Unit-3 2D problem

Strain-Displacement [B] matrix [Gradient matrix]

WKT

Strain in x-direction

$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{\partial N_1}{\partial x}\cdot u_1 + \frac{\partial N_2}{\partial x} u_2 + \frac{\partial N_3}{\partial x} u_3$$

Strain in y-direction

$$\varepsilon_y = \frac{\partial v}{\partial y} = \frac{\partial N_1}{\partial y}\cdot v_1 + \frac{\partial N_2}{\partial y} v_2 + \frac{\partial N_3}{\partial y}\cdot v_3$$

Strain along xy direction.

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\partial N_1}{\partial y}\cdot u_1 + \frac{\partial N_2}{\partial y}\cdot u_2 + \frac{\partial N_3}{\partial y} u_3 + \frac{\partial N_1}{\partial x}\cdot v_1 + \frac{\partial N_2}{\partial x}\cdot v_2 + \frac{\partial N_3}{\partial x}\cdot v_3$$

In matrix form

$$\{\varepsilon\} = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{\partial N_1}{x} & 0 & \frac{\partial N_2}{\partial x} & 0 & \frac{\partial N_3}{\partial x} & 0 \\ 0 & \frac{\partial N_1}{\partial y} & 0 & \frac{\partial N_2}{\partial y} & 0 & \frac{\partial N_2}{\partial y} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial y} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial y} & \frac{\partial N_2}{\partial x} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ v_1 \\ v_2 \\ v_3 \end{Bmatrix}$$

$$N_1 = \frac{p_1 + q_1 x + r_1 y}{2A} \; ; \; N_2 = \frac{p_2 + q_2 x + r_2 y}{2A} \; ; \; N_3 = \frac{p_3 + q_3 x + r_3 y}{2A}$$

$$\frac{\partial N_1}{\partial x} = \frac{q_1}{2A} \; ; \; \frac{\partial N_2}{\partial x} = \frac{q_2}{2A} \; ; \; \frac{\partial N_3}{\partial x} = q_3/2A$$

$$\frac{\partial N_1}{\partial y} = r_1/2A \; ; \; \frac{\partial N_2}{\partial y} = \frac{r_2}{2A} \; ; \; \frac{\partial N_3}{\partial y} = r_3/2A$$

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \frac{1}{2A} \begin{bmatrix} q_1 & 0 & q_2 & 0 & q_3 & 0 \\ 0 & r_1 & 0 & r_2 & 0 & r_3 \\ r_1 & q_1 & r_2 & q_2 & r_3 & q_3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

This is of the form

$$\{\varepsilon\} = [B]\{u\}$$

Stress- Displacement Relation

$$\{\sigma\} = [D] \cdot \{\varepsilon\}$$

$$\{\sigma\} = [D] \cdot [B] \cdot \{u\}$$

Max. Normal stress

$$\sigma_1 = \frac{1}{2}\left[(\sigma_x + \sigma_y) + \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}\right]$$

$$\sigma_2 = \frac{1}{2}\left[(\sigma_x + \sigma_y) - \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}\right]$$

principle angle $\theta_p = \frac{1}{2}\tan^{-1}\left[\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right]$

WKT stiffness matrix [K] for CST

$$[K] = \int_v [B]^T [D][B] \cdot dv$$

$$= [B]^T [D][B] \cdot A \cdot t$$

where t = Thickness of body in case of plane stress

$= 1$ in case of plane strain