



**LECTURE NOTES**

**ON**

**19ASB201 AERO MECHANICS OF SOLIDS**

**II YEAR AEROSPACE ENGINEERING**



## UNIT – 1: SIMPLE STRESSES AND STRAINS

### INTRODUCTION AND REVIEW

#### Preamble

Engineering science is usually subdivided into number of topics such as

1. Solid Mechanics
2. Fluid Mechanics
3. Heat Transfer
4. Properties of materials and soon Although there are close links between them in terms of the physical principles involved and methods of analysis employed.

The solid mechanics as a subject may be defined as a branch of applied mechanics that deals with behaviours of solid bodies subjected to various types of loadings. This is usually subdivided into further two streams i.e Mechanics of rigid bodies or simply Mechanics and Mechanics of deformable solids.

The mechanics of deformable solids which is branch of applied mechanics is known by several names i.e. strength of materials, mechanics of materials etc.

#### **Mechanics of rigid bodies:**

The mechanics of rigid bodies is primarily concerned with the static and dynamic behaviour under external forces of engineering components and systems which are treated as infinitely strong and undeformable Primarily we deal here with the forces and motions associated with particles and rigid bodies.

#### **Mechanics of deformable solids :**

#### **Mechanics of solids:**

The mechanics of deformable solids is more concerned with the internal forces and associated changes in the geometry of the components involved. Of particular importance are the properties of the materials used, the strength of which will determine whether the components fail by breaking in service, and the stiffness of which will determine whether the amount of deformation they suffer is acceptable. Therefore, the subject of mechanics of materials or strength of materials is central to the whole activity of engineering design. Usually the objectives in analysis here will be the determination of the stresses, strains, and deflections produced by loads. Theoretical analyses and experimental results have an equal roles in this field.

#### **Analysis of stress and strain :**

**Concept of stress :** Let us introduce the concept of stress as we know that the main problem of engineering mechanics of material is the investigation of the internal resistance of the body, i.e. the nature of forces set up within a body to balance the effect of the externally applied forces.

The externally applied forces are termed as loads. These externally applied forces may be due to any one of the reason.

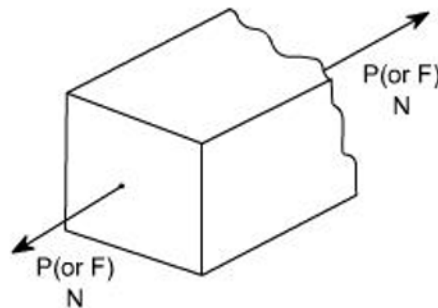


- (i) due to service conditions
- (ii) due to environment in which the component works
- (iii) through contact with other members
- (iv) due to fluid pressures
- (v) due to gravity or inertia forces.

As we know that in mechanics of deformable solids, externally applied forces acts on a body and body suffers a deformation. From equilibrium point of view, this action should be opposed or reacted by internal forces which are set up within the particles of material due to cohesion.

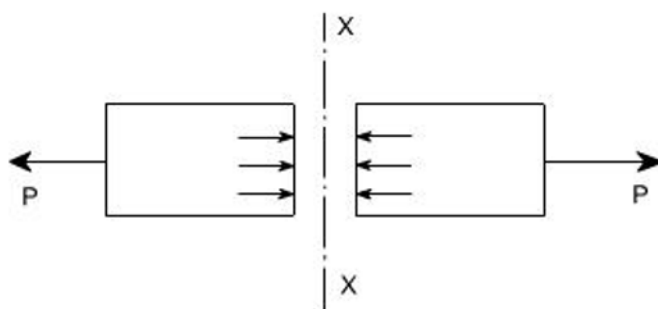
These internal forces give rise to a concept of stress. Therefore, let us define a stress Therefore, let us define a term stress

**Stress:**



Let us consider a rectangular bar of some cross – sectional area and subjected to some load or force (in Newtons )

Let us imagine that the same rectangular bar is assumed to be cut into two halves at section XX. The each portion of this rectangular bar is in equilibrium under the action of load P and the internal forces acting at the section XX has been shown



Now stress is defined as the force intensity or force per unit area. Here we use a symbol  $\sigma$  to represent the stress.

$$\sigma = \frac{P}{A}$$

Where A is the area of the X – section



Here we are using an assumption that the total force or total load carried by the rectangular bar is uniformly distributed over its cross – section.

But the stress distributions may be far from uniform, with local regions of high stress known as stress concentrations.

If the force carried by a component is not uniformly distributed over its cross – sectional area, A, we must consider a small area, 'ΔA' which carries a small load ΔP, of the total force 'P', Then definition of stress is

$$\sigma = \frac{\delta F}{\delta A}$$

As a particular stress generally holds true only at a point, therefore it is defined mathematically as

$$\sigma = \lim_{\delta A \rightarrow 0} \frac{\delta F}{\delta A}$$

### Units :

The basic units of stress in S.I units i.e. (International system) are N / m<sup>2</sup> (or Pa)

$$\text{MPa} = 10^6 \text{ Pa}$$

$$\text{GPa} = 10^9 \text{ Pa}$$

$$\text{KPa} = 10^3 \text{ Pa}$$

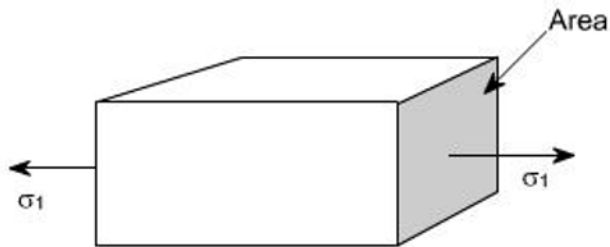
Some times N / mm<sup>2</sup> units are also used, because this is an equivalent to MPa. While US customary unit is pound per square inch psi.

### TYPES OF STRESSES :

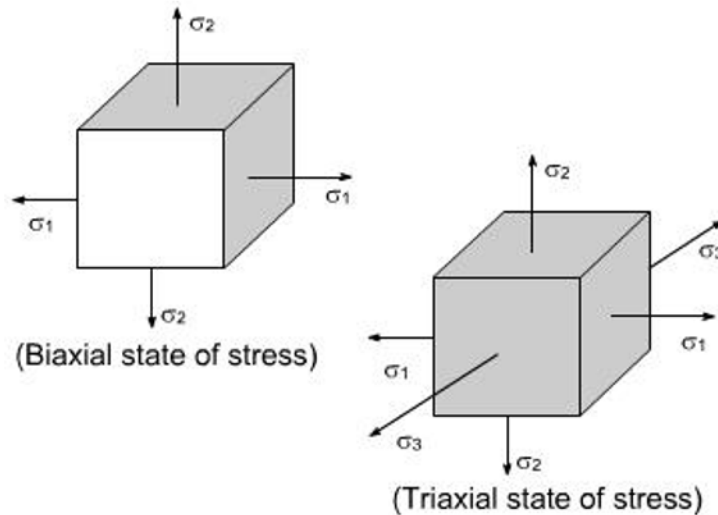
only two basic stresses exists : (1) normal stress and (2) shear stress. Other stresses either are similar to these basic stresses or are a combination of these e.g. bending stress is a combination tensile, compressive and shear stresses. Torsional stress, as encountered in twisting of a shaft is a shearing stress.

Let us define the normal stresses and shear stresses in the following sections.

**Normal stresses :** We have defined stress as force per unit area. If the stresses are normal to the areas concerned, then these are termed as normal stresses. The normal stresses are generally denoted by a Greek letter ( σ )

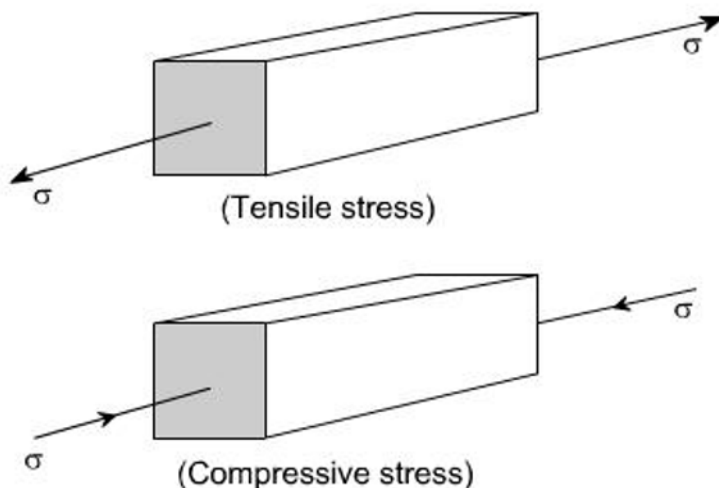


This is also known as uniaxial state of stress, because the stresses acts only in one direction however, such a state rarely exists, therefore we have biaxial and triaxial state of stresses where either the two mutually perpendicular normal stresses acts or three mutually perpendicular normal stresses acts as shown in the figures below :

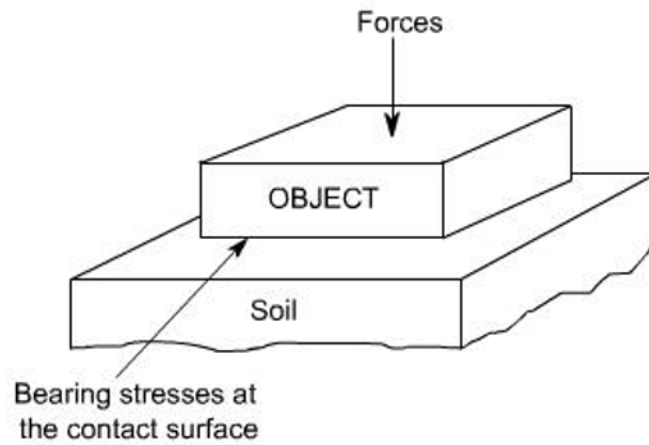


**Tensile or compressive stresses :**

The normal stresses can be either tensile or compressive whether the stresses acts out of the area or into the area

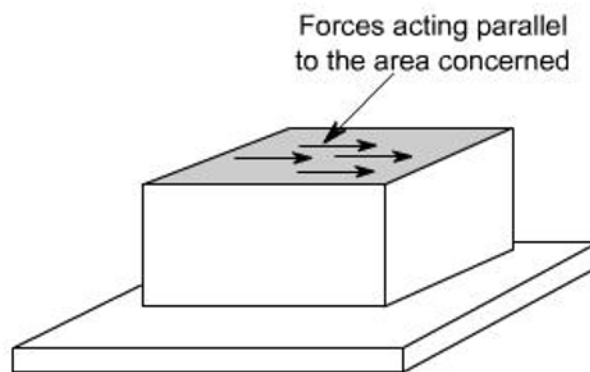


**Bearing Stress :** When one object presses against another, it is referred to a bearing stress ( They are in fact the compressive stresses ).



### Shear stresses :

Let us consider now the situation, where the cross – sectional area of a block of material is subject to a distribution of forces which are parallel, rather than normal, to the area concerned. Such forces are associated with a shearing of the material, and are referred to as shear forces. The resulting force intensities are known as shear stresses.



The resulting force intensities are known as shear stresses, the mean shear stress being equal to

$$\tau = \frac{P}{A}$$

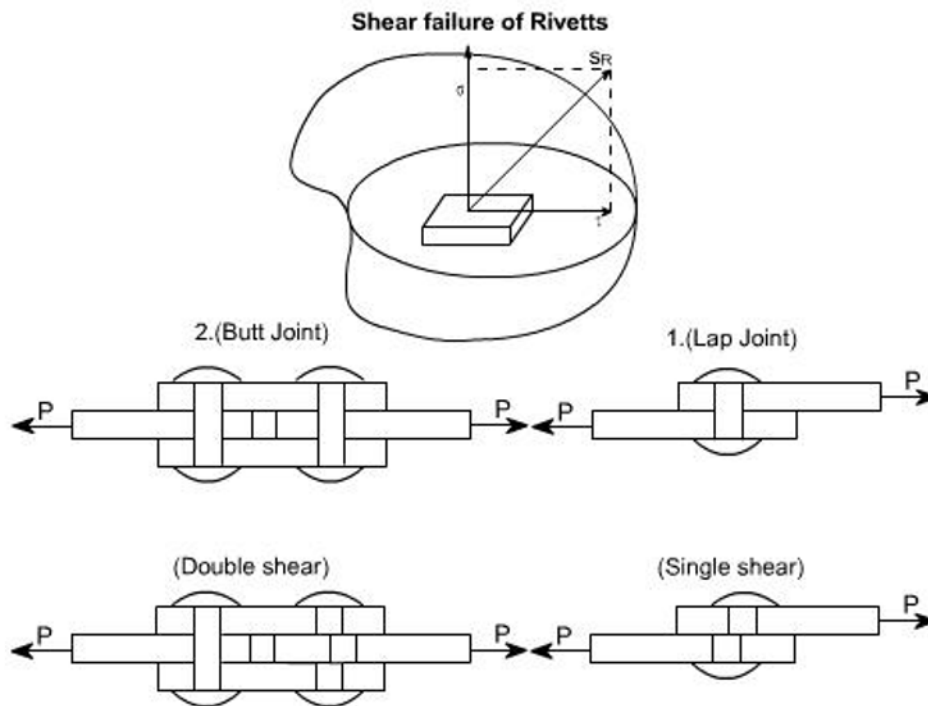
Where P is the total force and A the area over which it acts.

As we know that the particular stress generally holds good only at a point therefore we can define shear stress at a point as

$$\tau = \lim_{\delta A \rightarrow 0} \frac{\delta F}{\delta A}$$

The greek symbol  $\tau$  ( tau ) ( suggesting tangential ) is used to denote shear stress.

However, it must be borne in mind that the stress ( resultant stress ) at any point in a body is basically resolved into two components one acts perpendicular and other parallel to the area concerned, as it is clearly defined in the following figure.11



The single shear takes place on the single plane and the shear area is the cross - sectional of the rivet, whereas the double shear takes place in the case of Butt joints of rivets and the shear area is the twice of the X - sectional area of the rivet.

### MECHANICAL PROPERTIES

In the course of operation or use, all the articles and structures are subjected to the action of external forces, which create stresses that inevitably cause deformation. To keep these stresses, and, consequently deformation within permissible limits it is necessary to select suitable materials for the Components of various designs and to apply the most effective heat treatment. i.e. a Comprehensive knowledge of the chief characteristics of the semi-finished metal products & finished metal articles (such as strength, ductility, toughness etc) are essential for the purpose.

For this reason the specification of metals, used in the manufacture of various products and structure, are based on the results of mechanical tests or we say that the mechanical tests conducted on the specially prepared specimens (test pieces) of standard form and size on special machines to obtained the strength, ductility and toughness characteristics of the metal.

The conditions under which the mechanical test are conducted are of three types

- (1) **Static:** When the load is increased slowly and gradually and the metal is loaded by tension, compression, and torsion or bending.
- (2) **Dynamic:** when the load increases rapidly as in impact
- (3) **Repeated or Fatigue:** (both static and impact type) . i.e. when the load repeatedly varies in the course of test either in value or both in value and direction Now let us consider the uniaxial tension test.

[ For application where a force comes on and off the structure a number of times, the material cannot withstand the ultimate stress of a static tool. In such cases the ultimate strength depends on no. of times the force is applied as the material works at a particular stress level. Experiments one conducted



to compute the number of cycles requires breaking to specimen at a particular stress when fatigue fluctuating load is acting. Such tests are known as fatigue tests]

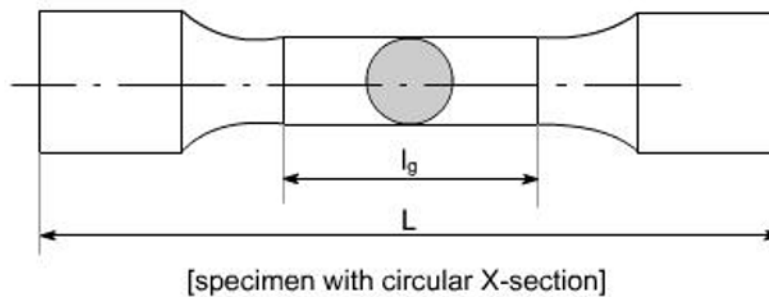
**Uniaxial Tension Test:** This test is of static type i.e. the load is increased comparatively slowly from zero to a certain value.

Standard specimens are used for the tension test.

There are two types of standard specimen's which are generally used for this purpose, which have been shown below:

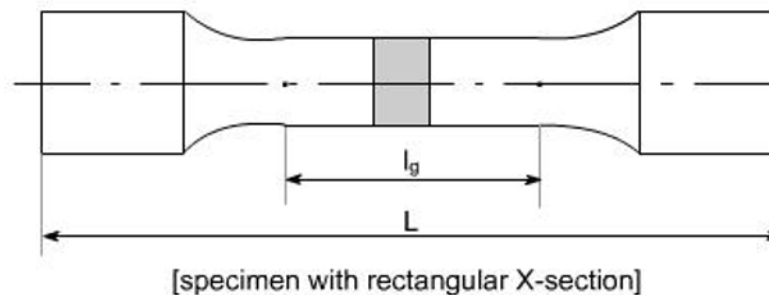
**Specimen I:**

This specimen utilizes a circular X-section.



**Specimen II:**

This specimen utilizes a rectangular X-section.

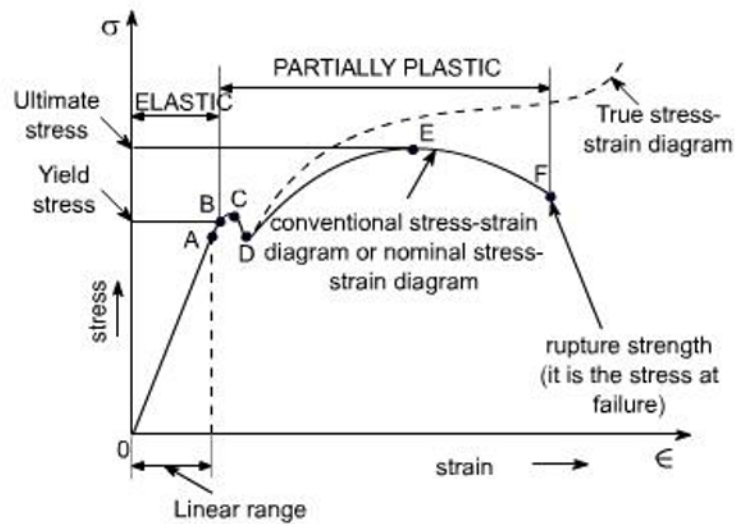


$l_g$  = gauge length i.e. length of the specimen on which we want to determine the mechanical properties. The uniaxial tension test is carried out on tensile testing machine and the following steps are performed to conduct this test.

- (i) The ends of the specimen's are secured in the grips of the testing machine.
- (ii) There is a unit for applying a load to the specimen with a hydraulic or mechanical drive.
- (iii) There must be a some recording device by which you should be able to measure the final output in the form of Load or stress. So the testing machines are often equipped with the pendulum type lever, pressure gauge and hydraulic capsule and the stress Vs strain diagram is plotted which has the following shape.

A typical tensile test curve for the mild steel has been shown below





### Nominal stress – Strain OR Conventional Stress – Strain diagrams:

Stresses are usually computed on the basis of the original area of the specimen; such stresses are often referred to as conventional or nominal stresses.

### True stress – Strain Diagram:

Since when a material is subjected to a uniaxial load, some contraction or expansion always takes place. Thus, dividing the applied force by the corresponding actual area of the specimen at the same instant gives the so called true stress.

### SALIENT POINTS OF THE GRAPH:

**(A)** So it is evident from the graph that the strain is proportional to strain or elongation is proportional to the load giving a st. line relationship. This law of proportionality is valid upto a point A.

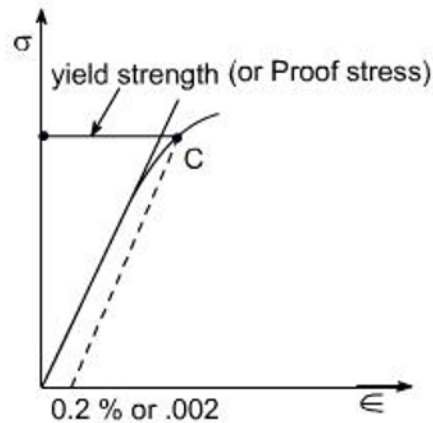
Or we can say that point A is some ultimate point when the linear nature of the graph ceases or there is a deviation from the linear nature. This point is known as **the limit of proportionality or the proportionality limit**.

**(B)** For a short period beyond the point A, the material may still be elastic in the sense that the deformations are completely recovered when the load is removed. The limiting point B is termed as **Elastic Limit**.

**(C) and (D)** - Beyond the elastic limit plastic deformation occurs and strains are not totally recoverable. There will be thus permanent deformation or permanent set when load is removed. These two points are termed as upper and lower yield points respectively. The stress at the yield point is called the yield strength.

A study a stress – strain diagrams shows that the yield point is so near the proportional limit that for most purpose the two may be taken as one. However, it is much easier to locate the former. For material which do not possess a well define yield points, In order to find the yield point or yield strength, an offset method is applied.

In this method a line is drawn parallel to the straight line portion of initial stress diagram by off setting this by an amount equal to 0.2% of the strain as shown as below and this happens especially for the low carbon steel.



**(E)** A further increase in the load will cause marked deformation in the whole volume of the metal. The maximum load which the specimen can with stand without failure is called the load at the ultimate strength.

The highest point 'E' of the diagram corresponds to the ultimate strength of a material.

$\sigma_u$  = Stress which the specimen can with stand without failure & is known as Ultimate Strength or Tensile Strength.

$\sigma_u$  is equal to load at E divided by the original cross-sectional area of the bar.

**(F)** Beyond point E, the bar begins to forms neck. The load falling from the maximum until fracture occurs at F.

[Beyond point E, the cross-sectional area of the specimen begins to reduce rapidly over a relatively small length of bar and the bar is said to form a neck. This necking takes place whilst the load reduces, and fracture of the bar finally occurs at point F]

**Note:** Owing to large reduction in area produced by the necking process the actual stress at fracture is often greater than the above value. Since the designers are interested in maximum loads which can be carried by the complete cross section, hence the stress at fracture is seldom of any practical value.

### Percentage Elongation:

The ductility of a material in tension can be characterized by its elongation and by the reduction in area at the cross section where fracture occurs.

It is the ratio of the extension in length of the specimen after fracture to its initial gauge length, expressed in percent.

$$\delta = \frac{(l_1 - l_g)}{l_g} \times 100$$

$l_1$  = gauge length of specimen after fracture(or the distance between the gage marks at fracture)

$l_g$ = gauge length before fracture(i.e. initial gauge length)

For 50 mm gage length, steel may here a % elongation □ of the order of 10% to 40%.