8. Illustrative Problem :

The simply supported beam shown below carries a vertical load that increases uniformly from zero at the one end to the maximum value of 6kN/m of length at the other end .Draw the shearing force and bending moment diagrams.

Solution

Determination of Reactions

For the purpose of determining the reactions R1 and R2 , the entire distributed load may be replaced by its resultant which will act through the centroid of the triangular loading diagram.

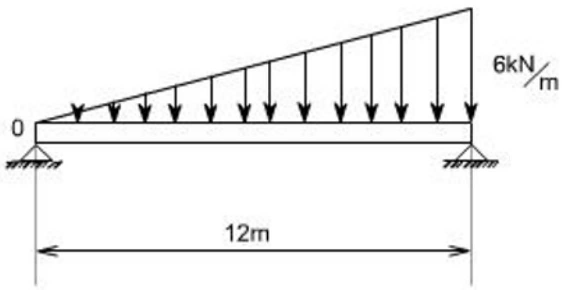
So the total resultant load can be found like this-

Average intensity of loading = (0 + 6)/2

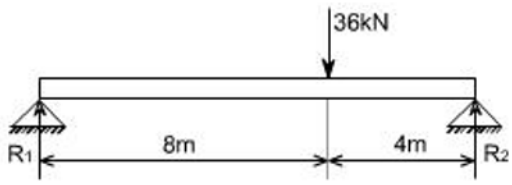
= 3 kN/m

Total Load = 3 x 12

= 36 kN



Since the centroid of the triangle is at a 2/3 distance from the one end, hence 2/3 x 3 = 8 m from the left end support.



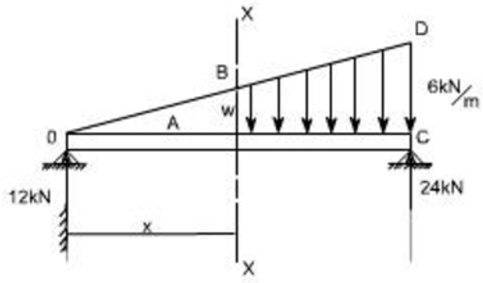
Now taking moments or applying conditions of equilibrium

36 x 8 = R2 x 12

R1 = 12 kN

R2 = 24 kN

Note: however, this resultant can not be used for the purpose of drawing the shear force and bending moment diagrams. We must consider the distributed load and determine the shear and moment at a section x from the left hand end.



Consider any X-section X-X at a distance x, as the intensity of loading at this X-section, is unknown let us find out the resultant load which is acting on the L.H.S of the X-section X-X, hence

So consider the similar triangles

OAB & OCD

$$\frac{w}{6} = \frac{x}{12}$$
$$w = \frac{x}{2} \ k\frac{N}{m}$$

In order to find out the total resultant load on the left hand side of the X-section

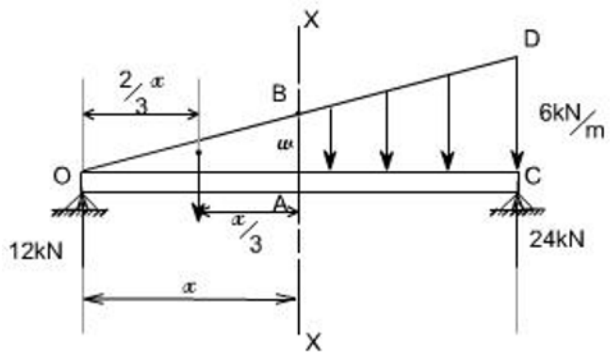
Find the average load intensity

$$= \frac{0 + \frac{x}{2}}{2}$$
$$= \frac{x}{4} \ k\frac{N}{m}$$

Therefore the total load over the length x would be

$$= \frac{x}{4} . x \ kN$$
$$= \frac{x^2}{4} \ kN$$

Now these loads will act through the centroid of the triangle OAB. i.e. at a distance 2/3 x from the left hand end. Therefore, the shear force and bending momemt equations may be written as



$$S.F_{at\ xx} = \left(12 - \frac{x^2}{4}\right) \text{ kN}$$

valid for all values of x ....(1)

$$B.M_{at\ xx} = 12x - \frac{x^2}{4}.\frac{x}{3}$$

$$B.M_{at\ xx} = 12x - \frac{x^3}{12} \text{ kN-m}$$

valid for all values of x ....(2)

$$S.F_{at\ x=0} = 12 \text{ kN}$$

$$S.F_{at\ x=12m} = 12 - \frac{12 \times 12}{4}$$

$$= -24 \text{ kN}$$

In order to find out the point where S.F is zero

$$\left(12 - \frac{x^2}{4}\right) = 0$$

$x = 6.92$ m (selecting the positive values)

Again

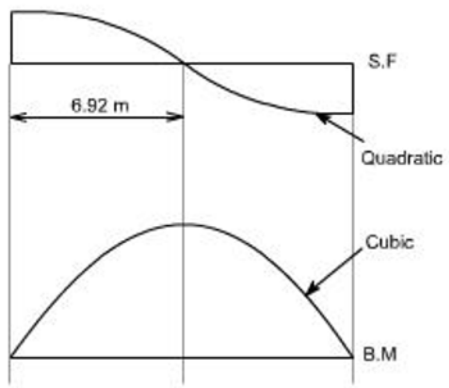
$$B.M_{at\ x=0} = 0$$

$$B.M_{at\ x=12} = 12 \times 12 - \frac{12^3}{12}$$

$$= 0$$

$$B.M_{at\ x=6.92} = 12 \times 6.92 - \frac{6.92^3}{12}$$

$$= 55.42 \text{ kN-m}$$



9. Illustrative problem :

In the same way, the shear force and bending moment diagrams may be attempted for the given problem