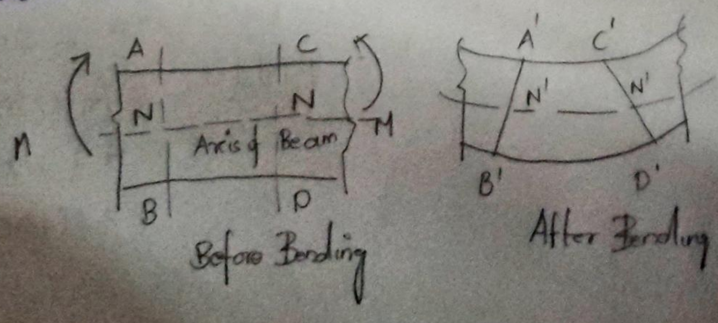
Pure Bending or Simple Bending

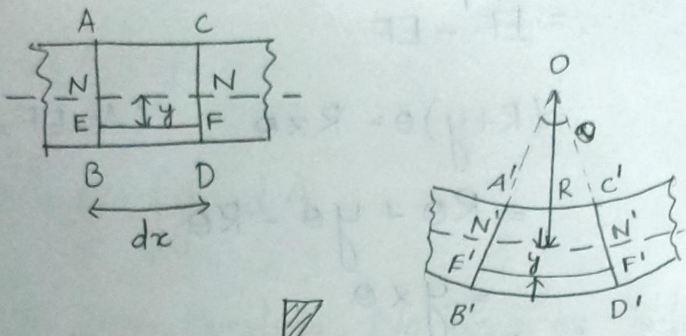
If a Length of a Beam is Subjected to Constant Bending Moment and No Shear force then the stresses will be Setup in that Length of Beam due to B.M only & the Length is said to be Pure Bending or Simple Bending. And the stresses is called Bending stress.

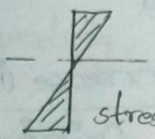
Assumptions made in the theory of Simple Bending

1) The Material of Beam is Homogenous & Isotropic
2) the Value of Young's Modulus of Elasticity is the same in Tension & Compression
3) The Transverse Section which were plane before Bending remains Plane after Bending
4) the Radius of Curvature is Large compared with the Dimensions of Cross-Section.
5) Each Layer of the Beam is free to Expand or Contract, Independently of the Layer above or Below it.



Expression for Bending stress.





stress Diagram

Small Length δx of beam subjected to Simple Bending.

Let A'B' and C'D' meet at O.

$R \rightarrow$ Radius of Neutral Layer N'N'

$\theta \rightarrow$ Angle at O by A'B' & C'D'.

Consider EF at Distance y below Neutral Layer, After Bending it is E'F'.

$$EF = \delta x$$

$$NN = \delta x$$

After Bending the Length of Neutral Layer remains Unchanged. But E'F' will increase

$$N'N' = NN = \delta x$$

$$N'N' = R \times \theta$$

$$E'F' = (R+y) \times \theta$$

$$\delta x = R \times \theta \quad (\text{as } N'N' = \delta x)$$

Increase in the Length of Layer EF

$= E'F' - EF$

$= (R+y)\theta - R\times\theta$ (As $EF = R\times\theta$)

$= R\theta + y\theta - R\theta$

$= y\times\theta$

Strain in EF $= \dfrac{\text{Increase in Length}}{\text{Original Length}}$

$= \dfrac{y\times\theta}{EF} = \dfrac{y\times\theta}{R\times\theta}$

$= \dfrac{y}{R}$

σ = Stress in Layer EF

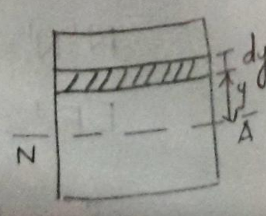
E = Youngs Modulus of Beam

$E = \dfrac{\text{Stress in EF}}{\text{Strain in EF}}$

$= \dfrac{\sigma}{(y/R)}$

$\sigma = E\times\dfrac{y}{R}$

$\dfrac{\sigma}{y} = \dfrac{E}{R}$



F = Stress on Layer $\times$ Area on Layer

$= \sigma \times dA$

$$= \frac{E}{R} \times y \times dA$$

$$= \int \frac{E}{R} y \cdot dA$$

$$= \frac{E}{R} \int y \cdot dA$$

In Pure Bending, No Force on Section of Beam.

$$\frac{E}{R} \int y \cdot dA = 0$$

$$\int y \cdot dA = 0$$

$$\text{Force} = \frac{E}{R} \times y \cdot dA$$

Moment about NA (Neutral Axis)

$$= \text{Force} \times y$$

$$= \frac{E}{R} \times y \cdot dA \times y$$

$$= \frac{E}{R} \cdot y^2 \cdot dA$$

$$= \frac{E}{R} \int y^2 \, dA$$

$$M = \frac{E}{R} \int y^2 \cdot dA \qquad I = \int y^2 \cdot dA$$

$$\therefore M = \frac{E}{R} \times I$$

$$\therefore \frac{M}{I} = \frac{E}{R}$$

From Above Equations

$$\boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}}$$

This is known as Bending Equation

M → Moment

R → Radius of Curvature

I → Moment of Inertia

σ → Stress

y → Distance

E → Youngs Modulus

1) A Steel Plate of Width 120 mm & of thickness 20mm is bent into a circular arc of radius 10m. Determine the Maximum stress Induced & the Bending Moment which will produce the Max. stress. $E = 2 \times 10^5$ N/mm²

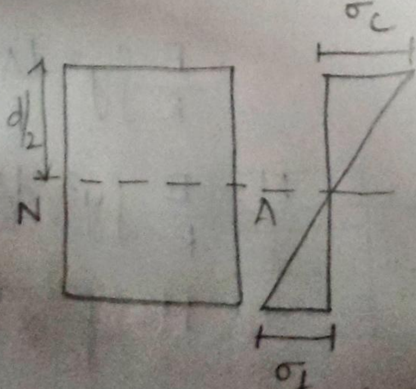
$b = 120$ mm

$t = 20$ mm

$$I = \frac{bt^2}{12} = \frac{120 \times 20^3}{12} = 8 \times 10^4 \text{ mm}^4$$

$R = 10\text{m} = 10 \times 10^3$ mm

$E = 2 \times 10^5$ N/mm²



$$\frac{\sigma}{y} = \frac{E}{R}$$

$$\sigma = \frac{E}{R} \cdot y$$

$$y_{max} = \frac{t}{2} = \frac{20}{2} = 10 \text{ mm}$$

$$\sigma_{max} = \frac{E}{R} \cdot y_{max}$$

$$= \frac{2\times10^5}{10\times10^3} \times 10 = 200 \text{ N/mm}^2$$

$$\frac{M}{I} = \frac{E}{R}$$

$$M = \frac{E}{R} \times I = \frac{2\times10^5}{10\times10^3} \times 8\times10^4$$

$$= 1.6 \text{ kNm}$$

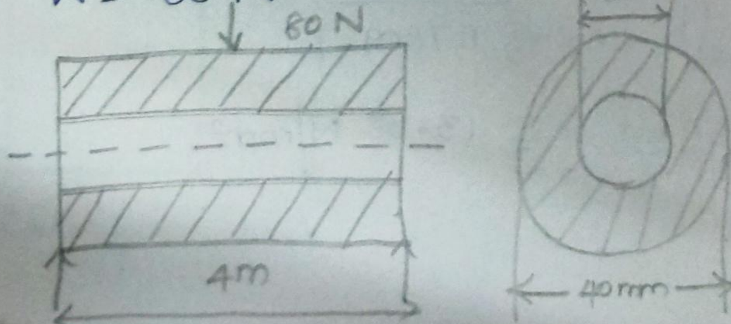
2) Calculate the Maximum stress Induced in a Cast Iron Pipe of External Diameter of 40 mm of Internal Dia 20 mm & Length 4 m when the Pipe is Supported at it ends carries a point Load of 80 N at its Centre.

$D = 40$ mm

$d = 20$ mm

$L = 4$ m $= 4000$ mm

$W = 80$ N



$$B.M = \frac{WL}{4}$$

$$= \frac{80 \times 4000}{4} = 8 \times 10^4 \ Nmm$$

$$M = 8 \times 10^4 \ Nmm$$

$$I = \frac{\pi}{64} [D^4 - d^4]$$

$$= \frac{\pi}{64} [40^4 - 20^4]$$

$$= \frac{\pi}{64} [2560000 - 160000]$$

$$= 117809.7 \ mm^4$$

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$y_{max} = \frac{D}{2} = \frac{40}{2} = 20 \ mm.$$

$$\frac{M}{I} = \frac{\sigma_{max}}{y_{max}}$$

$$\sigma_{max} = \frac{M \times y_{max}}{I}$$

$$= \frac{8 \times 10^4 \times 20}{117809.7}$$

$$= 13.58 \ N/mm^2.$$

Section Modulus:

Section Modulus is defined as the ratio of Moment of Inertia of a Section about the Neutral Axis to the distance of Outermost Layer of (from) the Neutral Axis. It is denoted by Z.

$$Z = \frac{I}{y_{max}}$$

$$\frac{M}{I} = \frac{\sigma}{y}$$

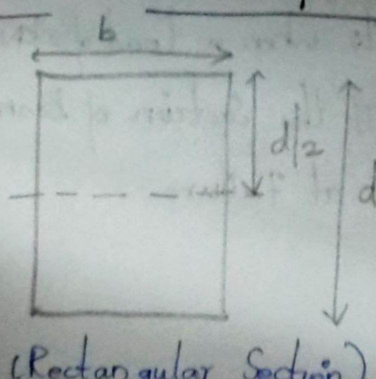
$$\frac{M}{I} = \frac{\sigma_{max}}{y_{max}}$$

$$M = \sigma_{max} \cdot \frac{I}{y_{max}}$$

$$\frac{I}{y_{max}} = Z$$

$$\boxed{M = \sigma_{max} \cdot Z}$$

Section Modulus for Various shapes



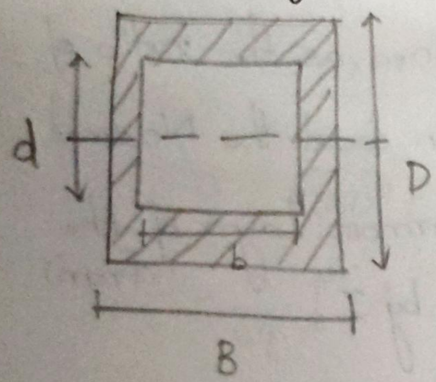
(Rectangular Section)

$$I = \frac{bd^3}{12}$$

$$y_{max} = d/2$$

$$Z = \frac{bd^2}{6}$$

Hollow Rectangular Section



$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$= \frac{1}{12}\left[BD^3 - bd^3\right]$$

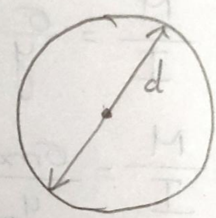
$$y_{max} = \frac{D}{2}$$

$$Z = \frac{1}{6D}\left[BD^3 - bd^3\right]$$

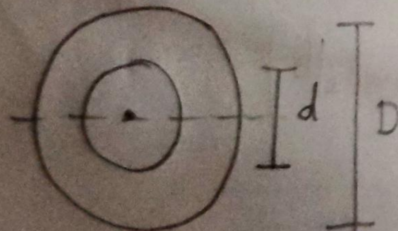
Circular Section

$$I = \frac{\pi}{64} d^4 \quad y_{max} = d/2.$$

$$Z = \frac{\pi}{32} d^3$$



Hollow Circular Section



$$I = \frac{\pi}{64}(D^4 - d^4)$$

$$y_{max} = D/2$$

$$Z = \frac{\pi}{32D}\left(D^4 - d^4\right]$$

1) A Cantilever of Length 2m fails when a Load of 2kN is applied at the free End. If the Section of Beam is 40×60 mm. Find the stress at Failure.

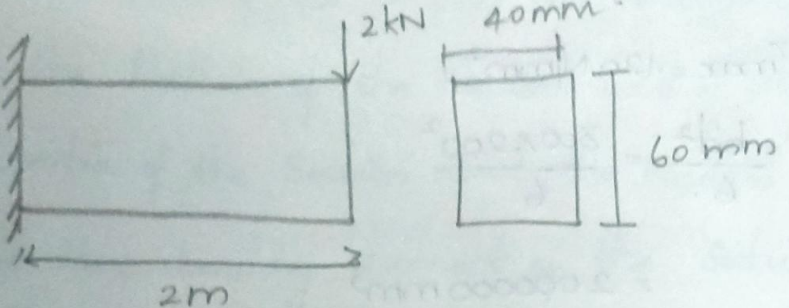
$L = 2m = 2\times10^3 mm$

$W = 2\,kN = 2000N$

Section of Beam 40mm x 60 mm

b = 40 mm

d = 60 mm.



$$Z = \frac{bd^2}{6} = \frac{40 \times 60^2}{6} = 24000 \text{ mm}^3.$$

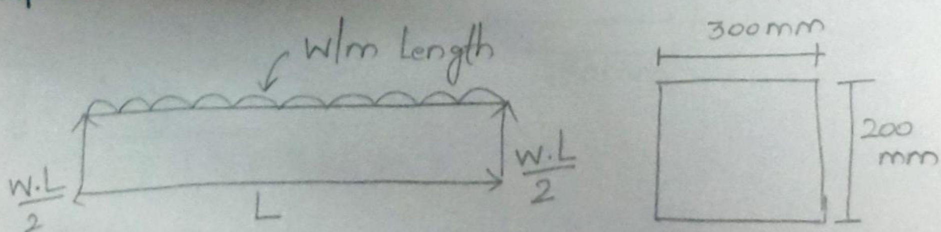
$$M = WL = 2000 \times 2 \times 10^3$$

$$= 4 \times 10^6 \text{ Nmm}.$$

$$M = \sigma_{max} . Z$$

$$\sigma_{max} = \frac{M}{Z} = \frac{4 \times 10^6}{24000} = 166.67 \text{ N/mm}^2$$

2) A Rectangular Beam 200 mm deep & 300mm Wide is Simply Supported over a Span of 8m. What uniformly distributed Load per metre the beam may Carry, if the bending stress is not to exceed 120 N/mm².



$d = 200\ mm$

$b = 300\ mm$

$L = 8\ m$

$\sigma_{max} = 120\ N/mm^2$

$$Z = \frac{bd^2}{6} = \frac{300 \times 200^2}{6}$$

$$= 2000000\ mm^3$$

$$M = \frac{w \times L^2}{8}$$

$$= 8w \cdot Nm$$

$$= 8w \times 1000\ Nmm.$$

$$= 8000\ w\ Nmm.$$

$$M = \sigma_{max} \cdot Z$$

$$8000 w = 120 \times 2000000$$

$$w = \frac{120 \times 2000000}{8000}.$$

$$= 30 \times 1000\ N/m$$

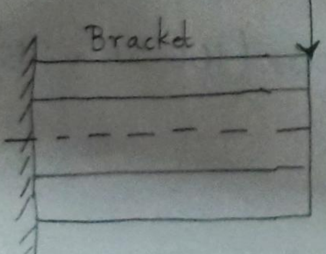
$$= 30\ kN/m.$$

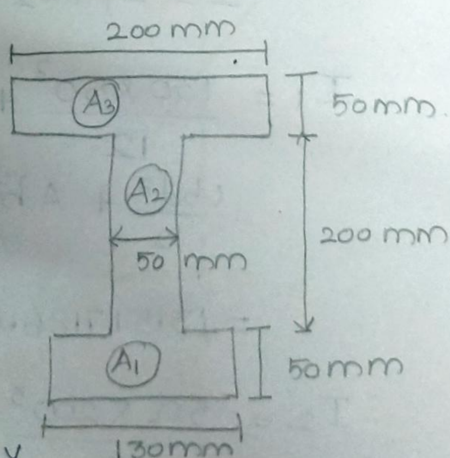
3) A Cast Iron Bracket Subject to bending has the Cross-Section of I-form with Unequal flanges. The Dimensions of the Section are shown in fig. Find the Position of the Neutral axis & Moment of Inertia of the Section about the Neutral Axis. If the Max. bending moment of the Section is 40 MN mm. Determine the Maximum Bending stress. What is Nature of Stress.

Max. B.M, $M = 40\ MN\ mm$

$= 40 \times 10^6\ N.mm.$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{(A_1 + A_2 + A_3)}$$





$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{(A_1 + A_2 + A_3)}$$

$A_1 = 130 \times 50 = 6500\ mm^2$

$y_1 = \frac{50}{2} = 25\ mm$

$A_2 = 200 \times 50 = 10000\ mm^2$

$y_2 = 50 + \frac{200}{2} = 50 + 100 = 150\ mm$

$A_3 = 200 \times 50 = 10000\ mm^2$

$y_3 = 50 + 200 + \frac{50}{2} = 275\ mm$

$$\bar{y} = \frac{(6500 \times 25) + (10000 \times 150) + (10000 \times 275)}{6500 + 10000 + 10000}$$

$$= \frac{4412500}{26500} = 166.51\ mm.$$

Neutral Axis is at Distance of 166.51 mm from Bottom face.

Moment of Inertia of Section about NA

$I = I_1 + I_2 + I_3.$

$$I_1 = \frac{130 \times 50^3}{12} + 6500 \times (166.25 - 25)^2$$

$$\left(\frac{bd^3}{12} + A h^2\right)$$

$$= 131517186.6\ mm^4.$$

$$I_2 = \frac{50 \times 200^3}{12} + 10000(166.51 - 150)^2$$

$$= 33605913.43\ mm^4$$

$$I_3 = \frac{200 \times 50^3}{12} + 10000(275 - 166.51)^2$$

$$= 119784134.3\ mm^4$$

$$I = I_1 + I_2 + I_3$$
$$= 284907234.9 \text{ mm}^4$$

Distance of CG from Upper Top fibre

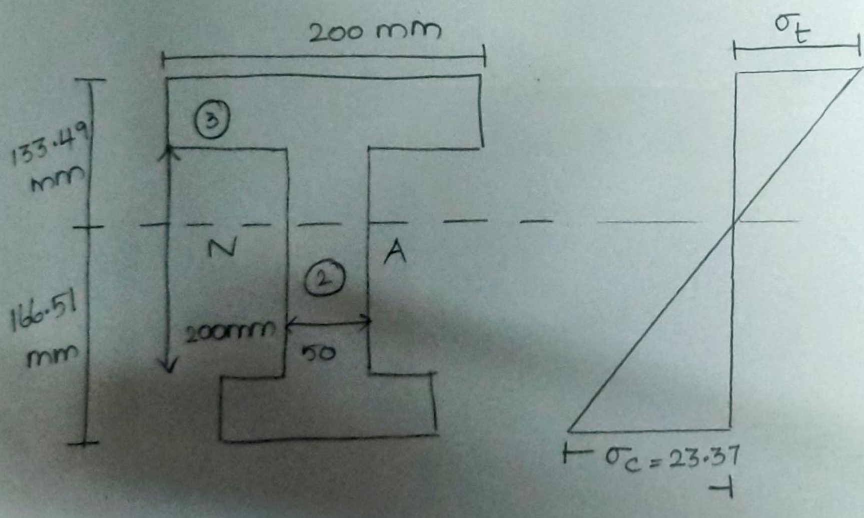
$$= 300 - \bar{y} = 300 - 166.51 = 133.49 \text{ mm.}$$

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma = \frac{My}{I} = \frac{40 \times 10^6}{284907234.9} \times 166.51$$

$$= 23.37 \text{ N/mm}^2$$

Max. Bending stress = 23.37 N/mm²



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