Type - IV RHS $= e^{ax+by} x^m y^n$ (or) $e^{ax+by} \cos(ax+by)$

$e^{ax+by} \sin(ax+by)$

$$P.I = \frac{1}{\phi(D, D')} e^{ax+by} x^m . y^n$$

Replace $D \to D+a$; $D' \to D+b$.

1. Solve: $(D^2 - 2DD' + D'^2)z = x^2 y^2 e^{x+y}$

The A.E is $m^2 - 2m + 1 = 0$

$(m-1)(m-1) = 0 \Rightarrow m = 1, 1$

The roots are equal.

$CF = f_1(y+x) + x f_2(y+x)$

$$P.I = \frac{1}{D^2 - 2DD' + D'^2} x^2 y^2 e^{x+y} \qquad D \to D+1,\ D' \to D'+1$$

$$= \frac{1}{(D+1)^2 - 2(D+1)(D'+1) + (D'+1)^2} x^2 y^2 e^{x+y}$$

$$= \frac{1}{D^2 + \cancel{2D} + \cancel{1} - 2DD' - \cancel{2D} - \cancel{2D'} - \cancel{2} + D'^2 + \cancel{2D'} + \cancel{1}} x^2 y^2 e^{x+y}$$

$$= \frac{1}{D^2 - 2DD' + D'^2} x^2 y^2 e^{x+y}$$

$$= e^{x+y} \frac{1}{D^2} \frac{1}{1 - \left(\frac{2D'}{D} - \frac{D'^2}{D^2}\right)} x^2 y^2$$

$$= e^{x+y} \frac{1}{D^2} \left[1 - \left(\frac{2D'}{D} - \frac{D'^2}{D^2}\right)\right]^{-1} x^2 y^2$$

$$= e^{x+y} \frac{1}{D^2}\left[1+\left(\frac{2D'}{D}-\frac{D'^2}{D^2}\right)\right]x^2y^2$$

$$= e^{x+y} \frac{1}{D^2}\left[x^2y^2 + \frac{2D'}{D}(x^2y^2) - \frac{D'^2}{D^2}(x^2y^2)\right]$$

$$= e^{x+y}\left[\frac{1}{D^2}(x^2y^2) + \frac{2D'}{D^3}(x^2y^2) - \frac{D'^2}{D^4}(x^2y^2)\right]$$

$$= e^{x+y}\left[\frac{x^4}{12}y^2 + \frac{2}{D^3}(x^2 2y) - \frac{1}{D^4}(2x^2)\right]$$

$$= e^{x+y}\left[\frac{x^4y^2}{12} + 4y\frac{x^5}{60} - 2\frac{x^6}{360}\right]$$

$$= e^{x+y}\left[\frac{x^4y^2}{12} + \frac{x^5y}{15} - \frac{x^6}{180}\right]$$

$$\therefore Z = C.F + P.I = f_1(y+x) + x f_2(y+x) + e^{x+y}\left[\frac{x^4y^2}{12} + \frac{x^5y}{15} - \frac{x^6}{180}\right]$$

5. Solve $r+s-6t = y\cos x$.

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y\cos x.$$

(ie) $(D^2 + DD' - 6D'^2) z = y\cos x$.

The A.E is $m^2 + m - 6 = 0$.

$m_1 = -3, m_2 = 2$.

$\therefore$ C.F $= f_1(y-3x) + f_2(y+2x)$

$$P.I = \frac{1}{D^2 + DD' - 6D'^2} y\cos x$$

$$= \frac{1}{(D-2D')(D+3D')} y\cos x.$$

$$= \frac{1}{D-2D'} \int (c+3x)\cos x \, dx, \quad y = c+3x, \quad D \to x, \; D' \to x$$

$u = c+3x \qquad v = \cos x$

$u' = 3 \qquad v_1 = \sin x, \; v_2 = -\cos x$

$$= \frac{1}{D-2D'}\left[(c+3x)(\sin x) - 3(-\cos x)\right]$$

$$= \frac{1}{D-2D'}\left[(c+3x)\sin x + 3\cos x\right]$$

$$= \frac{1}{D-2D'}\left[y \sin x + 3\cos x\right]$$

$D \to c$

$D' \to x$

$$= \int \left[(c_1 - 2x)\sin x + 3\cos x\right] dx$$

$u = c_1 - 2x \quad V = \sin x$

$u' = -2 \quad V_1 = -\cos x$

$V_2 = -\sin x$

$$= \left[(c_1 - 2x)(-\cos x) - 2\sin x + 3\sin x\right]$$

$$= -y\cos x + 2\sin x + 3\cos x$$

$$P.I = -y\cos x + \sin x$$

$$\therefore z = C.F + P.I = f_1(y-3x) + f_2(y+2x) + \sin x - y\cos x$$