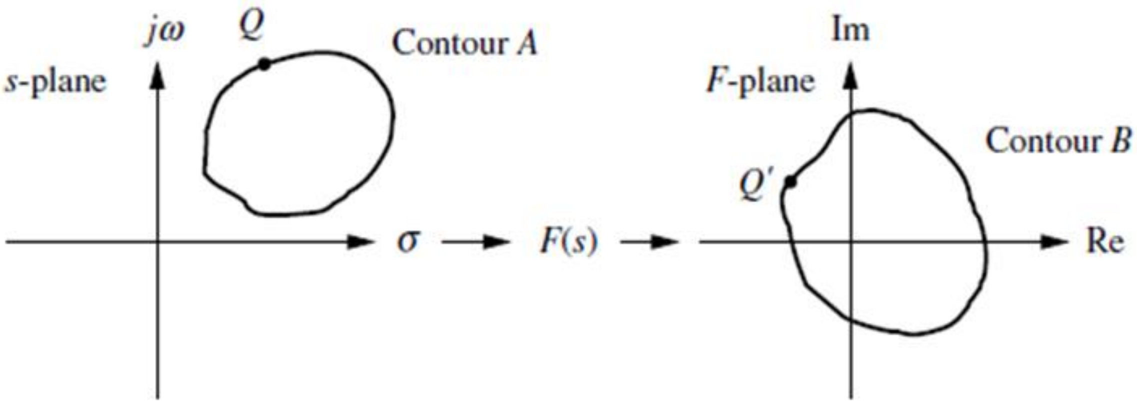
The Nyquist stability criterion determines the stability of a closed-loop system from its open-loop frequency response and open-loop poles.

Mathematical background:

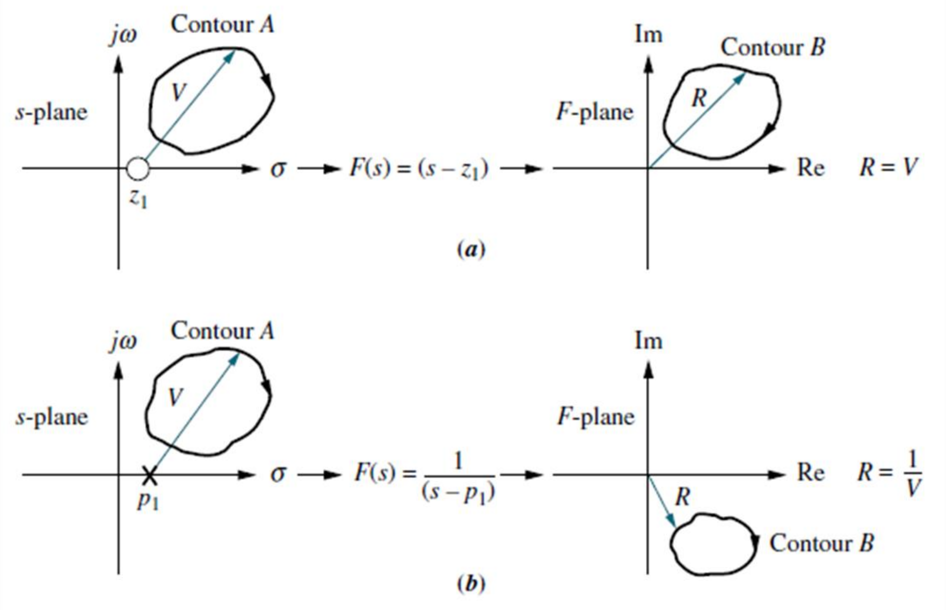
If we take a complex number s = x+jy on the s-plane and substitute it into a function, F(s), another complex number results. This process is called mapping. The collection of points, called a contour.



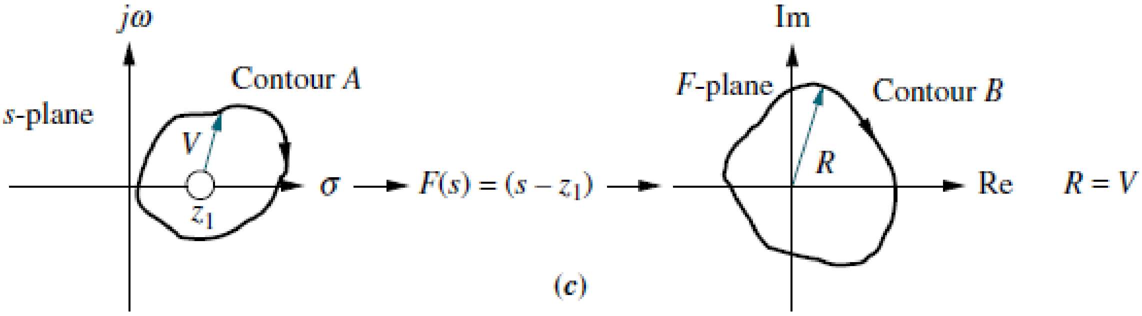
Mapping contour A through function $F(s)$ to contour B

Contour A can be mapped through $F(s)$ into contour B by substituting each point of contour A into the function $F(s)$ and plotting the resulting complex numbers. For example, point Q in Figure 10.21 maps into point Q' through the function $F(s)$.

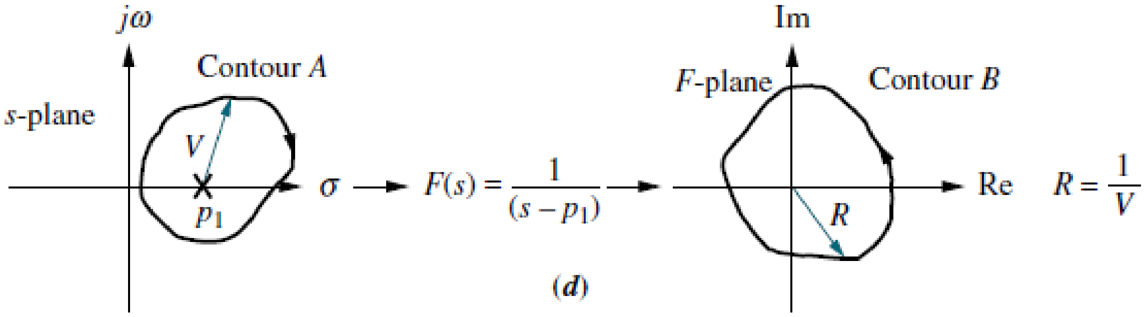
If we assume a clockwise direction for mapping the points on contour **A**, the contour **B** maps a clockwise direction if **F(s)** has just zeros or has just poles that are not encircled by the contour.



The **contour B** maps in a counter clockwise direction if **F(s)** has just poles that are encircled by the contour, Also, you should verify that, if the pole or zero of **F(s)** is enclosed by **contour A**, the mapping encircles the origin.



(c)



(d)

The Nyquist stability test is obtained by applying the Cauchy principle of argument to the complex function.

Cauchy's Principle of Argument

Let F(s) be an analytic function in a closed region of the complex plane given in Figure except at a finite number of points (namely, the poles of F(s)). It is also assumed that F(s) is analytic at every point on the contour. Then, as travels around the contour in the - plane in the clockwise direction, the function F(s) encircles the origin in the -plane in the same direction N times (see Figure, with given by

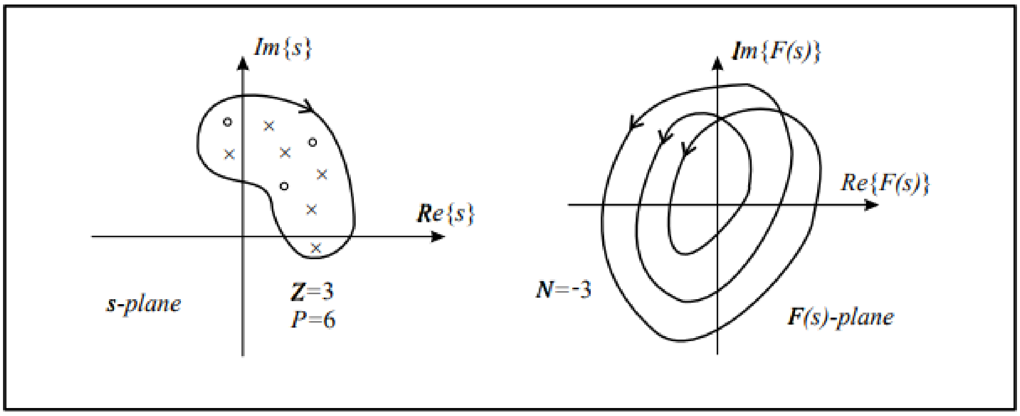
$$N = Z - P$$

where Z and P stand for the number of zeros and poles (including their multiplicities) of the function $F(s)$ *inside the contour.*

The above result can be also written as

$$\arg\{F(s)\} = (Z - P)2\pi = 2\pi N$$

which justifies the terminology used, "the principle of argument".



Cauchy's principle of argument