## SNS College of Technology



(An Autonomous Institution)

Coimbatore – 35

### DEPARTMENT OF MATHEMATICS **UNIT- II FOURIER TRANSFORM**



CONVOLUTION OF TWO FUNCTION

Convolution Theorem of Two functions The convolution of two functions (121) and grow is defenced as,

$$(1+9)(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} 1(x) g(x-t) dt$$

Convolution Theorem:

The fowerer transform of the convolution from and gran 95 the product of their foroises toansforms

Publishes based on Convolution:

J. fraluate of dx using transforms.

pand the fourier cosine transform of  $f(x) = e^{-dx}$ and  $g(x) = e^{-bx}$  and Evaluate  $\int_{0}^{\infty} \frac{dx}{6x^2 + a^2} (x^2 + b^2)$ 

Evaluate parseval's Identity  $\int_{-ax}^{a} \frac{dx}{(x^2 + a^2)(x^2 + b^2)}$ proof: Consider  $\int_{-ax}^{a} \int_{-ax}^{a} \frac{dx}{(x^2 + a^2)(x^2 + b^2)}$ 

 $F_c[S] = F_c[S[N]] = F_c[e^{-\alpha N}] = \sqrt{\frac{\alpha}{\pi}} \frac{\alpha}{\alpha^2 + \alpha^2}$ 

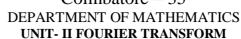
 $F_{c}[g(x)] = F_{c}[s] = F_{c}[e^{-bx}] = \sqrt{\frac{a}{h^{2}+s^{2}}}$ 

# \* X

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we know that

$$\int_{0}^{\infty} F_{c} \left[ f(x) \right] F_{c} \left[ g(x) \right] ds = \int_{0}^{\infty} f(x) \cdot g(x) dx$$

$$\int \sqrt{\frac{a}{m}} \frac{a}{a^2 + 5^2} \sqrt{\frac{a}{m}} \frac{b}{b^2 + 5^2} = \int e^{-ax} e^{-bx} dx$$

$$\frac{2}{\pi} \int_{-\infty}^{\infty} \frac{ab}{(a^2+s^2)(b^2+s^2)} ds = \int_{-\infty}^{\infty} e^{-(a+b)x} dx$$

$$\frac{2ab}{\pi} \int_{-\infty}^{\infty} \frac{ds}{(a^2+s^2)(b^2+s^2)} = \left[\frac{e^{-(a+b)\pi}}{-(a+b)}\right]_{-\infty}^{\infty}$$

$$= \frac{1}{a+b} \begin{bmatrix} 0 - 1 \end{bmatrix}$$
$$= \frac{1}{a+b}$$

$$\int_{0}^{\infty} \frac{ds}{(s^{2}+a^{2})(s^{2}+b^{2})} = \frac{\pi}{aab(a+b)}$$

a]. Evaluate 
$$\int_{0}^{\infty} \frac{2a}{(2a+a^2)(2a+b^2)} dx$$
 using Transform.

Soln: consider 
$$f(x) = e^{-ax}$$
 and  $g(x) = e^{-bx}$ 

$$F_{S}[S(x)] = F_{S}[e^{-\alpha x}] = \sqrt{\frac{2}{\pi}} \frac{3}{s^{2}+\alpha^{2}}$$

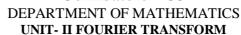
$$F_{3}[g(n)] = F_{3}[e^{-bn}] = \sqrt{\frac{g}{\pi}} \frac{g}{g^{2} + b^{2}}$$

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$$\int_{0}^{\infty} \frac{S}{\Pi} \frac{S}{S^{2}+\alpha^{2}} \left[ \frac{R}{\Pi} \frac{S}{S^{2}+b^{2}} ds \right] = \int_{0}^{\infty} \frac{e^{-\alpha \pi} e^{-b\pi}}{e^{-\alpha \pi} e^{-b\pi}} dx$$

$$\frac{2}{\Pi} \int_{0}^{\infty} \frac{S^{2}}{(S^{2}+\alpha^{2})(S^{2}+b^{2})} ds = \int_{0}^{\infty} \frac{e^{-(\alpha+b)\pi}}{(\alpha+b)} d\pi$$

$$\frac{2}{\Pi} \int_{0}^{\infty} \frac{S^{2}}{(S^{2}+\alpha^{2})(S^{2}+b^{2})} ds = \left[ \frac{e^{-(\alpha+b)\pi}}{-(\alpha+b)} \right]_{0}^{\infty}$$

$$= \frac{1}{(\alpha+b)} \left[ 0 - 1 \right]$$

$$\Rightarrow \int_{0}^{\infty} \frac{S^{2}}{(S^{2}+\alpha^{2})(S^{2}+b^{2})} d\pi = \frac{\Pi}{2(\alpha+b)}$$

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