3. Find the F.T of the function $f(x) = \begin{cases} 1-x^2, & |x|<1 \\ 0, & |x|>1 \end{cases}$ &

hence deduce that $\int_0^\infty \frac{\sin s - s\cos s}{s^3} \cos\frac{s}{2}\, ds = \frac{3\pi}{16}$.

$$F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx}\, dx.$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} (1-x^2)\, e^{isx}\, dx.$$

$u = 1-x^2$	$V = e^{isx}$
$u' = -2x$	$V_1 = \frac{e^{isx}}{is}$
$u'' = -2$	$V_2 = \frac{e^{isx}}{(is)^2}$
$u''' = 0$	$V_3 = \frac{e^{isx}}{(is)^3}$

$$= \frac{1}{\sqrt{2\pi}} \left[(1-x^2)\frac{e^{isx}}{is} + 2x\frac{e^{isx}}{(is)^2} - 2\frac{e^{isx}}{(is)^3} \right]_{-1}^{1}$$

$$= \frac{1}{\sqrt{2\pi}} \left[0 + \frac{2e^{is}}{-s^2} - \frac{2e^{is}}{-is^3} \right] - \left[0 - \frac{2e^{-is}}{-s^2} - \frac{2e^{-is}}{-is^3} \right]$$

$$= \frac{1}{\sqrt{2\pi}}\, 2 \left[-\left(\frac{e^{is}}{s^2} \mp \frac{e^{is}}{is^3} \right) \mp \left(\frac{e^{-is}}{s^2} + \frac{e^{-is}}{is^3} \right) \right]$$

$$= \sqrt{\frac{2}{\pi}}\left[-\left(\frac{e^{is}}{s^2}+\frac{e^{-is}}{s^2}\right)+\frac{e^{is}}{is^3}-\frac{e^{-is}}{is^3}\right]$$

$$= \sqrt{\frac{2}{\pi}}\left[-\left(\frac{\cos s + i\sin s + \cos s - i\sin s}{s^2}\right)+\left(\frac{\cos s + i\sin s - \cos s + i\sin s}{is^3}\right)\right]$$

$$= \sqrt{\frac{2}{\pi}}\left[\frac{-2\cos s}{s^2}+\frac{2i\sin s}{is^3}\right]$$

$$= \frac{2\sqrt{2}}{\sqrt{\pi}}\left[\frac{\sin s - s\cos s}{s^3}\right]$$

Inverse Fourier transform:

$$f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(s)\, e^{-isx}\, ds$$

$$1-x^2 = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \frac{2\sqrt{2}}{\sqrt{\pi}}\left[\frac{\sin s - s\cos s}{s^3}\right][\cos sx - i\sin sx]\, ds.$$

$$1-x^2 = \frac{2}{\pi}\int_{-\infty}^{\infty} \underbrace{\frac{(\sin s - s\cos s)\cos sx}{s^3}}_{E} ds - i\int_{-\infty}^{\infty}\underbrace{\frac{\sin s - s\cos s}{s^3}\sin sx}_{0}\, ds$$

$$1-x^2 = \frac{2}{\pi}\, 2\int_0^{\infty} \frac{\sin s - s\cos s}{s^3}\cos sx\, ds$$

Put $x = \frac{1}{2}$

$$1-\frac{1}{4} = \frac{4}{\pi}\int_0^{\infty} \frac{\sin s - s\cos s}{s^3}\cos\frac{s}{2}\, ds$$

$$\frac{3}{4}\times\frac{\pi}{4} = \int_0^{\infty}\frac{\sin s - s\cos s}{s^3}\cos\frac{s}{2}\, ds$$

$$\int_0^{\infty}\frac{\sin s - s\cos s}{s^3}\cos\frac{s}{2}\, ds = \frac{3\pi}{16}.$$