

8. Find the Laplace transform of  $x(t) = \sinh at$

$$\begin{aligned}
 L[x(t)] &= x(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt \\
 &= \int_{-\infty}^{\infty} \left( \frac{e^{at} - e^{-at}}{2} \right) e^{-st} dt \quad \left| \begin{array}{l} \sinh at = \\ \frac{e^{at} - e^{-at}}{2} \end{array} \right. \\
 &= \frac{1}{2} \left[ \int_{-\infty}^{\infty} e^{at} e^{-st} dt + \int_{-\infty}^{\infty} e^{-at} e^{-st} dt \right] \\
 &= \frac{1}{2} [L[e^{at}] - L[e^{-at}]] \\
 &= \frac{1}{2} \left[ \frac{1}{s-a} - \frac{1}{s+a} \right] \\
 &= \frac{1}{2} \left[ \frac{s+a - s+a}{s^2 - a^2} \right] \\
 &= \frac{a}{s^2 - a^2}
 \end{aligned}$$

9. Find the L.T of  $x(t) = e^{a \cosh at}$

$$\begin{aligned}
 L[x(t)] &= x(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt \\
 &= \int_{-\infty}^{\infty} \frac{e^{at} + e^{-at}}{2} e^{-st} dt \\
 &= \frac{1}{2} \left[ \int_{-\infty}^{\infty} e^{at} e^{-st} dt + \int_{-\infty}^{\infty} e^{-at} e^{-st} dt \right] \\
 &= \frac{1}{2} [L[e^{at}] + L[e^{-at}]] \\
 &= \frac{1}{2} \left[ \frac{1}{s-a} + \frac{1}{s+a} \right] \\
 &= \frac{1}{2} \left[ \frac{2s}{s^2 - a^2} \right] \\
 &= \frac{s}{s^2 - a^2}
 \end{aligned}$$