31.10. Change in Diameter and Volume of a Thin Spherical Shell due to an Internal Pressure

Consider a thin spherical shell subjected to an internal pressure.

Let d = Diameter of the shell,
p = Intensity of internal pressure and
t = Thickness of the shell.

We have already discussed in the last article that the stress in a spherical shell,

$$\sigma = \frac{pd}{4t}$$

and strain in any one direction,

$$\varepsilon = \frac{\sigma}{E} - \frac{\sigma}{mE} \qquad \text{...}(\because \sigma_1 = \sigma_2 = \sigma)$$

$$= \frac{pd}{4tE} - \frac{pd}{4tEm} = \frac{pd}{4tE}\left(1 - \frac{1}{m}\right)$$

∴ Change in diameter,

$$\delta d = \varepsilon \cdot d = \frac{pd}{4tE}\left(1 - \frac{1}{m}\right) \times d = \frac{pd^2}{4tE}\left(1 - \frac{1}{m}\right)$$

We also know that original volume of the sphere,

$$V = \frac{\pi}{6} \times (d)^3$$

and final volume due to pressure,

$$V + \delta V = \frac{\pi}{6} \times (d + \delta d)^3$$

where $(d + \delta d)$ = Final diameter of the shell.

∴ Volumetric strain,

$$\frac{\delta V}{V} = \frac{(V + \delta V) - V}{V} = \frac{\frac{\pi}{6}(d + \delta d)^3 - \frac{\pi}{6} \times d^3}{\frac{\pi}{6} \times d^3}$$

$$= \frac{d^3 + (3d^2 \cdot \delta d) - d^3}{d^3}$$

...(Ignoring second and higher power of δd)

$$= \frac{3 \cdot \delta d}{d} = 3\varepsilon$$

and

$$\delta V = V \cdot 3\varepsilon = \frac{\pi}{6}(d)^3 \times 3 \times \frac{pd}{4tE}\left(1 - \frac{1}{m}\right) = \frac{\pi p d^4}{8tE}\left(1 - \frac{1}{m}\right)$$

Example 31.10. *A spherical shell of 2 m diameter is made up of 10 mm thick plates. Calculate the change in diameter and volume of the shell, when it is subjected to an internal pressure of 1.6 MPa. Take E = 200 GPa and 1/m = 0.3.*

Solution. Given: Diameter of shell (d) = 2 m = 2×10^3 mm ; Thickness of plates (t) = 10 mm ; Internal pressure (p) = 1.6 MPa = 1.6 N/mm^2 ; Modulus of elasticity (E) = 200 GPa = 200×10^3 N/mm^2 and Poisson's ratio ($1/m$) = 0.3.

Change in diameter

We know that change in diameter,

$$\delta d = \frac{pd^2}{4tE}\left(1 - \frac{1}{m}\right) = \frac{1.6 \times (2 \times 10^3)^2}{4 \times 10 \times (200 \times 10^3)}(1 - 0.3)$$

$$= \mathbf{0.56\ mm} \qquad \textbf{Ans.}$$

Change in volume

We also know that change in volume,

$$\delta V = \frac{\pi p d^4}{8tE}\left(1 - \frac{1}{m}\right) = \frac{\pi \times 1.6 \times (2 \times 10^3)^4}{8 \times 10 \times (200 \times 10^3)}(1 - 0.3) \text{ mm}^3$$

$$= \mathbf{3.52 \times 10^6 \text{ mm}^3} \qquad \textbf{Ans.}$$

31.11. Riveted Cylindrical Shells

Sometimes, boilers of the desired capacity are made of cylindrical shape by joining different plates usually by rivets. This is generally done : (*i*) by bending the plates to the required diameter and then joining them by a butt joint and (*ii*) by joining individually fabricated shells by a lap joint as shown in Fig. 31.5 (*a*) and (*b*). A little consideration will show that in this case, the plate is weakened by the rivet hole.

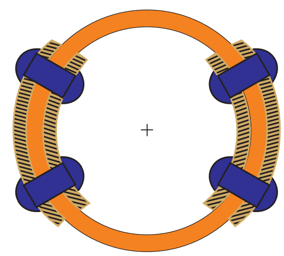

Fig. 31.5 (*a*) Joining by Butt Joint

The circumferential stress in a riveted cylindrical shell,

$$\delta_c = \frac{pd}{2t\eta}$$

Similarly, longitudinal stress,

$$\delta_l = \frac{pd}{4t\eta}$$

where η is the efficiency of the riveted joint.

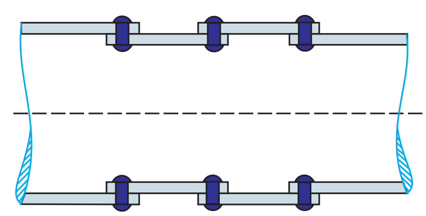

Fig. 31.5 (*b*) Joining by Lap Joint

NOTES: 1. If the efficiency of the joint is different *i.e.*, the joint has different longitudinal efficiency and circumferential efficiency, then the respective values should be used in the above relation.

2. For designing the shell *i.e.*, determining the thickness of shell, the efficiency of the joint should also be considered.

EXAMPLE 31.11. *A boiler shell of 2 m diameter is made up of mild steel plates of 20 mm thick. The efficiency of the longitudinal and circumferential joints is 70% and 60% respectively. Determine the safe pressure in the boiler, if the permissible tensile stress in the plate section through the rivets is 100 MPa. Also determine the circumferential stress in the plate and longitudinal stress through the rivets.*

SOLUTION. Given: Diameter of boiler (d) = 2 m = 2×10^3 mm ; Thickness (t) = 20 mm ; Longitudinal efficiency (η_l) = 70% = 0.7 ; Circumferential efficiency (η_c) = 60% = 0.6 and permissible stress (σ) = 100 MPa = 100 N/mm^2.

Safe pressure in boiler

Let p = Safe pressure in boiler in N/mm^2

We know that permissible stress in boiler (σ),

$$100 = \frac{pd}{2t\eta_l} = \frac{p \times (2 \times 10^3)}{2 \times 20 \times 0.7} = \frac{500p}{7}$$

$$p = \frac{100 \times 7}{500} = 1.4 \text{ N/mm}^2 = \mathbf{1.4 \text{ MPa}} \qquad \textbf{Ans.}$$