Type - iv $f_1(x, p) = f_2(y, q)$

For this type, there is no singular integral

1. Solve $q^2 - p = y - x$

$$q^2 - y = p - x = k$$

$q^2 - y = k$ | $p - x = k$

$q^2 = k + y$ | $p = x + k$

$q = \sqrt{k+y}$

$$z = \int p\,dx + \int q\,dy$$

$$z = \int (x+k)\,dx + \int \sqrt{k+y}\,dy$$

$$= \frac{x^2}{2} + kx + \frac{(k+y)^{3/2}}{\frac{3}{2}} + c$$

$$= kx + \frac{x^2}{2} + \frac{2}{3}(k+y)^{3/2} + c$$

which is the complete integral.

2. Solve $\sqrt{p} + \sqrt{q} = x + y$

$$\sqrt{p} - x = y - \sqrt{q} = k.$$

$\sqrt{p} - x = k.$ | $y - \sqrt{q} = k$

$\sqrt{p} = k + x$ | $\sqrt{q} = y - k$

$p = (k+x)^2$ | $q = (y-k)^2$

$$z = \int p\,dx + \int q\,dy$$

$$z = \int (k+x)^2\,dx + \int (y-k)^2\,dy$$

$$z = \frac{(k+x)^3}{3} + \frac{(y-k)^3}{3} + c.$$ which is the complete integral.

3. Find the complete integral of $xp - yq = y^2 - x^2$

$$xp + x^2 = y^2 + yq = k$$

$xp + x^2 = k$	$y^2 + yq = k$
$xp = k - x^2$	$yq = k - y^2$
$p = \frac{k - x^2}{x}$	$q = \frac{k - y^2}{y}$
$p = \frac{k}{x} - x$	$q = \frac{k}{y} - y$

$$z = \int p\,dx + \int q\,dy$$

$$= \int\left(\frac{k}{x} - x\right)dx + \int\left(\frac{k}{y} - y\right)dy$$

$$= k\log x - \frac{x^2}{2} + k\log y - \frac{y^2}{2} + c.$$

$$z = k\log xy - \left(\frac{x^2+y^2}{2}\right) + c.$$ which is the complete integral