partial t}}$$

$$\frac{\partial H_z}{\partial x} = -\varepsilon(-V_0 f')$$

$$\frac{\partial H_z}{\partial x} = V_0 \varepsilon f'$$

$$\frac{\partial H_z}{\partial x} = \frac{1}{\sqrt{\mu\varepsilon}} \varepsilon f'$$

$$\boxed{\frac{\partial H_z}{\partial x} = \sqrt{\frac{\varepsilon}{\mu}} f'}$$

Integrating

$$H_z = \int \sqrt{\varepsilon/\mu}\, f'\, dx$$

$$H_z = \sqrt{\varepsilon/\mu}\, f$$

$$H_z = \sqrt{\frac{\varepsilon}{\mu}}\, E_y$$

$$\frac{E_y}{H_z} = \sqrt{\mu/\varepsilon}$$

lll'y

$$\frac{E_z}{H_y} = -\sqrt{\mu/\varepsilon}.$$

$\therefore \frac{E}{H} = \sqrt{\mu/\varepsilon}$ - characteristic

Impedance.

It is the ratio of ~~characteristic impedance (or)~~ square root of permeability to the dielectric constant of the medium and it is denoted by η

$$\eta = \frac{E}{H} = \sqrt{\mu/\varepsilon}$$

Free space, $\varepsilon_r = \mu_r = 1$ $\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

$$\eta_0 = 120\pi$$

$$\boxed{\eta_0 = 377\Omega}$$

Dot product

$$E \cdot H = 0.$$

Cross product

$$\boxed{E \times H = \vec{x}\,\eta H^2}$$