$$= \frac{A}{2}\left[L[e^{j\omega_0 t}\cdot u(t)] + L[e^{-j\omega_0 t}\cdot u(t)]\right]$$

$$e^{-at} = \frac{1}{s+a}$$
$$e^{at} = \frac{1}{s-a}$$

$$= \frac{A}{2}\left[\frac{1}{s-j\omega_0} + \frac{1}{s+j\omega_0}\right]$$

$$= \frac{A}{2}\left[\frac{s+\cancel{j\omega_0}+s-\cancel{j\omega_0}}{(s-j\omega_0)(s+j\omega_0)}\right]$$

$$= \frac{A}{2}\left[\frac{2s}{s^2-(j\omega_0)^2}\right] = \frac{A}{\cancel{2}}\left[\frac{\cancel{2}s}{s^2+\omega_0^2}\right]$$

$$X(s) = \frac{As}{s^2+\omega_0^2}$$

6. Find the Laplace transform of $x(t) = e^{-at}\cos\omega t\,.\,u(t)$

$$L[x(t)] = X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$= \int_{-\infty}^{\infty} e^{-at}\cos\omega t\,.\,u(t)\,.\,e^{-st}\,dt$$

$$= \int_{-\infty}^{\infty} e^{-at}\left(\frac{e^{j\omega t}+e^{-j\omega t}}{2}\right)u(t)\, e^{-st}\,dt$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} e^{-at}(e^{j\omega t}+e^{-j\omega t})\,u(t)\,e^{-st}\,dt$$

$$= \frac{1}{2}\left[\int_{-\infty}^{\infty} e^{-at}e^{j\omega t}\,.\,u(t)\,e^{-st}\,dt + \int_{-\infty}^{\infty} e^{-at}e^{-j\omega t}\,u(t)\,e^{-st}\,dt\right]$$

$$= \frac{1}{2}\left[\int_{-\infty}^{\infty} e^{(j\omega-a)t}\,.\,u(t)\,e^{-st}\,dt + \int_{-\infty}^{\infty} e^{-(j\omega+a)t}\,.\,u(t)\,e^{-st}\,dt\right]$$

$$= \frac{1}{2}\left[L[e^{(j\omega-a)t}\,.\,u(t)] + L[e^{-(j\omega+a)t}\,.\,u(t)]\right]$$

$$= \frac{1}{2}\left[\frac{1}{s-j\omega+a} + \frac{1}{s+j\omega+a}\right]$$

$$= \frac{1}{2}\left[\frac{s+\cancel{j\omega}+a+s-\cancel{j\omega}+a}{s^2+\omega^2+a^2+2as}\right] = \frac{1}{\cancel{2}}\left[\frac{\cancel{2}(s+a)}{s^2+\omega^2+a^2+2as}\right]$$

$$= \frac{s+a}{s^2+\omega^2+a^2+2as} = \frac{s+a}{(s+a)^2+\omega^2}$$