

Solving ~~the~~ of difference Equation:-

Formulas!

$$Z[Y_n] = F(z)$$

$$Z[Y_{n+1}] = zF(z) - Zy_0$$

$$Z[Y_{n+2}] = z^2F(z) - z^2y_0 - Zy_1$$

$$Z[Y_{n+3}] = z^3F(z) - z^3y_0 - z^2y_1 - Zy_2.$$

$$\text{here } y_0 = y(0)$$

$$y_1 = y(1)$$

$$y_2 = y(2)$$

1. Solve $Y_{n+2} + 4Y_{n+1} + 3Y_n = 2^n$. with $y_0 = 0$ and $y_1 = 1$ using z-transform.

$$Y_{n+2} + 4Y_{n+1} + 3Y_n = 2^n.$$

Taking z-transform on both sides,

$$z[Y_{n+2}] + 4z[Y_{n+1}] + 3z[Y_n] = z[2^n]$$

$$z^2F(z) - z^2y_0 - Zy_1 + 4[zF(z) - Zy_0] + 3F(z) = \frac{z}{z-2}$$

$$z^2(F(z)) - 0 - z + 4zF(z) + 3F(z) = \frac{z}{z-2}$$

$$(z^2 + 4z + 3)F(z) = \frac{z}{z-2} + z$$

$$F(z) = \frac{z^2 - z - 2z}{(z-2)(z^2 + 4z + 3)}$$

$$= \frac{z^2 - z}{(z-2)(z^2 + 4z + 3)}$$

$$F(z) = \frac{z^2 - z}{(z-2)(z+1)(z+3)}$$

$$\frac{F(z)}{z} = \frac{z-1}{(z-2)(z+1)(z+3)} \rightarrow \textcircled{1}$$

$$\frac{z-1}{(z-2)(z+1)(z+3)} = \frac{A}{z-2} + \frac{B}{z+1} + \frac{C}{z+3}$$

$$= \frac{A(z+1)(z+3) + B(z-2)(z+3) + C(z-2)(z+1)}{(z-2)(z+1)(z+3)}$$

$$z-1 = A(z+1)(z+3) + B(z-2)(z+3) + C(z-2)(z+1)$$

$$\text{When } z = -1, \quad -1-1 = B(-1-2)(-1+3)$$

$$-2 = B(-3)(2)$$

$$-6B = -2$$

$$\boxed{B = 1/3}$$

$$\text{When } z = 2, \quad 2-1 = A(2+1)(2+3)$$

$$1 = A(3)(5)$$

$$\boxed{A = 1/15}$$

$$\text{When } z = -3, \quad -3-1 = C(-3-2)(-3+1)$$

$$-4 = C(-5)(-2)$$

$$C = 4/10$$

$$\boxed{C = -2/5}$$

$$\textcircled{1} \Rightarrow \frac{F(z)}{z} = \frac{1}{15} \frac{1}{z-2} + \frac{1/3}{z+1} + \frac{-2/5}{z+3}$$

$$F(z) = \frac{1}{15} \frac{z}{z-2} + \frac{1}{3} \frac{z}{z+1} - \frac{2}{5} \frac{z}{z+3}$$

Taking z^{-1} on both sides,

$$z^{-1}[F(z)] = \frac{1}{15} z^{-1} \left(\frac{z}{z-2} \right) + \frac{1}{3} z^{-1} \left(\frac{z}{z+1} \right) - \frac{2}{5} z^{-1} \left(\frac{z}{z+3} \right)$$

$$y_n = \frac{1}{15} 2^n + \frac{1}{3} (-1)^n - \frac{2}{5} (-3)^n$$

2] Solve using z-transforms

$$y_{n+2} - 3y_{n+1} - 10y_n = 0 \text{ with } y_0 = 1, y_1 = 0.$$

$$y_{n+2} - 3y_{n+1} - 10y_n = 0.$$

$$z[y_{n+2}] - 3z[y_{n+1}] - 10z[y_n] = 0.$$

$$z^2 F(z) - z^2 y_0 - z F(z) - 3[z F(z) - z y_0] - 10 F(z) = 0$$

$$z^2 F(z) - z^2 y_0 - z F(z) - 3z F(z) + 3z y_0 - 10 F(z) = 0$$

$$z^2 F(z) - z^2 - 3z F(z) + 3z - 10 F(z) = 0.$$

$$[z^2 - 3z - 10] F(z) = z^2 - 3z$$

$$F(z) = \frac{z^2 - 3z}{z^2 - 3z - 10}$$

$$\frac{F(z)}{z} = \frac{z-3}{(z-5)(z+2)} \rightarrow \textcircled{1}$$

$$\frac{z-3}{(z+2)(z-5)} = \frac{A}{z-5} + \frac{B}{z+2}$$

$$= \frac{A(z+2) + B(z-5)}{(z-5)(z+2)}$$

$$z-3 = A(z+2) + B(z-5)$$

$$\text{When } z=5 \Rightarrow 5-3 = A(5+2) + B(5-5)$$

$$2 = A(7)$$

$$A = 2/7$$

$$z=-2 \Rightarrow -2-3 = A(-2+2) + B(-2-5)$$

$$-5 = -B(7)$$

$$B = 5/7$$

$$\textcircled{1} \Rightarrow \frac{F(z)}{z} = \frac{2/7}{z-5} + \frac{5/7}{z+2}$$

$$F(z) = \frac{2}{7} \left(\frac{z}{z-5} \right) + \frac{5}{7} \left(\frac{z}{z+2} \right)$$

$$z^{-1}(F(z)) = \frac{2}{7} z^{-1} \left(\frac{z}{z-5} \right) + \frac{5}{7} z^{-1} \left(\frac{z}{z+2} \right)$$

$$= \frac{2}{7} (5)^n + \frac{5}{7} (-2)^n$$

3. Solve the difference eqn using z-transform

$$y_{n+2} + by_{n+1} + ay_n = 2^n \text{ with } y_0 = y_1 = 0.$$

$$y_{n+2} + by_{n+1} + ay_n = 2^n$$

$$z[y_{n+2}] + bz[y_{n+1}] + az[y_n] = z[2^n]$$

$$z^2 F(z) - z^2 y_0 - zy_1 + b[zF(z) - zy_0] + aF(z) = \frac{z}{z-2}$$

$$z^2 F(z) + bz + aF(z) = \frac{z}{z-2}$$

$$(z^2 + bz + a)F(z) = \frac{z}{z-2}$$

$$F(z) = \frac{z}{(z-2)(z^2 + bz + a)}$$

$$= \frac{z}{(z-2)(z+3)^2}$$

$$\frac{F(z)}{z} = \frac{1}{(z-2)(z+3)^2} \rightarrow \textcircled{1}$$

$$= \frac{A}{z-2} + \frac{B}{z+3} + \frac{C}{(z+3)^2}$$

$$= \frac{A(z+3)^2 + B(z+3)(z-2) + C(z-2)}{(z-2)(z+3)^2}$$

$$1 = A(z+3)^2 + B(z-2)(z+3) + C(z-2)$$

$$\text{When } z=2 \Rightarrow 1 = A(2+3)^2 \Rightarrow 1 = A(5) \quad A = \frac{1}{25}$$

When $z = -3$

$$\Rightarrow 1 = c(-3-2)$$

$$c = -1/5$$

When $z = 0$.

$$1 = 9A + (-6B) - 2C$$

$$1 = 9A(1/25) - 6B - 2(-1/5)$$

$$1 = \frac{9}{25} - 6B + \frac{2}{5}$$

$$6B = \frac{9+10-25}{25} = \frac{-6}{25}$$

$$B = \frac{-1}{25}$$

$$\textcircled{1} \Rightarrow \frac{f(z)}{z} = \frac{1/25}{z-2} + \frac{-1/25}{z+3} + \frac{-1/5}{(z+3)^2}$$

$$F(z) = \frac{1}{25} \left(\frac{z}{z-2} \right) - \frac{1}{25} \left(\frac{z}{z+3} \right) - \frac{1}{5} \left(\frac{z}{(z+3)^2} \right)$$

$$z^{-1}(F(z)) = \frac{1}{25} z^{-1} \left(\frac{z}{z-2} \right) - \frac{1}{25} z^{-1} \left(\frac{z}{z+3} \right) - \frac{1}{5} z^{-1} \left(\frac{z}{(z+3)^2} \right)$$

$$= \frac{1}{25} (2)^n - \frac{1}{25} (-3)^n - \frac{1}{5} n(-3)^{n-1}$$

$$= \frac{1}{25} (2)^n - \frac{1}{25} (-3)^n + \frac{1}{15} n(-3)^n$$