# UNIT - 4

## TORSION

### PART-A

1. **Write torsional equation** **( AU April/May 2017)**

   $T/J=C\theta/L=q/R$

   T-Torque

   J- Polar moment of inertia

   C-Modulus of rigidity

   L- Length

   q- Shear stress R- Radius

2. **Define torsional rigidity.** **(AU Nov/Dec 2016)**

   The torque required to introduce unit angle of twist in unit length is called torsional rigidity or stiffness of shaft.

3. **State any two functions of springs.** **(AU Nov/Dec 2015)**

   1. To measure forces in spring balance, meters and engine indicators.

   2. To store energy.

4. **Write down the expression for power transmitted by a shaft.** **(AU Nov/Dec 2014)**

   $P=2\pi NT/60$

   Where, N-speed in rpm

   T-torque

5. **Write down the expression for torque transmitted by hollow shaft**

   T= /16)*Fs*((D4-d4)/d4 Where, T-torque

   q- Shear stress D-outer diameter d- Inner diameter

**6. Write down the equation for maximum shear stress of a solid circular section in diameter 'D' when subjected to torque 'T' in a solid shaft.**

T=π/16 * Fs*D3 where, T-torque

q - Shear stress D – diameter

**7. What is composite shaft?**

Sometimes a shaft is made up of composite section i.e. one type of shaft is sleeved over other types of shaft. At the time of sleeving, the two shafts are joined together, that the composite shaft behaves like a single shaft.

**8. What is a spring?**

A spring is an elastic member, which deflects, or distorts under the action of load and regains its original shape after the load is removed.

**9. What are the various types of springs?**

i. Helical springs ii. Spiral springs

iii. Leaf springs iv. Disc spring or Belleville springs

**10. Classify the helical springs.**

Close – coiled or tension helical spring.

Open –coiled or compression helical spring.

**11. What is spring index (C)?**

The ratio of mean or pitch diameter to the diameter of wire for the spring is called the spring index.

**12. What are the assumptions made in Torsion equation**

The material of the shaft is homogeneous, perfectly elastic and obeys Hooke"s law.

Twist is uniform along the length of the shaft

The stress does not exceed the limit of proportionality

The shaft circular in section remains circular after loading o Strain and deformations are small.

**13. What is solid length?**

The length of a spring under the maximum compression is called its solid length. It is the product of total number of coils and the diameter of wire.

Ls = nt x d

Where, nt = total number of coils.

**14. Define spring rate (stiffness).**

The spring stiffness or spring constant is defined as the load required per unit deflection of the spring.

K= W/y Where , W - load

y- Deflection.

**15. Define pitch.**

Pitch of the spring is defined as the axial distance between the adjacent coils in uncompressed state. Mathematically

Pitch=free length n-1

**16. Define helical springs..**

The helical springs are made up of a wire coiled in the form of a helix and are primarily intended for compressive or tensile load.

**17. What are the differences between closed coil & open coil helical springs? Closed coil spring**

The spring wires are coiled very closely, each turn is nearly at right angles to the axis of helix . Helix angle is less ($7^0$ to $10^o$)

**Open coil spring**

The wires are coiled such that there is a gap between the two consecutive turns. Helix angle is large (>10o)

**18. Write the assumptions in the theory of pure torsion.**

1. The material is homogenous and isotropic.

2. The stresses are within elastic limit

3. C/S which are plane before applying twisting moment remain plane even after the application of twisting moment.

4. Radial lines remain radial even after applying torsional moment.

5. The twist along the shaft is uniform

**19. Define : Polar Modulus**

Polar modulus is defined as the ratio of polar moment of inertia to extreme radial distance of the fibre from the centre.

**20. Write the equation for the polar modulus for solid circular section**

$$Z_p = \frac{\pi d^3}{16}$$

**21. Define Torsion**

When a pair of forces of equal magnitude but opposite directions acting on body, it tends to twist the body. It is known as twisting moment or torsion moment or simply as torque.

Torque is equal to the product of the force applied and the distance between the point of application of the force and the axis of the shaft.

**22. Define polar modulus**

It is the ratio between polar moment of inertia and radius of the shaft.
£ = polar moment of inertia = J/R

**23. Why hollow circular shafts are preferred when compared to solid circular shafts?**

- The torque transmitted by the hollow shaft is greater than the solid shaft.
- For same material, length and given torque, the weight of the hollow shaft will be less compared to solid shaft.

## PART-B

**1. A solid shaft is to transmit 300kW at 100rpm if the shear stress is not exceed 80N/mm². Find the diameter of the shaft. If this shaft were to be replaced by hollow shaft of same material and length with an internal diameter of 0.6 times the external diameter, what percentage saving in weight is possible.**

**[Madras Univ., Oct 96] (AU April/May 2017)**

**Given Data:** $P = 300kW$

$N = 100rpm$

$\tau = 80N/mm^2$

$d = 0.6\ D_1$

[ $d_1$ – Inner diameter of hollow shaft $D_1$- Outer diameter of hollow shaft]

**To find:**

1. Diameter of the solid shaft(D)
2. % of saving in weight

**@ Solution:**

We know that,

Power,

$$\text{Power}, P = \frac{2\pi N T}{60}$$

$$300 = \frac{2 \times \pi \times 100 \times T}{60}$$

$$T = 28.6kN - m = 28.6 \times 10^3\ N - m$$

$$\boxed{T = 28.6 \times 10^6\ N - mm} \quad .........(1)$$

Torque for solid shaft (considering shear stress)

$$T = \frac{\pi}{16} \times \tau \times D^3$$

$$28.6 \times 10^6 = \frac{\pi}{16} \times 80 \times D^3$$

$$\boxed{\text{Solid shaft diameter}, D = 122.1mm}$$

Solid shaft is replaced by hollow shaft

In hollow shaft, Inner diameter, d

Outer diameter, $D_1$

Torque transmitted by hollow shaft

$$T = \frac{\pi}{16} \times \tau \left[ \frac{D^4_1 - d^4}{D_1} \right]$$

Where, d = 0.6 $D_1$ (Given)

$$\Rightarrow \qquad T = \frac{\pi}{16} \times 80 \left[ \frac{(D_1)^4 - (0.6D_1)^4}{D_1} \right]$$

$$= \frac{\pi}{16} \times 80 \times \frac{D^4_1}{D_1} \left[ 1 - (0.6)^4 \right]$$

$$= \frac{\pi}{16} \times 80 \times D^3_1 [0.8704] \qquad ..........(2)$$

We know that,

Torque transmitted by hollow shaft is equal to torque transmitted by solid shaft when the solid shaft is replaced by hollow shaft.

Equating (1) and (2),

$$\Rightarrow \qquad 28.6 \times 10^6 = \frac{\pi}{16} \times 80 \times D^3_1 [0.8704]$$

$$\boxed{\text{External diameter, } D_1 = 127.8 \text{ mm}}$$

We know that, $\qquad d = 0.6 D_1 = 0.6 \times 127.8$

$$\boxed{\text{Internal Diameter, } d = 76.68 \text{ mm}}$$

% of saving in weight

$$= \frac{\text{Weight of solid shaft} - \text{Weight of hollow shaft}}{\text{weight of solid shaft}} \times 100$$

$$.......(3)$$

Weight of solid shaft

$$= \text{Area of solid shaft} \times \text{Density} \times \text{Length}$$

$$= \frac{\pi}{4} \left( D_1^2 - d^2 \right) \times p \times L$$

$$= \frac{\pi}{4} \times \left[ (127.8)^2 - (76.68)^2 \right] \times p \times L$$

$$\boxed{\text{Weight of hollow Shaft} = 8209.7 \times p \times L}$$

$(3) \Rightarrow$

$$\%\,\text{of saving in weight} = \frac{11709.03\,\text{pL} - 8209.7\,\text{pL}}{11709.03\,\text{pL}} \times 100$$

$$\boxed{\%\,\text{of Saving in weight} = 29.8\%}$$

**Result:** Percentage of saving in weight = 29.8%

2. **A hollow shaft is to transmit 200 kW at 80 rpm. If the shear stress is not to exceed 70 MN/m² and internal diameter is 0.5 of the external diameter. Find the external and internal diameters assuming that maximum torque is 1.6 times the mean. (AU April/May 2016)**

**Given Data:** $P = 200\text{kW}$

$N = 80\text{rpm}$

$T = 70\text{MN/m}^2$

$= 70 \times 10^6\ \text{N/m}^2 = 70\ \text{N/mm}^2$

$D = 0.5\ D$

$T_{max} = 1.6T_{mean}$

**To find:** External diameter (D)

Internal diameter (d)

**Solution:** We know that,

Power, P

$$= \frac{2\pi N T}{60}$$

$$200 = \frac{2 \times \pi \times 80 \times T}{60}$$

$$\Rightarrow \quad \text{Torque, } T = 23.87\,\text{kN.m}$$

$$T = 23.87 \times 10^3\,\text{N} - \text{m}$$

$$= 23.87 \times 10^6\,\text{N} - \text{mm}$$

$$\boxed{T = T_{mean} = 23.87 \times 10^6\,\text{N} - \text{mm}}$$

We know that,

$$T_{max} = 1.6T_{mean}$$

$$T_{max} = 1.6 \times 23.87 \times 10^6\,\text{N} - \text{mm}$$

$$T_{max} = 38.19 \times 10^6\,\text{N} - \text{mm}$$

We know that,

$$T_{max} = \frac{\pi}{16} \times \tau \left[\frac{D^4 - d^4}{D}\right]$$

Substitute $d = 0.5D$

$$\Rightarrow \qquad T_{max} = \frac{\pi}{16} \times \tau \left[\frac{D^4 - (0.5D)^4}{D}\right]$$

$$38.19 \times 10^6 = \frac{\pi}{16} \times 70 \times D^4 \left[\frac{1 - (0.5)^4}{D}\right]$$

$$\Rightarrow \qquad \boxed{D = 143.6\,mm}$$

$$\Rightarrow \qquad d = 0.5\,D = 71.82mm$$

**Result:** Outer diameter, D = 143.6 mm

Inner diameter, d = 71.82 mm

**3. A shaft is required to transmit a power of 210kW at 200rpm. The maximum torque may be 1.5 times the mean torque. The shear stress in the shaft should not exceed 45N/mm² and the twist 1° per metre length. Determine the diameter required if**

**(i) The shaft is solid**

**(ii) The shaft is hollow with external diameter twice the internal diameter. Take modulus of rigidity = 80 N/mm² (AU Nov/Dec 2015)**

**Solution:**

$$\text{Power}(P) = \frac{2\pi NT}{60}$$

$$T = \frac{P \times 60}{2\pi N} = \frac{210 \times 60}{2 \times \pi \times 200}$$

$$T = T_{mean} = 10.02\,KNm$$

$$= 10.02 \times 10^6\,Nmm$$

**WKT** $T_{max} = 1.5\ T_{mean}$

$T_{max} = 15.03 \times 10^6$ Nmm

**For solid shaft**

$$T = \frac{\pi}{16} \tau D^3$$

$$15.03 \times 10^6 = \frac{\pi}{16} \times 45 \times D^3$$

$$D = 119.37\,mm$$

**For Hollow shaft [d = D/2]**

$$T_{max} = \frac{\pi}{16}\tau\left[\frac{D^4 - d^4}{D}\right]$$

$$15.03\times10^6 = \frac{\pi}{16}\times45\times\left[\frac{D^4 - \left(\frac{D}{16}^4\right)}{D}\right]$$

$$15.03\times10^6 = 8.835\times\frac{15}{16}D^3$$

$$D = 12.197\,mm$$

$$d = \frac{D}{2} = 60.98mm$$

4. **A close coiled helical spring is to carry a load of 100N and the mean coil diameter is to be 8times that of the wire diameter. Calculate these diameters, if the maximum stress is to be 10 N/mm²**
**(AU Nov/Dec 2014)**

**Given :** Load (W) = 100N

Mean coil dia (D) = 8d

Max Shear stress ($\tau$) = 10 N/mm$^2$

WKT

$$\tau = \frac{8WD}{\pi d^3}$$

$$10 = \frac{8\times100\times \cancel{8d}}{\pi \cancel{d^3}}$$

$$d^2 = \frac{8\times100\times8}{\pi\times10}$$

$$d = 14.27\,mm$$

$$D = 8d = 114.18\,mm$$

5. **A circular shaft of 100mm diameter is required to transmit torque. Find the safe torque if the shear stress is not to exceed 100Mpa.**

**Given:-**

Dia, D = 100mm,

Shear stress, $\tau$ = 100Mpa

$$= 100\ \text{MN/m}^2 = 100 \times 10^6\ \text{N}/10^4\ \text{mm}^2$$

$$\tau = 100\ \text{N/mm}^2$$

**Required:-**

Tonque, T =?

**Solution:**

Considering shear stress

$$T = \frac{\pi}{4} \times \tau \times D^3$$

$$= \frac{\pi}{4} \times 100 \times (100)^3$$

$$T = 19.635 \times 10^4\ \text{N / mm.}$$

**Result:-**

The sate torque, T = 19.635 × $10^6$ N/mm.

6. **A solid shaft of diameter 100mm is required to transmit 150kW at 120rpm. If the length of the shaft is 4m and modulus of rigidity for the shaft is 75Gpa. Find the angle of twist.**

**Given:**

D = 100mm; P = 150KN; N = 120Kpm; λ = 4m = 4000mm;
c = 75 Gpa = 75 × $10^8$ Gpa C = 75 × $10^9$ N/m² = 75 × $10^3$ N/mm².

**Solution:**

For solid shaft, (considering angle of twist)

$$\frac{T}{J} = \frac{C\theta}{\ell}. \qquad J = \frac{\pi}{32}(D)^4.$$

$$J = 9.81 \times 10^6\ \text{mm}^4$$

$$\text{Power, } P = \frac{2\pi NT}{60}$$

$$T = 11.93 \times 10^6\ \text{N.mm}$$

All values applied in equation (i)

$$\frac{11.93 \times 10^6}{9.81 \times 10^6} = \frac{75 \times 10^3 \times \theta}{4000}$$

$$\theta = 0.06\ \text{radians}$$

$$\theta = 3.7^\circ.$$

**7. A hollow circular shaft of external diameter 40mm and internal diameter 20mm transmits a torque of 15kNm. Find the maximum shear stress induced in the shaft.**

**Given:**

D = 40mm

d = 20mm

T = 15kN.m

**Required:**

T = ?

**Solution:**

Consider shear stress,

For hollow section,

$$T = \frac{\pi}{16} \times \tau \left[\frac{D^4 - d^4}{D}\right]$$

$$15 \times 10^6 = \frac{\pi}{16} \times \tau \left[\frac{(40)^4 - (20)^4}{40}\right]$$

$$15 \times 10^6 = 11780.97\tau$$

$$\tau = 1273.24\, N/mm^2.$$

**8. A hollow Shaft of external diameter 100mm and internal diameter 50mm is required to transist torque from one end to shaft can transmit, if the shear stress is not to exceed 50 Mpa.**

**Given:-**

D = 100mm

d = 50mm

$\tau = 50$ Mpa $= 50 \times 10^6$ N/m$^2$.

$\tau = 50$ N/mm$^2$.

**Solution:**

Torque,

$$T = \frac{\pi}{16} \times \tau \left[ \frac{D^4 - d^4}{D} \right]$$

$$= \frac{\pi}{16} \times 50 \left[ \frac{(100)^4 - (50)^4}{100} \right]$$

$$T = 9.2 \times 10^6 \text{ N.mm}$$

$$\boxed{T = 9.2 \text{KN.m}}$$

**Result:**

Torque, T = 9.2 KN.mℓ.

**9. A hollow shaft of external diameter 120mm transmits 300kWpower at 200rpm. Determine the maximum internal diameter if the maximum stress in the shaft is not to exceed 60N/mm$^2$.**

**Given:**

External Dia, $D_0$ = 120mm

Power, P = 300kN

Speed, N = 200 K.P.M

Max. Shear stress, $\tau$ = 60 N/mm$^2$.

**Solution:**

Let, $D_i$ = Internal dia of shaft

$$P = \frac{2\pi NT}{60}$$

$$T = \frac{300 \times 60}{2\pi \times 200} = 14.32 \text{KN.m.}$$

We know, the equation,

$$T = \frac{\pi}{16} \times \tau \times \frac{\left(D_0{}^4 - D_i{}^4\right)}{D_0}$$

$$14.32 \times 10^6 = \frac{\pi}{10} \times 60 \times \frac{\left(120^4 - D_i\right)}{120}$$

$$\left(D_i\right)^4 = 61.458 \times 10^6$$

$$\left(D_0\right)^4 = \left(61.458 \times 10^6\right)^{1/4}$$

**10. Two shafts of the same material and of same lengths are subjected to the same torque, if the first shaft is of a solid circular section and the second shaft is of hollow circular section, whose internal diameter is 2/3 of the outside diameter and the maximum shear stress developed in each shaft is the same, compare the weights of the shafts.**

**Solution:**

Let,

T – Torque transmitted by each shaft

$\tau$ – Max. shear stress developed in each shaft.

$D_S$ = outer diameter of solid shaft

$D_O$= Outer diameter of hollow shaft

$D_i$ = Internal diameter of hollow shaft = $\frac{2}{3}D_O$

$W_l$ = Weight of solid shaft

$W_H$ = weight of hollow shaft

L = length of each shaft

W = Weight density of the material each shaft.

Torque transmitted by the solid shaft,

$$T = \frac{\pi}{16}\tau.D^2$$

$$= \frac{\pi}{16}\tau\left[\frac{D_o^{\,4} - \frac{16}{81}D_o^{\,4}}{D_o}\right]$$

$$= \frac{\pi}{16}\tau \times \left[\frac{D_o^{\,4} - 0.196\,D_o^{\,4}}{D_o}\right]$$

$$T = \frac{\pi}{16}\tau\left[\frac{0.804\,\cancel{D_o^{\,4}}}{\cancel{D_o^{\,4}}}\right]$$

$$T - \frac{\pi}{16}\tau\,0.804\,D_o^{\,3} \qquad \text{.............(ii)}$$

Equation (i) & (ii)

$$\frac{\pi}{16}\tau D_s^3 = \frac{\pi}{16}\tau\left[0.804 D_o^3\right]$$

$$D_s^3 = 0.804 D_o^3$$

$$D_s = \left(0.804 D_o^3\right)^{1/3}$$

$$D_S = \left(0.804\right)^{1/3} . D_o$$

$$\boxed{D_s = 0.93 D_o}$$

Now weight of solid shaft,

$W_S$ =weight density × Volume of solid shaft

$W_H = \omega \times A_H \times 1$

$$= \omega \times \frac{\pi}{4}\left[D_o^2 - D_i^2\right] \times L$$

$$= \omega \times \frac{\pi}{4}\left[D_o^2 - \left(\frac{2}{3}D_o\right)^2\right] \times L$$

$$= \omega \times \frac{\pi}{4}\left[D_o^2 - \frac{4}{9}D_o^2\right] \times L$$

$$W_H = \omega \times \frac{\pi}{4} \times 0.556 D_o^2 \times L ........(iii)$$

Dividing Eqn (i) & (ii)

$$\frac{W_S}{W_H} = \frac{\omega \times \frac{\pi}{4} \times D_S^2 \times L}{\omega \times \frac{\pi}{4} \times 0.556 D_o^2 \times L}$$

$$= \frac{D_S^2}{0.556 D_o^2}$$

We know, $D_S = 0.93\ D_o$

$$= \frac{\left(0.93 D_o\right)^2}{0.556 D_o^2}$$

$$= \frac{0.865 D_o^2}{0.556 D_o^2}$$

$$= 1.55$$

**11. A closely coiled helical spring is to carry a load of 500N. Its mean diameter is to be 10 times that of the wire diameter calculate these diameter if the maximum shear stress in the materials of the spring is to be 80 N/mm$^2$ . Also calculate the number of coils in the usually wired helical shoring if the Stiffness of the spring is 20 N/mm deflection and modulus of rigidity = 8.6 × 10$^4$ N/mm$^2$.**

**Given:**

Load on spring, W = 500N

Max. Shear stress, $\tau$ = 80 N/mm$^2$.

**Required:**

Diameter of wire, d =?

Mean diameter of coil, D = ?

i.e., D = 10d.

**Solution:**

$$\tau = \frac{16WR}{\pi d^3}$$

$$80 = \frac{16 \times 500 \times \left(\frac{D}{2}\right)}{\pi d^3}$$

$$80 = \frac{8000\left(10\frac{d}{2}\right)}{\pi d^3}$$

$$80 \times \pi d^3 = 8000 \times 5d$$

$$d^3 = \frac{8000 \times 5}{80 \times \pi} = 159.25$$

$$\boxed{d = 12.6\text{mm}}$$

$$\text{Stiffness}(K) = \frac{\text{Load}}{\text{Deflection}}$$

$$\therefore K = \frac{W}{\Delta}$$

$$20 = \frac{500}{\Delta}$$

$$\therefore \Delta = \frac{500}{20}$$

$\boxed{\Delta = 25\text{mm}}$

We know, the general equation,

$$\Delta = \frac{64\,W.R^3.n}{C.d^4}$$

$$25 = \frac{64 \times 500 \times (63)^3 \times n}{8.4 \times 10^4 \times (12.6)^4}$$

$n = 6.6$

Say, 7 No 3.

**Result:-**

**(i)** Wire diameter, d = 12.6mm

**(ii)** Mean coiled diameter, D = 26mm.

**(iii)** Deflection, $\Delta$ = 25mm

**(iv)** no. of coils, n = 7

**12. A closely coiled helical spring of round wire 100mm in diameter having 10 complete turns with a mean diameter of 120mm is subjected to an axial load of 200N. Determine**

i) **The deflection of the spring**

ii) **Maximum shear stress in the wire**

iii) **Stiffness of the spring.**

**Take C = 8x10$^4$ N/mm$^2$**

**Given:**

Dia of wire, d= 10mm

No. of turns, n= 10

Mean dia of coil, D = 120mm

Radius of coil, R = D/2 = 60mm

Axial load, W = 200N

Modulus of rigidity, C = 8 × 10$^4$ N/mm$^2$

**Required:**

$\Delta$ = Deflection of the spring =?

$\tau$ = maximum shear stress in the wire =?

k = Stiffness of the spring =?

**Solution:**

**(i) Deflection,**

$$\Delta = \frac{64 W R^3 \times n}{c d^4}$$

$$\Delta = \frac{64 \times 200 \times (60)^3 \times 10}{8 \times 10^4 \times 10^4} = 34.5\,\text{mm}$$

**(ii) Shear stress,**

$$\tau = \frac{16 WR}{\pi d^3}$$

$$\tau = \frac{16 \times 200 \times 60}{\pi \times 10^3} = 61.1\,\text{N/mm}^2$$

**(iii) Stiffness of the spring, K**

K = W/δ

**13. A close coiled helical spring of 100mm mean diameter is made up of 10mm diameter rod and has 20 turns. The spring carries an axial load of 200N. Determine the shearing stress taking the values of modulus of rigidity = 8.4x10$^4$ N/mm$^2$ Determine the deflection when carrying this load. Also calculate the stiffness of the spring and the frequency of free vibration for a mass hanging from it.**

**Given:**

Mean dia of coil, D = 100mm

Mean radius of coil, R = D/2 = 100/2 = 50mm

Diameter of rod, d = 10mm

No. of turns, n = 20

Axial load, W = 200N

Modulus of rigidity, C = 8.4 × 10$^4$ N/mm$^2$.

**Required:**

$\tau$ =? ; $\Delta$ =?; K =?; g = ?

**Solution:**

$$\left.\begin{matrix}\text{Shear}\\ \text{stress}\end{matrix}\right\}\tau = \frac{16WR}{\pi d^3} = \frac{16\times 200\times 50}{\pi\times 10^3} = 50.93\,N/mm^3$$

$$\left.\begin{matrix}\text{Deflection of}\\ \text{the spring}\end{matrix}\right\},\Delta = \frac{64WR^3\times n}{6d^4} = \frac{64\times 200\times (50)^3\times 20}{8.4\times 10^4\times 10^4} = 38.095\,N/mm^2$$

$$\left.\begin{matrix}\text{Stifness of}\\ \text{the spring}\end{matrix}\right\},\quad K = \frac{\text{Load on spring}}{\text{Deflection on spring}} = \frac{200}{38.095}$$

$$K = 5.25\,N/mm.$$

$$\left.\begin{matrix}\text{Frequency}\\ \text{of free vibration}\end{matrix}\right\},\quad g = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta}}$$

$$= \frac{1}{2\pi}.\sqrt{\frac{981}{3.8095}}$$

$g =$ centre of graduing

$\Delta =$ deflection in

Cycles/sec

**14. The stiffness of a closely coiled helical spring is 1.5N/mm of compression under a maximum load of 60N. The maximum shearing stress produced in the wire of the spring is 125 N/mm². The solid length of the spring (when the coils are touching) is given as 5cm. Find (i) Diameter of wire (ii) mean diameter of the coils and (iii) number of coils required. Take C = 4.5x10⁴ N/mm²**

**Given:**

Stiffness of spring, K = 1.5 N/mm.

Load on spring, W = 60N

Max. shear stress, $\tau = 125 N/mm^2$

Solid length of spring = 50mm

Modulus of rigidity, $C = 4.5 \times 10^4 N/mm^2$.

**Required:**

Dia. Of wire, d = ?

Mean dia of coil, D = ?

Mean radius of coil, R = D/2.

No. of coils, n =?

**Solution:-**

$$K = \frac{C d^4}{64 R^3 n}$$

$$1.5 = \frac{4.5 \times 10^4 \times d^4}{64 \times R^3 \times n}$$

$$d^4 = \frac{1.5 \times 64 \times R^3 \times n}{4.5 \times 10^4}$$

$$\boxed{d^4 = 0.002133 R^3 .n} \ldots\ldots(i)$$

$$\tau = \frac{16 W \times R}{\pi d^3}$$

$$12.5 = \frac{16 \times 60 \times R}{\pi d^3}$$

$$\boxed{R = 0.40906 d^3} \quad \ldots.(ii)$$

Substituting the value of R in equation (i), we get

$$d^4 = 0.02133 \times \left(0.40906 d^3\right) \times n$$

$$= 0.002133 \times \left(0.40906^3\right) \times d^9 \times n = 0.00014599 \times d^9 \times n$$

$$\frac{d^9 .n}{d^4} = \frac{1}{0.00014599} \text{ or } d^5 .n = \frac{1}{0.00014599} \qquad \rightarrow (iii)$$

$$\boxed{d^5 .n = \frac{1}{0.000146}} \ldots..(iii)$$

Solid length, = n × d

$$50 = n \times d$$

$$\boxed{n = \frac{50}{d}} \ldots\ldots(iv)$$

Substituting eqn (iv) in (iii)

$$d^3 \times \frac{50}{d} = \frac{1}{0.00016}$$

$$d^4 = 137$$

$$d = (137)^{1/4}$$

$$\boxed{d = 3.42 \text{ mm}}$$

Substituting, d= 3.42mm value in eqn (iv)

$$n = \frac{50}{d} = \frac{50}{3.42} = 14.62 \text{ say } 15$$

$$\boxed{n = 15}$$

Substitute, d = 3.42mm value in eqn (ii)

$$R = 0.40906\,d^3 = 0.40906(3.42)^2$$

$$\boxed{R = 16.36\,\text{mm}}$$

Mean dia of coil, D = 2R (R=D/2)

$$= 2 \times 16.36 = 32.72$$

$$\boxed{D = 32.72\text{mm}}$$

**15. An open coil helical spring made of 10mm diameter wire and of mean diameter 10cm has 12coils, angle of helix being 15°. Determine the axial deflection and intensities of bending and shear stress under a load of 500N. Take C as 80kN/mm2 and E = 200kN/mm$^2$. [Nov/Dec 2014]**

**Given Data:**

Diameter of wire (d) = 10mm = 0.01 m

Mean coil diameter (D) = 10cm = 0.1 m

No.of coils (n) = 12 radius (R) = $\frac{D}{2}$ = 0.05m

Angle of helix (∝) = 15°

Axial load (W) = 500N

C = 80 KN / mm$^2$ = 80 × 10$^9$ N/m$^2$

E = 200 kN/mm$^2$ = 200 ×10$^9$ N/m$^2$

**To find:**

(i) Axial Deflection (δ\)

(ii) Bending Stress ($\sigma_b$)

(iii) Shear stress (τ)

**Solution:**

(i) Axial Deflection (δ)

$$\delta = 2WR^3 n\pi \sec\alpha \left[\frac{\cos^2\alpha}{C\,I_P} + \frac{Sin^2\alpha}{EI}\right]$$

$$\delta = 2\times 500\times 0.05^3\times 12\times\pi\sec 15\times$$

$$\left[\frac{\cos^2 15}{80\times 10^9\times\frac{\pi}{32}\times(0.01)^4} + \frac{Sin^2 15}{200\times 10^9\times\frac{\pi}{64}\times(0.01)^4}\right]$$

$$\boxed{\delta = 0.061\,mm}$$

$$\sigma_b = \frac{32\,M}{T\,d^3}$$

M = W R sin ∝ = 500 × 0.05 × sin 15

M= 6.47 Nm

$$\sigma_b = \frac{32\times 6.47}{\pi\times 0.01^3} = 65.90\,W/m^2$$

**(iii) Shear stress (τ)**

$$\tau = \frac{16T}{\pi d^3}$$

τ = W R cos ∝ = 500 × 0.05 × cos 15

τ= 24.14 Nm

$$\tau = \frac{16\times 24.14}{(\pi\times 0.01^3)} = 122.94\,MN/m^2$$

90 = $\sigma_x$ = sin θ, 30 = $\sigma_{x=}$ cos θ

Thus,

$$\sigma_x = \sqrt{90^2+30^2}$$
$$= 94.87\,MN/m^2 \quad (Ans)$$

$$\frac{\sigma_x\sin\theta}{\sigma_x\cos\theta} = \frac{90}{30} = 3$$

Or, tan θ = 3

Or, θ = 71°33’ (Ans)