Inverse z-transform:

Methods:

i) Partial fraction method

ii) Cauchy's Residue method.

Partial fraction method:

I : $\frac{1}{(z+a)(z+b)} = \frac{A}{z+a} + \frac{B}{z+b}$

II : $\frac{1}{(z+a)^2(z+b)} = \frac{A}{z+a} + \frac{B}{(z+a)^2} + \frac{C}{z+b}$

III : $\frac{1}{(z^2+a)(z+b)} = \frac{Az+B}{z^2+a} + \frac{C}{z+b}$

Problems:

1. Find $z^{-1}\left[\frac{10z}{z^2-3z+2}\right]$

$$\frac{10z}{z^2-3z+2} = \frac{10z}{(z-1)(z-2)} \Rightarrow F(z) = \frac{10z}{(z-1)(z-2)}$$

$$\frac{F(z)}{z} = \frac{10}{(z-1)(z-2)} = \frac{A}{z-1} + \frac{B}{z-2}$$

$$10 = A(z-2) + B(z-1)$$

$z=1$, $A=-10$

$z=2$, $B=10$.

$$\therefore \frac{F(z)}{z} = \frac{-10}{z-1} + \frac{10}{z-2}$$

$$\Rightarrow F(z) = \frac{-10z}{z-1} + \frac{10z}{z-2}.$$

$$z^{-1}[F(z)] = z^{-1}\left[\frac{-10z}{z-1}\right] + z^{-1}\left[\frac{10z}{z-2}\right]$$

$$= -10\, z^{-1}\left[\frac{z}{z-1}\right] + 10\, z^{-1}\left[\frac{z}{z-2}\right]$$

$$= -10(1) + 10(2)^n$$

$$= -10 + 10(2)^n$$

$$= 10\left[2^n - 1\right]$$

2. Find $Z^{-1}\left[\frac{z^2-3z}{(z-5)(z+2)}\right]$

$$F(z) = \frac{z^2-3z}{(z-5)(z+2)} = \frac{z(z-3)}{(z-5)(z+2)}$$

$$\frac{F(z)}{z} = \frac{z-3}{(z-5)(z+2)} = \frac{A}{z-5} + \frac{B}{z+2} = \frac{A(z+2)+B(z-5)}{(z-5)(z+2)}$$

$$z-3 = A(z+2) + B(z-5)$$

$z=5$, $A = \frac{2}{7}$; $z=-2$, $B=\frac{5}{7}$

$$\therefore \frac{F(z)}{z} = \frac{2}{7}\frac{1}{z-5} + \frac{5}{7}\frac{1}{z+2}$$

$$F(z) = \frac{2}{7}\frac{z}{z-5} + \frac{5}{7}\frac{z}{z+2}$$

$$Z^{-1}[F(z)] = \frac{2}{7}Z^{-1}\left[\frac{z}{z-5}\right] + \frac{5}{7}Z^{-1}\left[\frac{z}{z+2}\right]$$

$$= \frac{2}{7}(5)^n + \frac{5}{7}(-2)^n$$

3. Find $Z^{-1}\left[\frac{z^3}{(z-1)^2(z-2)}\right]$

$$F(z) = \frac{z^3}{(z-1)^2(z-2)}$$

$$\frac{F(z)}{z} = \frac{z^2}{(z-1)^2(z-2)} = \frac{A}{z-1} + \frac{B}{(z-1)^2} + \frac{C}{z-2}$$

$$= \frac{A(z-1)(z-2) + B(z-2) + C(z-1)^2}{(z-1)^2(z-2)}$$

$$z^2 = A(z-1)(z-2) + B(z-2) + C(z-1)^2$$

$z=1$, $B=-1$

$z=2$, $C=4$

$z=0$, $0 = A(-1)(-2) - 1(-2) + 4(1) \Rightarrow A = -3$.

$$\frac{F(z)}{z} = \frac{-3}{z-1} - \frac{1}{(z-1)^2} + \frac{4}{z-2}.$$

$$F(z) = -3\frac{z}{z-1} - \frac{z}{(z-1)^2} + 4\frac{z}{z-2}$$

$$z^{-1}[F(z)] = -3z^{-1}\left[\frac{z}{z-1}\right] - z^{-1}\left[\frac{z}{(z-1)^2}\right] + 4z^{-1}\left[\frac{z}{z-2}\right]$$

$$= -3(1) - n + 4(2)^n$$

$$= 4(2)^n - 3 - n$$

4. Find $z^{-1}\left[\dfrac{z^2}{(z+2)(z^2+4)}\right]$

$$F(z) = \frac{z^2}{(z+2)(z^2+4)}$$

$$\frac{F(z)}{z} = \frac{z}{(z+2)(z^2+4)} = \frac{A}{z+2} + \frac{Bz+C}{z^2+4}$$

$$z = A(z^2+4) + (z+2)(Bz+C)$$

$z = -2$, $\quad A = -\frac{1}{4}$

$z = 0$, $\quad 0 = 4A + 2C \Rightarrow 0 = 4\left(\frac{-1}{4}\right) + 2C \Rightarrow C = 1/2$

$z = 1$, $\quad 1 = \frac{-1}{4}(5) + 3\left(B + \frac{1}{2}\right)$

$\quad 1 + \frac{5}{4} = 3B + \frac{3}{2} \Rightarrow \frac{9}{4} - \frac{3}{2} = 3B \Rightarrow \frac{3}{4} = 3B \Rightarrow B = \frac{1}{4}$

$$\therefore \frac{F(z)}{z} = \frac{-\frac{1}{4}}{z+2} + \frac{\frac{1}{4}z + \frac{1}{2}}{z^2+4}$$

$$z^{-1}[F(z)] = -\frac{1}{4}z^{-1}\left[\frac{z}{z+2}\right] + \frac{1}{4}z^{-1}\left[\frac{z^2}{z^2+4}\right] + \frac{1}{2}z^{-1}\left[\frac{z}{z^2+4}\right]$$

$$= -\frac{1}{4}(-2)^n + \frac{1}{4}2^n\cos\frac{n\pi}{2} + \frac{1}{4}z^{-1}\left[\frac{2z}{z^2+4}\right]$$

$$= -\frac{1}{4}(-2)^n + \frac{1}{4}2^n\cos\frac{n\pi}{2} + \frac{1}{4}2^n\sin\frac{n\pi}{2}$$

$$\because \quad z\left[a^n\cos\frac{n\pi}{2}\right] = \frac{z^2}{z^2+a^2}$$

$$z\left[a^n\sin\frac{n\pi}{2}\right] = \frac{az}{z^2+a^2}$$