

A) Solve $y(n+2) - 4y(n+1) + 4y(n) = 0$ with $y(0) = 1, y(1) = 0$
using z-transform

$$y(n+2) - 4y$$

$$y_{n+2} - 4y_{n+1} + 4y_n = 0.$$

$$z[y_{n+2}] - 4z[y_{n+1}] + 4z[y_n] = 0.$$

$$z^2 F(z) - z^2 y(0) - zy(1) - 4[zF(z) - zy(0)] + 4F(z) = 0$$

$$z^2 F(z) - z^2 - 0 - 4[zF(z) - z] + 4F(z) = 0$$

$$z^2 F(z) - 4zF(z) + 4F(z) - z^2 + 4z = 0$$

$$(z^2 - 4z + 4)F(z) = z^2 - 4z$$

$$F(z) = \frac{z^2 - 4z}{z^2 - 4z + 4}$$

$$= \frac{z^2 - 4z}{(z-2)^2}$$

$$\frac{F(z)}{z} = \frac{z-4}{(z-2)^2} \rightarrow \textcircled{1}$$

$$\frac{z-4}{(z-2)^2} = \frac{A}{z-2} + \frac{B}{(z-2)^2}$$

$$= \frac{A(z-2) + B}{(z-2)^2}$$

$$z-4 = A(z-2) + B$$

When $z = 2$

$$2-4 = A(2-2) + B \Rightarrow \boxed{B = -2}$$

$z = 0$

$$0-4 = A(0-2) + B$$

$$-4 = -2A - 2$$

$$-2A = -2 \Rightarrow \boxed{A = 1}$$

$$\textcircled{1} \Rightarrow \frac{F(z)}{z} = \frac{1}{z-2} + \frac{-2}{(z-2)^2}$$

$$F(z) = \frac{z}{z-2} - 2 \frac{z}{(z-2)^2}$$

$$z^{-1}[F(z)] = z^{-1}\left[\frac{z}{z-2}\right] - 2 z^{-1}\left[\frac{z}{(z-2)^2}\right]$$

$$= (2)^n - 2n2^{n-1}$$

$$= 2^n - n2^n$$

$$= 2^n(1-n)$$

2) Solve $y_{n+3} - 3y_{n+1} + 2y_n = 0$. with $y(0)=4, y(1)=0$
and $y(2)=8$.

$$y_{n+3} - 3y_{n+1} + 2y_n = 0$$

$$z[y_{n+3}] - 3z[y_{n+1}] + 2z[y_n] = 0$$

$$z^3 F(z) - z^3 y(0) - z^2 y(1) - z y(2) - 3[z F(z) - z y(0)] + 2F(z) = 0$$

$$z^3 F(z) - 4z^3 - 0 - 8z - 3[z F(z) - 4z] + 2F(z) = 0$$

$$z^3 F(z) - 4z^3 - 8z - 3z F(z) + 12z + 2F(z) = 0$$

$$(z^3 - 3z + 2) F(z) - 4z^3 + 4z = 0$$

$$(z^3 - 3z + 2) F(z) = 4z^3 - 4z$$

$$\frac{F(z)}{z} = \frac{4z^2 - 4}{z^3 - 3z + 2}$$

$$z^2 + 2z - 2 = 0$$

$$(z+2)(z-1) = 0$$

1 0 -3 2
1 1 -2 0
1 1 -2 0
1 1 -2 0

$$= \frac{4z^2 - 4}{(z-1)^2(z+2)} = \frac{4[z^2-1]}{(z-1)^2(z+2)}$$

$$= \frac{4[(z+1)(z-1)]}{(z-1)^2(z+2)}$$

$$\frac{F(z)}{z} = \frac{4(z+1)}{(z-1)(z+2)} \rightarrow \textcircled{1}$$

$$\frac{4(z+1)}{(z-1)(z+2)} = \frac{A}{(z-1)} + \frac{B}{(z+2)}$$

$$= \frac{A(z+2) + B(z-1)}{(z-1)(z+2)}$$

$$4(z+1) = A(z+2) + B(z-1)$$

When $z=1$

$$4(1+1) = A(1+2)$$

$$3A = 8$$

$$\boxed{A = \frac{8}{3}}$$

When $z = -2$

$$4(-2+1) = B(-2-1)$$

$$-4 = -3B$$

$$\boxed{B = \frac{4}{3}}$$

$$\textcircled{1} \Rightarrow \frac{F(z)}{z} = \frac{\frac{8}{3}}{(z-1)} + \frac{\frac{4}{3}}{(z+2)}$$

$$F(z) = \frac{8}{3} \left(\frac{z}{z-1} \right) + \frac{4}{3} \left(\frac{z}{z+2} \right)$$

$$z^{-1} [F(z)] = \frac{8}{3} z^{-1} \left(\frac{z}{z-1} \right) + \frac{4}{3} z^{-1} \left(\frac{z}{z+2} \right)$$

$$= \frac{8}{3} (1) + \frac{4}{3} (-2)^n$$

$$= \frac{8}{3} + \frac{4}{3} (-2)^n.$$