SNS COLLEGE OF TECHNOLOGY

Coimbatore-35

(An Autonomous Institution)

Accredited by NBA – AICTE and Accredited by NAAC – UGC with ‘A+’ Grade

Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF MECHANICAL ENGINEERING

16ME401 FINITE ELEMENT ANALYSIS

IV YEAR VII SEM

UNIT III TWO DIMENSIONAL PROBLEMS

TOPIC – Constant Strain Triangular element -Formula

SNS *Design Thinkers*

Dr.M.SUBRAMANIAN, Professor& Mechanical

Formulae used

For constant strain triangle (CST) element,

Shape function, $N_1 + N_2 + N_3 = 1$

$$\text{Co-ordinate, } x = N_1 x_1 + N_2 x_2 + N_3 x_3$$

$$\text{Co-ordinate, } y = N_1 y_1 + N_2 y_2 + N_3 y_3$$

or

$$\text{Co-ordinate, } x = (x_1 - x_3) N_1 + (x_2 - x_3) N_2 + x_3$$

$$\text{Co-ordinate, } y = (y_1 - y_3) N_1 + (y_2 - y_3) N_2 + y_3$$

Area of the triangular element, $A = \frac{1}{2}\begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}$

Strain–Displacement matrix for CST element is,

$$[B] = \frac{1}{2A}\begin{bmatrix} q_1 & 0 & q_2 & 0 & q_3 & 0 \\ 0 & r_1 & 0 & r_2 & 0 & r_3 \\ r_1 & q_1 & r_2 & q_2 & r_3 & q_3 \end{bmatrix}$$

where, $q_1 = y_2 - y_3$; $q_2 = y_3 - y_1$; $q_3 = y_1 - y_2$

$r_1 = x_3 - x_2$; $r_2 = x_1 - x_3$; $r_3 = x_2 - x_1$

Stress-Strain relationship matrix for plane stress problem,

$$[D] = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

where, $\nu \rightarrow$ Poisson's ratio

$E \rightarrow$ Young's modulus

Stress-Strain relationship matrix for plane strain problem,

$$[D] = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$

Element stiffness matrix for CST element,

$$[K] = [B]^T [D] [B] A t$$

Element stress, $\{\sigma\} = [D] [B] \{u\}$

$$\Rightarrow \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [D][B] \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix}$$

where, σ_x, σ_y $\rightarrow$ Normal stresses

τ_{xy} $\rightarrow$ Shear stress

u, v $\rightarrow$ Nodal displacements

Maximum normal stress, $\sigma_{max} = \sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

Minimum normal stress, $\sigma_{min} = \sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

Principal angle, $\tan 2\theta_P = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$

Element strain, $\{e\} = [B]\{u\} = [B]\begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix}$

Temperature effects

$$\left.\begin{array}{r}\text{Initial strain, } \{e_0\} \\ \text{(For plane stress problems)}\end{array}\right\} = \begin{Bmatrix} \alpha \Delta T \\ \alpha \Delta T \\ 0 \end{Bmatrix}$$

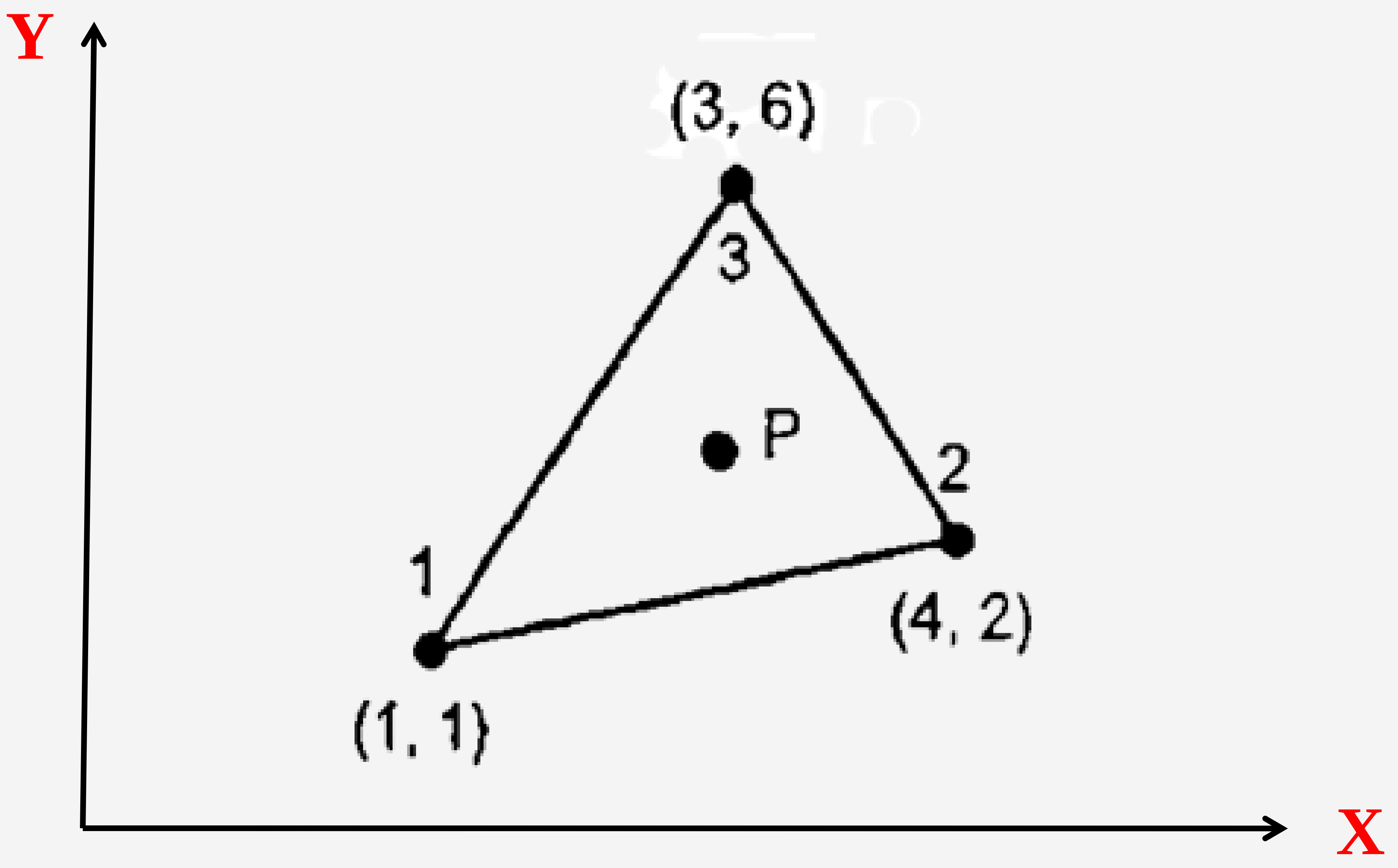
$$\left.\begin{array}{r}\text{Initial strain, } \{e_0\} \\ \text{(For plane strain problems)}\end{array}\right\} = (1+\nu) \begin{Bmatrix} \alpha \Delta T \\ \alpha \Delta T \\ 0 \end{Bmatrix}$$

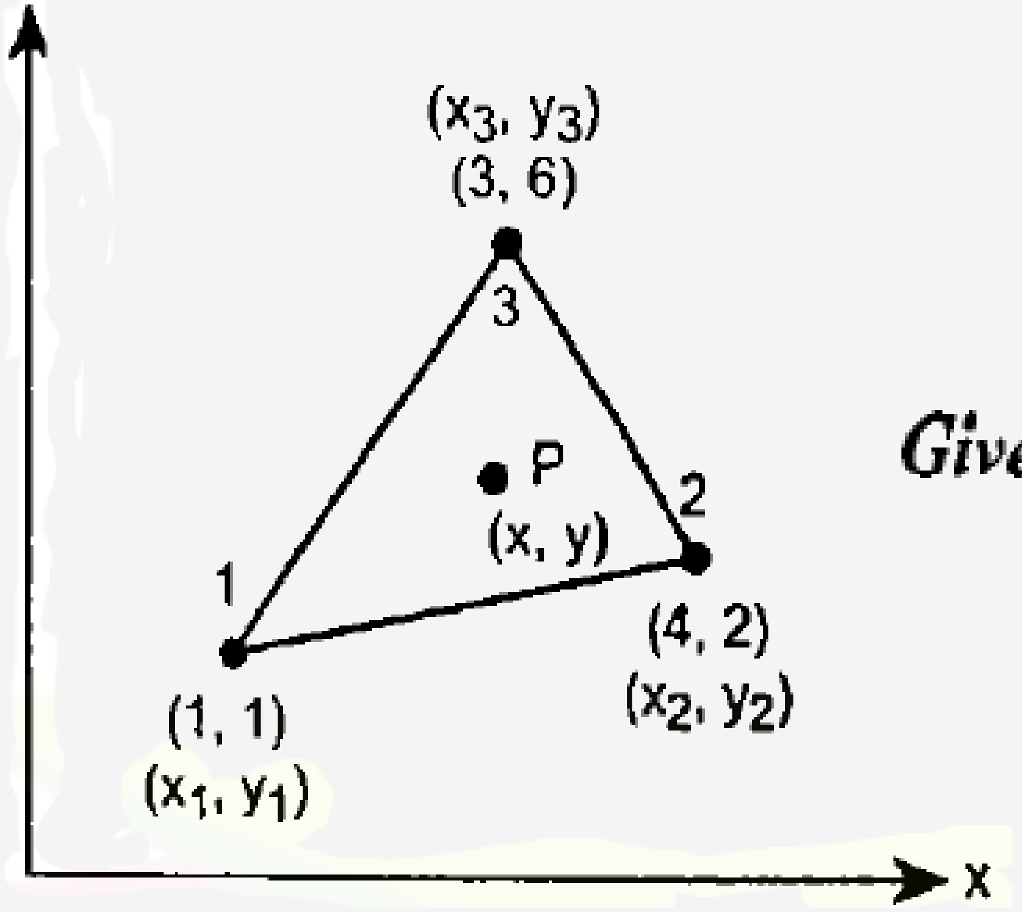
where, $\alpha \rightarrow$ Coefficient of thermal expansion

$\nu \rightarrow$ Poisson's ratio

Element temperature force, $\{F\} = [B]^T [D] \{e_0\}\, t\, A$

Determine the x and y co-ordinates of point P for the triangular element shown in Fig. . The shape functions N_1 and N_2 are 0.2 and 0.3 respectively.





Given: $x_1 = 1$ $y_1 = 1$ $x_2 = 4$ $y_2 = 2$

$x_3 = 3$ $y_3 = 6$ $N_1 = 0.2$ $N_2 = 0.3$

To find: x and y co-ordinates of point P.

Solution: We know that,

Sum of shape function is equal to one.

$$\Rightarrow \quad N_1 + N_2 + N_3 = 1$$

$$0.2 + 0.3 + N_3 = 1$$

$$\boxed{N_3 = 0.5}$$

x co-ordinate at point P is,

$$x = N_1 x_1 + N_2 x_2 + N_3 x_3$$

$$= 0.2 \times (1) + 0.3 \times (4) + 0.5 \times (3)$$

$$x = 2.9$$

y co-ordinate at point P is,

$$y = N_1 y_1 + N_2 y_2 + N_3 y_3$$

$$= 0.2 \times (1) + 0.3 \times (2) + 0.5 \times (6)$$

$$y = 3.8$$

Thank you