UNIT 15 SPRINGS

Structure

15.1 INTRODUCTION

The primary function of a spring is to deflect or distort under load and to recover its original shape when the load is released. During deflection or distortion, it absorbs energy and release the same as and when required. Springs are used in many engineering applications such as automobiles and railway buffers in order to cushion, absorb or control energy due to shock and vibrations. Springs will suffer a sizeable change in form without being distorted permanently when the loads are applied. Springs are generally classified as leaf springs or helical springs. Leaf springs consist of a number of thin curved plates, each of same thickness and width but of different lengths, all bent to the same curvature. Helical springs are formed by coiling thick spring wire into a helix. Helical springs are classified into two groups. When the helix angle is less than about 10°, it is named as close-coiled helical spring. In such springs, the wire experiences too little bending or direct shear stress and their effect is neglected. Torsional stresses are predominant in such springs. If, however the helix angle is significant, then the wire experiences both torsional and bending stresses. Such type of spring is termed as open-coiled helical spring.

Objectives

After studying this unit, you should be able to

- differentiate between close coiled, open coiled and leaf springs,
- calculate the stresses in springs,
- calculate the deflection, proof load and stiffness of springs, and
- identify the areas of application of springs to practical situations.

Definitions

The various terms used in this unit are as follows :

Proof load

It is the greatest load that the spring can carry without getting permanently distorted.

Proof stress

It is the maximum stress in the spring when subjected to proof load.

Proof resilience

It is the strain energy stored in the spring when it has been subjected to the maximum load i.e. proof load.

Spring constant (stiffness of the spring)

It is the load per unit deflection. It is expressed in N/m or kN/m.

15.2 CLOSE COILED HELICAL SPRINGS

A helical spring is a wire wound in spiral form, which can undergo considerable deflection without getting permanently distorted. A helical spring is said to be close-coiled when the obliquity of the wire is small, i.e. the pitch of the coils is very small. Each turn can be regarded as practically lying in planes at right angles to the axis of the helix. Hence, close-coiled helical spring is a torsion spring under axial load it may be assumed that the spring is subjected to torsion only, neglecting the effects of bending and disect shear.

15.2.1 Stresses in Springs

When Subjected to Axial Load

Figure 15.1 (a) shows a close-coiled helical spring subjected to axial load W.

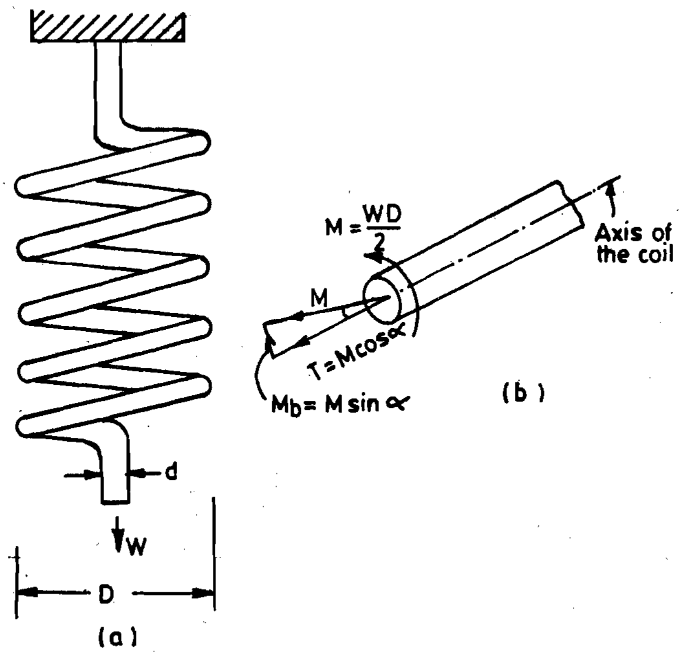

Figure 15.1 : Close Coiled Helical Spring (Subjected to Axial Load)

Let D = mean coil diameter

d = wire diameter

n = number of coils

l = length of the wire

δ = axial deflection

α = angle of helix

Moment at any point $= W \times \left(\frac{D}{2}\right)$

The axis of the coil is inclined at an angle α.

Resolving the moment, along and at right angles to the axis of the spring,

Vector component along the axis of the spring $= T = M \cos \alpha$

Vector component normal to the axis of the spring $M_b = M \sin \alpha$

The component normal to the axis of the spring causes bending and the helix angle α being small, it can be neglected and $\cos \alpha$ tends to unity for small angles.

$\therefore$ The vector component along the axis, T will be equal to M.

$$\therefore\ T = M = W \times \left(\frac{D}{2}\right) \qquad (15.1)$$

Total length of the spring = $(\pi D)\, n$

Total volume of the spring = $\left(\frac{\pi}{4} d^2\right)(\pi D n)$

Using torsion formula, $\frac{T}{J} = \frac{f_s}{r}$,

We get, shear stress, $f_s = \left(\frac{T}{J}\right) r$

$$= T \times \frac{\left(\frac{d}{2}\right)}{\left(\frac{\pi d^4}{32}\right)}$$

$$= \frac{16\,T}{\pi d^3}$$

On substituting, we get $T = W\left(\frac{D}{2}\right)$

$$f_s = \frac{16}{\pi d^3} \times \left(\frac{WD}{2}\right)$$

$$\therefore\ f_s = \frac{8\,WD}{\pi d^3} \qquad (15.2)$$

Alternatively,

Considering the equilibrium of one part of the spring, the spring is subjected to two types of forces,

(i) due to direct shear force W which is assumed to be uniformly distributed over the cross-section of the wire and

(ii) the moment WR acting in the plane of the section, which will cause torrional stress.

$\therefore$ Direct shear stress = $\frac{W}{\frac{\pi}{4} d^2} = \frac{4W}{\pi d^2}$

$$\text{Torsional shear stress} = \frac{WR \times \left(\frac{d}{2}\right)}{\left(\frac{\pi d^4}{32}\right)} = \frac{16WR}{\pi d^3}$$

Total shear stress, i.e. maximum shear stress

$$= \frac{4W}{\pi d^2} + \frac{16WR}{\pi d^3}$$

$$= \frac{16WR}{\pi d^3}\left(1 + \frac{d}{4R}\right)$$

The above equation is only for straight shafts, whereas in the case of springs, the wire is curved hence, the above equation is only approximate.

Whal gave a more exact relationship as

$$(f_s)_{max} = \frac{8WD}{d^3} \times K$$

where K is called the Whal's correction factor.

When the wire diameter is very small, compared to the mean radius of spring, $\frac{d}{4R}$ being too small and can be neglected.

$$\therefore \quad (f_s)_{max} = \frac{16WR}{\pi d^3}.$$

When Subjected to Axial Couple

Figure 15.2 (a) shows a close coiled helical spring under an axial couple M.

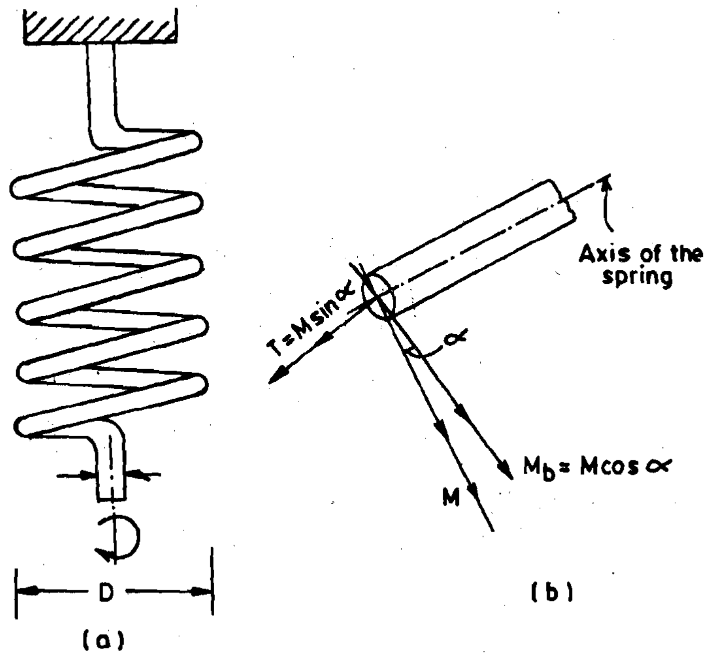

Figure 15.2 : Close Coiled Helical Spring (Subjected to Axial Couple)

In this case, the component $M \sin \alpha$ is negligible, as α is small and, therefore, the spring will be subjected to pure bending as can be seen from Figure 15.2 (b).

$$M = M_b$$

$$\text{Strain Energy, } U = \int_0^l \frac{M^2 ds}{2EI}$$

$$= \frac{M^2 l}{2EI}$$

$$= \frac{M^2 (\pi D n)}{2E\left(\frac{\pi d^4}{64}\right)}$$

$$= \frac{32 M^2 D n}{E d^4} \qquad (15.3)$$

Rotation of this spring,

$$\phi = \frac{\delta U}{\delta M} = \left(\frac{64 M D n}{E d^4}\right) \text{ radians} \qquad (15.4)$$

15.2.2 Strain Energy

When Subjected to Axial Load

Neglecting strain energy due to direct shear W, strain energy stored,

$$U = \frac{f_s^2}{4G} \times (\text{Volume of the spring})$$

where G = modulus of rigidity.

$$\therefore \quad U = \frac{\left(\frac{8WD}{\pi d^3}\right)^2}{4G} \times \left(\frac{\pi d^2}{4}(\pi D n)\right)$$

$$\therefore \quad U = \frac{4W^2D^3n}{Gd^4} \qquad (15.5)$$

15.2.3 Deflection of Springs

Let δ is the axial deflection.

Then, work done by the load = $\frac{1}{2}(W \times \delta)$

Equating the work done to the strain energy stored in the spring,

$$\frac{1}{2}(W\delta) = \frac{4W^2D^3n}{Gd^4} \qquad (15.6)$$

$$\therefore \quad \delta = \frac{8WD^3n}{Gd^4}$$

$$= \frac{64WR^3n}{Gd^4} \qquad (15.6a)$$

15.2.4 Stiffness of Springs

The stiffness of the spring is defined as the load required to produce unit deflection.

From Eq. (15.6a), for unit deflection, $\delta = 1$.

$$\therefore \text{Stiffness of the spring} = \frac{W}{\delta} = \frac{Gd^4}{64R^3n} \qquad (15.7)$$

This is also termed as spring constant k.

15.2.5 Proof Load

Proof load is the maximum load carrying capacity of the spring, without getting permanently distorted.

From Eq. (15.2), we have $\quad f_s = \frac{8WD}{\pi d^3}$

Proof Load, $\quad W_{max} = \frac{\pi d^3}{8D} \times (f_s)_{max}$

$$W_{max} = \frac{\pi d^3}{16R} \times (f_s)_{max} \qquad (15.8)$$

where $(f_s)_{max}$ is the allowable shear stress of the material of the spring.

Hence, using Eq. (15.8), you can determine the proof load of the spring.

Example 15.1

A close coiled helical spring is made of 5 mm diameter wire. It is made up of 30 coils, each of mean diameter 75 mm. If the maximum stress in the spring is not to exceed 200 MPa, then determine

(a) the proof load

(b) the extension of the spring when carrying this load.

Take $G = 80$ GPa.

Solution

Here, we have $\quad d = 5$ mm $\qquad n = 30$

$D = 75$ mm, $\qquad R = 37.5$ mm

$(f_s)_{max} = 200$ MPa $\qquad G = 80$ GPa

Using Eq. (15.2), we get

$$\therefore \quad W = \frac{\pi d^3}{8D} f_s$$

Thus, proof load $= \dfrac{\pi(5)^3}{8 \times 75} \times 200 = 131 \text{ N}$

Deflection $\delta = \dfrac{64\, WR^3 n}{Gd^4}$

$$\delta = \frac{64 \times 131 \times (37.5)^3 \times 30}{(80 \times 10^3)\,(5)^4} = 265.5 \text{ mm}$$

Example 15.2

A helical spring in which the slope of the helix may be assumed small, is required to transmit a maximum pull of 1 kN and to extend 10 mm for 200 N load. If the mean diameter of the coil is to be the 80 mm, find the suitable diameter for the wire and number of coils required. Take $G = 80$ GPa and allowable shear stress as 100 MPa.

Solution

Shear stress, $f_s = \dfrac{8WD}{\pi d^3}$

$$\therefore \quad d^3 = \frac{8WD}{\pi f_s} = \frac{8 \times 1000 \times 80}{\pi \times 100}$$

Here, we have $W = 1000$ N D = 80 mm

$f_s = 100$ MPa

∴ Diameter of spring wire = 12.68 mm.

Now $\delta = 10$ mm for $W = 200$ N.

$$\therefore \text{ Spring constant, } k = \frac{W}{\delta} = \frac{200}{10} = 20 \text{ N/mm}$$

$$= \frac{20}{10^{-3}} = 2 \times 10^4 \text{ N/m}$$

We have, $C = 80$ GPa

$$\delta = \frac{8WD^3 n}{Gd^4}$$

$$\therefore \quad n = \frac{Gd^4}{8\left(\dfrac{W}{\delta}\right)D^3} = \frac{(80 \times 10^3)\,(12.68)^4}{8 \times 20 \times (80)^3}$$

$$= 25.28$$

Number of coils required = 25.28 say 26.

Example 15.3

A truck weighing 30 kN and moving at 6 km/hr has to be brought to rest by a buffer. Find how many springs each of 20 coils will be required to store the energy of motion during a compression of 200 mm. The spring is made out of 20 mm diameter steel rod coiled to a mean diameter of 200 mm. Take $G = 100$ GPa.

Solution

Weight of truck = 30 kN.

Mass of the truck $= \left(\dfrac{30 \times 10^3}{9.81}\right)$ kg

Velocity of the truck = 6 km/hr

$$= \left(\frac{6 \times 10^3}{60 \times 60}\right) \text{ m/s}$$

Kinetic Energy of the truck $= \frac{1}{2} mv^2$

$$= \frac{1}{2}\left(\frac{30 \times 10^3}{9.81}\right)\left(\frac{6000}{60 \times 60}\right)^2$$

$$= 4248 \text{ N m}$$

For the spring, let N = number of springs.

$n = 20$	$\delta = 200$ mm
$d = 20$ mm	$G = 100$ GPa
$D = 200$ mm	$R = 100$ mm

Using the relationship, $\delta = \frac{64\, WR^3 n}{Gd^4}$

$$\therefore \quad W = \frac{200 \times (100 \times 10^3)\,(20)^4}{64 \times (100)^3 \times 20} = 2500 \text{ newton}$$

Strain energy stored in one spring $= \frac{1}{2} W\delta$

$$= \frac{1}{2} \times 2500 \times (200 \times 10^{-3})$$

Strain energy stored in N springs $= \left(\frac{1}{2} \times 2500 \times 200 \times 10^{-3}\right) N$

$$= 250\, N$$

Equating the strain energy stored to kinetic energy of the truck,

$$250\, N = 4248$$

$$\therefore \quad N = \frac{4248}{250} = 16.99$$

$\therefore$ Provide 17 springs.

Example 15.4

A close coiled helical spring of circular section has coils of 75 mm mean diameter. When loaded with an axial load of 250 N, it is found to extend 160 mm and when subjected to a twisting couple of 3 N m, there is an angular rotation of 60° degrees. Determine the Poisson's ratio for the material.

Solution

Here, we have

$D = 75$ mm	$W = 250$ N
$\delta = 160$ mm	$T = 3$ N m

$$\phi = 60^o \Rightarrow \left(60 \times \frac{\pi}{180}\right) \text{radians.}$$

Let the Poisson's ratio be ν.

Using the relationship, $\delta = \frac{8WD^3 n}{Gd^4}$ for an axial load,

$$\therefore \quad G = \frac{8WD^3 n}{\delta d^4}$$

$$= \frac{8 \times 250 \times (75 \times 10^{-3})^3 n}{160 \times 10^{-3} \times d^4 \times 10^{-12}}$$

Therfore, $$G = \frac{12.5 \times 75^3 \times 10^6\, n}{d^4} \text{ N/mm}^2 \qquad \text{(A)}$$

For an axial torque, $\phi = \frac{64TDn}{Ed^4}$

Thus, we get $$E = \frac{64TDn}{\delta^4}$$

$$= \frac{64 \times 3 \times (75 \times 10^{-3})\, n}{\left(60 \times \frac{\pi}{180}\right)(d^4 \times 10^{-12})}$$

Therefore, $$E = \frac{576 \times 75 \times 10^9\, n}{\pi d^4} \qquad \text{(B)}$$

But we know the relationship $E = 2G\,(1 + \nu)$

Subsitituting the values of E and G from (A) and (B),

$$\frac{576 \times 75 \times 10^9\, n}{\pi d^4} = 2 \times \frac{12.5 \times 75^3 \times 10^6\, n}{d^4}\,(1 + \nu)$$

$$\therefore\ (1 + \nu) = \frac{576 \times 75 \times 10^9}{25\pi \times 75^3 \times 10^6} = 1.304$$

or $$\therefore\ \nu = 0.304$$

Example 15.5

Two close coiled helical springs are compressed between two parallel plates by a load of 1 kN. The springs have a wire diameter of 10 mm and the radii of coils are 50 and 75 mm. Each spring has 10 coils and is of the same initial length. If the smaller spring is placed inside the larger one such that both the springs are compressed by same amount, calculate

(a) the total deflection, and

(b) the maximum stress in each spring.

Take $G = 40$ GPa for both the springs.

Solution

Deflections

Here, we denote spring 1 as larger and spring 2 as smaller.

$d_1 = 10$ mm	$d_2 = 10$ mm
$R_1 = 75$ mm	$R_2 = 50$ mm
$n_1 = 10$	$n_2 = 10$
$W = 1000$ N	$G = 40 \times 10^3$ MPa

Let W_1 and W_2 be the load carried by spring 1 and 2 respectively.

Since deflection of both the springs is same, we get

$$\frac{64\, W_1 R_1^3\, n_1}{G d_1^4} = \frac{64\, W_2 R_2^3\, n_2}{G d_2^4}$$

$$\therefore\ \frac{W_1}{W_2} = \left(\frac{R_2}{R_1}\right)^3 \left(\frac{d_1}{d_2}\right)^4 \left(\frac{n_2}{n_1}\right)$$

$$= \left(\frac{50}{75}\right)^3 \left(\frac{10}{10}\right)^4 \left(\frac{10}{10}\right)$$

$$= 0.296$$

$$\therefore\ W_1 = 0.296\, W_2 \qquad \text{(A)}$$

Also, we have $$W_1 + W_2 = 1000 \text{ N} \qquad \text{(B)}$$

From (A) and (B), we get, $W_1 = 228.5$ N, and $W_2 = 771.5$ N.

Thus $$\delta_1 = \delta_2 = \frac{64 \times 228.5 \times 75^3 \times 10}{(40 \times 10^3)\,(10^4)} = \frac{64 \times 771.5 \times 50^3 \times 10}{(40 \times 10^3)\,(10^4)} = 154 \text{ mm}$$

$$(f_s)_{max} \text{ in spring } 1 = \frac{16 W_1 R_1}{\pi d_1^3}$$

$$= \frac{16 \times 228.5 \times 75}{\pi (10)^3} = 87.3 \text{ N/mm}^2$$

$$(f_s)_{max} \text{ in spring } 2 = \frac{16 W_2 R_2}{\pi (d_2)^3}$$

$$= \frac{16 \times 771.5 \times 50}{\pi (10)^3} = 196.46 \text{ N/mm}^2$$

SAQ 1

A close coiled helical spring made out of 8 mm diameter wire has 18 coils. Each coil has 80 mm mean dia. If the maximum allowable stress on the spring is 140 MPa, determine the maximum allowable load on the spring, the elongation of the spring and the total strain energy stored in the spring at that load. Also determine the stiffness of the spring. Take G = 82 GPa.

SAQ 2

A close coiled helical spring has to absorb 70 N m of energy when compressed to 58 mm. The coil diameter is ten times the wire diameter. If there are 12 coils, estimate the diameters of coil and wire and the maximum shear stress.
Take $G = 87000$ N/mm^2.

SAQ 3

A close coiled helical spring is to be made of 5 mm diameter wire for which E = 200 GPa and G = 80 GPa. It is required to extend 28 mm for an axial load of 100 N and to twist 0.22 radian for an axial couple of 1 N m. Find the mean diameter of the coils and the number of coils required.

SAQ 4

A close coiled helical spring of circular section extends 1 m when subjected to an axial load of W newtons, and there is an angular rotation of one radian when a torque of T newton metre is independently applied about the axis of the spring. If Poisson's ratio is ν and the mean diameter of the coil is D, show that

$$\frac{T}{W} = \frac{D^2 (1 + \nu)}{4}$$