Z-transform

Definition : [Two sided (or) bilateral]

Let $\{f(n)\}$ be a sequence defined for all integers then its z-transform is defined to be

$$F(z) = Z\{f(n)\} = \sum_{n=-\infty}^{\infty} f(n) z^{-n}$$

where z is an arbitrary complex number.

Definition : [one-sided (or) unilateral]

Let $\{f(n)\}$ be a sequence defined for all positive integers then the z-transform of $\{f(n)\}$ is defined to be

$$F(z) = Z\{f(n)\} = \sum_{n=0}^{\infty} f(n) z^{-n}$$

Definition : Z-transform for discrete values of t.

If $f(t)$ is a function defined for discrete values of t, where $t = nT$, $n = 0, 1, 2, \ldots$ T being the sampling period, then z-transform of $f(t)$ is defined as

$$F(z) = Z\{f(t)\} = \sum_{n=0}^{\infty} f(nT) z^{-n}.$$

Note!

1. If $f(n)$ given then replace 'n' by 'n'.
2. If $f(t)$ given then replace 't' by nT

If $Z[f(n)] = F(z)$ then $Z^{-1}[F(z)] = f(n)$

1. $Z[a^n] = \frac{z}{z-a} \Rightarrow Z^{-1}\left[\frac{z}{z-a}\right] = a^n$

2. $Z[(-a)^n] = \frac{z}{z+a} \Rightarrow (-a)^n = Z^{-1}\left[\frac{z}{z+a}\right]$

3. $Z(n) = \frac{z}{(z-1)^2}$

4. $Z[n a^n] = \frac{az}{(z-a)^2}$

5. $Z[n a^{n-1}] = \frac{z}{(z-a)^2}$

6. $Z[a^{n-1}] = \frac{1}{z-a}$

7. $Z[(-a)^{n-1}] = \frac{1}{z+a}$

8. $Z[n(-a)^{n-1}] = \frac{z}{(z+a)^2}$

9. $Z[(n-1)a^{n-2}] = \frac{1}{(z-a)^2}$

10. $Z[(n-1)(-a)^{n-2}] = \frac{1}{(z+a)^2}$

11. $Z\left[\frac{1}{n+1}\right] = -z \log(1 - 1/z)$

12. $Z\left[\cos\frac{n\pi}{2}\right] = \frac{z^2}{z^2+1}$ 13. $Z\left[\sin\frac{n\pi}{2}\right] = \frac{z}{z^2+1}$

14. $Z\left[\frac{1}{n!}\right] = e^{1/z}$

15. $Z[n^2] = \frac{z(z+1)}{(z-1)^3}$

1. Find the z-transform of 1 (or) $z(1)$

or Prove that $z(1) = \frac{z}{z-1}$, $|z| > 1$

We know that, $z\{f(n)\} = \sum_{n=0}^{\infty} f(n) z^{-n}$

$$z(1) = \sum_{n=0}^{\infty} (1) z^{-n}$$

$$= \sum_{n=0}^{\infty} \frac{1}{z^n} = \sum_{n=0}^{\infty} \left(\frac{1}{z}\right)^n$$

$$= 1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \dots$$

$$= \left(1 - \frac{1}{z}\right)^{-1}$$

$$= \left(\frac{z-1}{z}\right)^{-1} = \frac{z}{z-1}$$

2. Find $z((-1)^n)$

We know that, $z\{f(n)\} = \sum_{n=0}^{\infty} f(n) z^{-n}$

$$z[(-1)^n] = \sum_{n=0}^{\infty} (-1)^n z^{-n}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{z^n} = \sum_{n=0}^{\infty} \left(\frac{-1}{z}\right)^n$$

$$= 1 + \left(\frac{-1}{z}\right)^1 + \left(\frac{-1}{z}\right)^2 + \left(\frac{-1}{z}\right)^3 + \dots$$

$$= 1 - \frac{1}{z} + \frac{1}{z^2} - \frac{1}{z^3} + \dots$$

$$= \left(1 + \frac{1}{z}\right)^{-1} = \left(\frac{z+1}{z}\right)^{-1}$$

$$z[(-1)^n] = \frac{z}{z+1}$$