Defn:

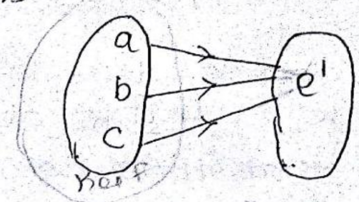
Morphism of groups:

Let $(G, *)$ and (H, Δ) be any two groups. A mapping $f: G \to H$ is said to be a homomorphism, if $f(a*b) = f(a) \Delta f(b)$ for any $a, b \in G$.

Kernel of a Homomorphism:

Let $f: G \to G'$ be a group homomorphism. The set of elts. of G which are mapped into e' (identity in G') is called the kernel of f and it is denoted by $\text{Ker}(f)$

$\text{Ker}(f) = \{x \in G / f(x) = e'\}$



Isomorphism:

A mapping f from a group $(G, *)$ to a group (G', Δ) is said to be an isomorphism if

i). f is a homomorphism

ii). f is 1–1 (injective)

iii). f is onto (surjective)

In otherwords, a bijective homomorphism is said to be an isomorphism.

Cosets:

Let H be a subgroup of G.

i). for any $a \in G$, the left coset of H denoted by $a * H = \{a * h, h \in H\}$, $\forall a \in G$

ii). The right coset of H is denoted by $H * a = \{h * a, h \in H\}$, $\forall a \in G$.

Problem:

1). Let $G = \{1, a, a^2, a^3\}$ $(a^4 = 1)$ be a group and $H = \{1, a^2\}$ is a subgroup of G under multiplication find the right cosets of H

Soln.

The right cosets of H in G,

$H * 1 = \{1, a^2\} = H$

$H * a = \{a, a^3\}$

$H * a^2 = \{a^2, a^4\} = \{a^2, 1\} = H$

$H * a^3 = \{a^3, a^5\} = \{a^3, a\} = H * a$

$\Rightarrow$ H and $H * a$ are two distinct right cosets of H in G

Here $G = \{1, a, a^2, a^3\}$ and $H = \{1, a^2\}$

$O(G) = 4$ and $O(H) = 2$

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$$I_G(H) = \frac{O(G)}{O(H)} = \frac{4}{2} = 2$$

Theorem:

Any two right (or left) cosets of H in G are either disjoint or identical

Proof:

Let $H*a$ and $H*b$ be two right cosets of a subgroup H of G.

Let $a, b \in G$.

We've to prove that either

$(H*a) \cap (H*b) = \phi$

or $H*a = H*b$

Suppose $(H*a) \cap (H*b) \neq \phi$.

Then $\exists$ an elt. $x \in (H*a) \cap (H*b)$

$\Rightarrow x \in H*a$ and $x \in H*b$.

Now, $x \in H*a$ (By previous theorem) $H*x = H*a \rightarrow (1)$

and $x \in H*b$

$\Rightarrow H*x = H*b$ (By previous theorem) $\rightarrow (2)$

From (1) and (2), $H*x = H*a = H*b$

$\therefore H*a = H*b$

$\therefore$ Either $(H*a) \cap (H*b) = \phi$ or $H*a = H*b$