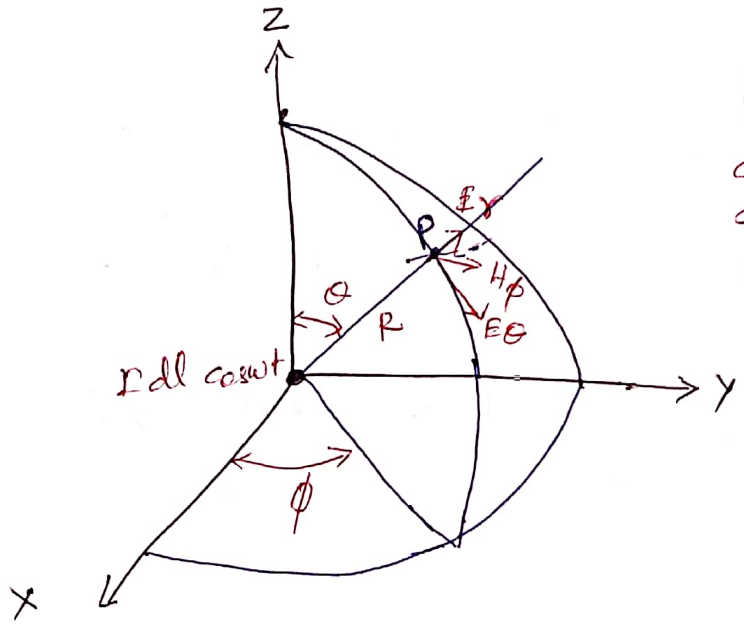
Radiation from a current Element

→ A current element Idl refers to a filamentary current I flowing along an elemental length dl.

→ An isolated current element is an unreal concept, but any physical antenna carrying current is considered to consists of large number of such elements joined end to end.

→ Therefore, if the EM field of this building block is known, the EM field of any actual antenna having a specified current distribution can be calculated.



(Fig) current element at the centre of a spherical Co-ordinate system.

Vector potential Determination

The first step is to obtain Vector potential A at Point 'p'.

$$A(r) = \frac{\mu}{4\pi} \int \frac{J(t-r/v)}{R} dv' \longrightarrow ①$$

$$\int J\,dv = I\,dl$$

$\therefore$ Eqn (1) becomes

$$A_z = \frac{\mu}{4\pi} \frac{I\,dl \cos \omega (t - \frac{r}{v})}{r} \rightarrow (2)$$

The vector potential has the same direction as the current element. In this case it is z direction and is retarded in time by r/v seconds.

__Magnetic Field strength__

$$\mu H = \nabla \times A \quad (\text{or})$$

$$H = \frac{1}{\mu}(\nabla \times A) \longrightarrow (3)$$

Transformation between rectangular to spherical co-ordinates.

$$\boxed{A_r = A_z \cos\theta, \quad A_\theta = -A_z \sin\theta \ \& \ A_\phi = 0} \ \& \ \partial/\partial\phi = 0$$

$$\therefore A_r = \frac{\mu}{4\pi} \frac{I\,dl \cos \omega (t - r/v)}{r} \cos\theta$$

$$A_\theta = -\frac{\mu}{4\pi} \frac{I\,dl \cos \omega (t - r/v)}{r} \sin\theta$$

$$A_\phi = 0$$

From eq (3)

$$H_r \vec{a_r} + H_\theta \vec{a_\theta} + H_\phi \vec{a_\phi} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} a_r & a_\theta r & a_\phi r\sin\theta \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix}$$

$$H_r \vec{a_r} + H_\theta \vec{a_\theta} + H_\phi \vec{a_\phi} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} a_r & a_\theta r & a_\phi r\sin\theta \\ \partial/\partial r & \partial/\partial\theta & 0 \\ A_r & rA_\theta & 0 \end{vmatrix}$$

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Electric Field strength

From Maxwell's eqn. $\nabla \times H = \varepsilon \dfrac{\partial E}{\partial t}$

$$E = \frac{1}{\varepsilon} \int \nabla \times H \, dt \longrightarrow (5)$$

Taking curl of eq (4) & integrating with respect to time, Electric field is obtained.

$$\cancel{E = \frac{1}{\varepsilon} \int} \qquad \nabla \times H = \frac{1}{r^2 \sin\theta} \begin{vmatrix} a_r & a_\theta r & a_\phi r\sin\theta \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ H_r & r H_\theta & r\sin\theta H_\phi \end{vmatrix}$$

$H_r \to 0$, $r H_\theta \to 0$

$$\nabla \times H = \frac{1}{r^2 \sin\theta} \begin{vmatrix} a_r & a_\theta r & a_\phi r\sin\theta \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ 0 & 0 & r\sin\theta H_\phi \end{vmatrix}$$

$$\therefore E_r = \frac{1}{\varepsilon} \int \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial\theta} \left(r\sin\theta \, H_\phi \right) dt$$

$$= \frac{1}{\varepsilon} \int \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial\theta} \left[\frac{I\,dl \sin^2\theta}{4\pi} \left(\frac{-\omega \sin\omega(t - r/v)}{rv} + \frac{\cos\omega(t - r/v)}{r^2} \right) \right] dt$$

$$= \frac{1}{\varepsilon} \int \frac{1}{r^2 \cancel{\sin\theta}} \left[\frac{I\,dl\, \cancel{2\sin\theta}\cos\theta}{\cancel{4\pi}\,2} \left(\frac{-\omega \sin\omega(t - r/v)}{rv} + \frac{\cos\omega(t - r/v)}{r^2} \right) \right] dt$$

$$= \frac{1}{\varepsilon r^2} \left[\frac{I\,dl \cos\theta}{2\pi} \cdot \left(- \frac{\cancel{\omega}(-\cos\omega(t - r/v))}{rv \times \cancel{\omega}} + \frac{\sin\omega(t - r/v)}{r^2 \omega} \right) \right]$$

$$= \frac{I\,dl \cos\theta}{2\pi\varepsilon} \left(\frac{\cos\omega t'}{r^2 v} + \frac{\sin\omega t'}{\omega r^3} \right)$$

Equating the coefficients of $\vec{a_r}$, $\vec{a_\theta}$ & $\vec{a_\phi}$ both sides

$a_r \quad \mu H_r = (\nabla \times H)_r = 0 \Rightarrow \boxed{H_r = 0}$

$a_\theta \quad \mu H_\theta = (\nabla \times H)_\theta = 0 \Rightarrow \boxed{H_\theta = 0}$

$a_\phi \quad \boxed{\mu H_\phi = (\nabla \times H)_\phi}$

$$\mu H_\phi = \frac{1}{r^{\cancel{2}} \cancel{\sin\theta}} \times \cancel{r} \cancel{\sin\theta} \left[\frac{\partial}{\partial r}(r A_\theta) - \frac{\partial A_r}{\partial \theta}\right]$$

$$H_\phi = \frac{1}{\mu r}\left[\frac{\partial}{\partial r}(r A_\theta) - \frac{\partial}{\partial \theta}(A_r)\right]$$

$$H_\phi = \frac{1}{\cancel{\mu} r}\left[\frac{\partial}{\partial r}\left(\cancel{r} \times \left(-\frac{\cancel{\mu}}{4\pi} \frac{I\,dl \cos\omega(t - r/v) \sin\theta}{\cancel{r}}\right)\right) - \frac{\partial}{\partial \theta}\left(\frac{\cancel{\mu}}{4\pi} \frac{I\,dl \cos\omega(t - r/v) \cos\theta}{r}\right)\right]$$

$$= \frac{1}{\cancel{\mu} r}\left[\frac{\partial}{\partial r}\left(\frac{I\,dl \cos\omega(t-r/v) \sin\theta}{4\pi}\right) - \frac{\partial}{\partial \theta}\left(\frac{I\,dl \cos\omega(t-r/v) \cos\theta}{4\pi r}\right)\right]$$

$$= \frac{1}{r}\left[\frac{I\,dl}{4\pi} \sin\omega(t - r/v) \times \left(-\frac{\omega}{v}\right) \sin\theta - \frac{I\,dl}{4\pi r} \cos\omega(t - r/v) . [-\sin\theta]\right]$$

$$\boxed{H_\phi = \frac{I\,dl \sin\theta}{4\pi}\left[\frac{-\omega \sin\omega(t - r/v)}{r v} + \frac{\cos\omega(t - r/v)}{r^2}\right]} \rightarrow (4)$$